Gravitation as a pressure force: a scalar ether theory, by M. Arminjon

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5. Energy balance and energy conservation

We summarize the discussion in Ref. 4, where a few additional references are discussed. The concept of energy is an essential one in most of modern physics but, surprisingly, it can hardly be defined in modern gravitation theory, i.e. in general relativity (GR). Indeed, an extremely important feature of the energy is that it should be conserved, also locally, in the sense that a *local balance equation without any source term* should apply to the total energy. Now according to GR and other relativistic theories of gravitation based on general covariance plus Einstein's "equivalence principle" (in the standard form: "in a local freely falling frame, the laws of non-gravitational physics are the same as in SR"), the general "conservation equation" is the equation

$$(T^{\mu\nu}{}_{;\,\nu})_{\,\mu = 0, \ldots, 3} \equiv \mathrm{div}_\gamma\, \mathbf{T} = 0 \tag{24}$$

for the energy-momentum tensor $\mathbf{T}$ [2]. But, as emphasized in most textbooks on GR, e.g. Landau & Lifchitz [23], this equation can not be considered as a true conservation equation (a balance equation without source term), for there is no Gauss theorem applying to the divergence of a second-order tensor in a curved Riemannian space. (In turn the main reason for this is that one can simply not define the integral of a vector field in a such space.) In more explicit terms: one cannot rewrite Eq. (24) in the form of a true conservation equation which would be valid for a generic coordinate system, i.e., which would be consistent with the principle of general covariance. One may, however, rewrite Eq. (24) in the form of a true conservation equation, if one accepts to restrict oneself to coordinate systems exchanging by *linear* coordinate transformations, as are Lorentz transformations in a flat space-time. Unfortunately, one may do this in many different ways, so that it is not clear which would be the correct way, even once one has specified the linear class of coordinate systems. Moreover, there is no reason to introduce such particular class, unless "the space-time has a particular symmetry", which means in fact that some *background metric* γ^0 on the space-time manifold, distinct from the physical metric γ, has some non-trivial group of isometries. The example of a such background metric that is relevant to the rewriting of Eq. (24) as a true conservation equation is that of a *flat* metric [4]: a such metric admits a particular class of coordinate systems, in which it reduces to the Galilean (Minkowskian) form. In summary, the search for a consistent concept of energy leads in GR to a contradiction with the very notion of "general relativity", since this search leads to restrict oneself to reference frames exchanging by Lorentz transformations of a flat background metric. In the present preferred-frame theory, on the other hand, we do have a flat background metric γ^0 and we do not even demand that the energy balance equation should be covariant by Lorentz transformations of this flat metric, so it would be hard to accept that the theory would not lead to a true conservation equation for the energy.

The obtainment of the energy equation for a mass point (pp. 42-43 in Ref. 4) is a modification of the elementary method used in classical mechanics to derive the energy equation in a force field deriving from a variable potential (the modifications are due to relativistic mechanics with a variable metric). Here, the assumed expression for $\mathbf{g}$ (Eq. 23)) derives from the potential $U' \equiv -c^2 \operatorname{Log} \beta$. Thus, one evaluates the rate of work per unit rest mass for a "free" mass point,

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[2] Semi-colon means covariant derivative with respect to the space-time metric γ.

$$dw / dt_{\mathbf{x}} \equiv \left(\mathbf{F}_g . \mathbf{v}\right) / m_0 = \gamma_v \mathbf{g}.(d\mathbf{x}/dt_{\mathbf{x}}) = c\,\mathbf{g}.(d\mathbf{x}/ds) \tag{25}$$

(where the point means scalar product **g**), using Newton's second law (19) (with $\mathbf{F}_0 = 0$), which involves the correct time-derivative (18). The result is

$$\frac{d}{dT}\left(c^2 \operatorname{Log}\gamma_v\right) = \mathbf{g}.\frac{d\mathbf{x}}{dT} = \left(\operatorname{grad}_{\mathbf{g}} U'\right).\frac{d\mathbf{x}}{dT} = U'_{,i}\frac{dx^i}{dT} \equiv \frac{dU'}{dT} - \frac{\partial U'}{\partial T}\ . \tag{26}$$

Using the definition of the potential U', one rewrites this as

$$\frac{d\left(\gamma_v \beta\right)}{dT} = \gamma_v \frac{\partial \beta}{\partial T}, \tag{27}$$

or, multiplying by $m_0 c^2$ with m_0 the rest mass:

$$\frac{d\left(E\beta\right)}{dT} = E\frac{\partial \beta}{\partial T}. \tag{28}$$

Equation (28) shows clearly that the total energy of the mass point must be defined as

$$e_{\mathrm{m}} \equiv E\beta = c^2 m_0 \gamma_v \beta, \tag{29}$$

which is a constant for a constant gravitational field. This is the total energy of the mass point, for it includes both its "purely material" energy E (i.e. the energy equivalent of the relativistic inertial mass, thus including the "kinetic" energy) and its "potential" energy in the gravitational field, which may be defined as $e_{\mathrm{gm}} = e_{\mathrm{m}} - E = E\,(\beta - 1)$. (It is hence negative, as in NG.) It turns out that just the same equation (28) may be derived also for a light-like particle.

The deduction of the energy equation (28) from Eq. (27) is trivial, but it rests on the essential assumption that the rest mass m_0 of the free mass point is conserved in the motion (an assumption that is already used in the derivation of Eq. (26)). If we now consider a *dust*, i.e. a continuum made of coherently moving, non-interacting particles, each of which conserves its rest mass, we may apply Eq. (27) pointwise in the continuum. The conservation of the rest-mass of the continuum is most easily expressed in terms of the "background" Euclidean metric $\mathbf{g}^0$ and the associated volume measure δV^0, for it is then expressed as the usual continuity equation for the density of the rest-mass with respect to δV^0, which is $\rho_{00} \equiv \delta m_0/\delta V^0$. The density, with respect to δV^0, of the total energy of the dust, is $\varepsilon_{\mathrm{m}} \equiv c^2 \rho_{00} \gamma_v \beta$, because

$$\delta e_{\mathrm{m}} = c^2 \delta m_0\, \gamma_v \beta = c^2 \rho_{00}\, \delta V^0\, \gamma_v \beta = \varepsilon_{\mathrm{m}}\, \delta V^0. \tag{30}$$

It turns out that ε_{m} is none other than the $T^0{}_0$ component of the energy-momentum tensor $\mathbf{T}$ for the dust, whose $T^i{}_0$ components are $T^i{}_0 = T^0{}_0\, u^i/c$, with $u^i \equiv dx^i/dT$ the "absolute velocity" (with respect to the preferred frame, and evaluated with the absolute time T) [3]. Using this and rewriting Eq. (27) with the help of the continuity equation, one gets the *local balance equation for the continuum:*

$$cT^{\mu}{}_{0,\mu} \equiv c\left(\operatorname{div}_{\gamma^0} \mathbf{T}\right)_0 = \frac{T^0{}_0}{\beta}\frac{\partial \beta}{\partial T} \qquad (x^0 = cT), \tag{31}$$

[3] In this paper, we shall use the energy units for tensor **T**, whereas mass units were used in Refs. 4 and 7.

where the identity applies when the spatial coordinates in the preferred frame are Cartesian (at least at the point considered), i.e. such that $g^0{}_{ij} = \delta_{ij}$ and $g^0{}_{ij,\,k} = 0$.

To rewrite this as a true conservation equation, we must use the field equation of the theory (Eq. (13)) so that the r.h.s. of Eq. (31), which in this form is a source term, be recast as a 4-divergence with respect to the flat metric γ^0. In other words, we have to make the gravitational energy and its flux appear. To do this, we adapt the reasoning that leads to a conservation equation in NG: we observe that, due to Eqs. (12) and (13), one has [4]

$$\frac{8\pi\, G}{c^2}\rho\, f_{,0} = -\frac{2}{c^2}\mathrm{div}_0\left(f_{,0}\,\mathbf{g}\right) - \frac{1}{2}\left[\left(\frac{f_{,0}}{f}\right)^2 + \frac{4}{c^4}\mathbf{g}^2\right]_{,0}, \quad \mathrm{div}_0 \equiv \mathrm{div}_{\,\mathbf{g}^0}, \quad \mathrm{g}^2 \equiv \mathbf{g}^0(\mathrm{g},\mathrm{g}), \quad x^0 = cT. \quad (32)$$

On the other hand, using the condition $\gamma_{0i} = 0$, the r.h.s. of Eq. (31) may be rewritten as [4]

$$\frac{T^0{}_0}{\beta}\frac{\partial \beta}{\partial t} = \frac{T^0{}_0}{2f}\frac{\partial f}{\partial t} = \frac{T^{00}}{2}\frac{\partial f}{\partial t} = \frac{c}{2}T^{00} f_{,0}. \quad (33)$$

Since $c^2\rho$ is "the mass-energy density" (in the preferred frame), it should be equal to T^{00}, or to $T^0{}_0$, or still to T_{00}. But Eqs. (32) and (33) show that the source term on the r.h.s. of Eq. (31) can be rewritten as a flat 4-divergence, if and only if $c^2\rho = T^{00}$. Therefore, we must precisely define $c^2\rho$ as the T^{00} component (in coordinates bound to the preferred frame, and such that $x^0 = cT$). This also means that the gravitational field reinforces itself, whereas, if we would assume $c^2\rho = T^0{}_0$ or $c^2\rho = T_{00}$, the gravitational field would have a weakening effect on itself [4]. We therefore obtain the *local conservation equation for the continuum:*

$$\left(\mathrm{div}_{\gamma^0}\,\mathbf{T}\right)_0 + \frac{1}{8\pi\, G}\left\{\left[\mathbf{g}^2 + \frac{c^4}{4}\left(\frac{f_{,0}}{f}\right)^2\right]_{,0} + \mathrm{div}_0\left(c^2 f_{,0}\,\mathbf{g}\right)\right\} = 0 \quad (x^0 = cT). \quad (34)$$

Although this equation has been derived for a dust, we assume that it holds true with **T** the *total* energy-momentum tensor of any kind of continuum (involving material particles and/or non-gravitational fields). This assumption is justified by the mass-energy equivalence and the universality of the gravitation force. Equation (34) rules the exchange between the total energy of matter, whose density is given by $\varepsilon_{\mathrm{m}} = T^0{}_0$ (cf. Eq. (30)), and the purely gravitational energy, whose density ε_{g} is defined by

$$\varepsilon_{\mathrm{g}} = \frac{1}{8\pi\, G}\left[\mathbf{g}^2 + \frac{c^4}{4}\left(\frac{f_{,0}}{f}\right)^2\right]. \quad (35)$$

This exchange occurs through the intermediate of the flux of the total matter energy, defined as the space vector with components $cT^i{}_0$, and the flux of the gravitational energy, defined as the space vector $c^3 f_{,0}\,\mathrm{g}/(8\pi G)$. Note that the total energy of matter contains also the negative potential energy of matter (and/or non-gravitational fields) in the gravitational field: for a dust, $T^0{}_0 = \varepsilon_{\mathrm{m}}$ is the density of the total energy of the individual particles, defined by $e_{\mathrm{m}} = c^2 m_0 \gamma_v \beta$ (cf. Eqs. (29) and (30)). Of course,

[4] Indices are raised and lowered with the help of the physical space-time metric γ, unless explicitly mentioned otherwise.

the local conservation of the energy implies a global conservation (of the global amount of *total energy*, i.e. gravitational energy plus total energy of matter), under asymptotic conditions ensuring that the global energy is finite [4].

6. Continuum dynamics and matter creation/destruction

By the foregoing induction (from a dust to a general continuum), we have got one scalar local equation for a continuous medium, i.e. the energy conservation, which, for a general continuum, is *substituted for the mass conservation.* However, for a point particle, we have four equations: the three equations of motion (19), plus the conservation of the rest-mass, and it is easy to convince oneself that one also needs exactly four independent dynamical equations for a continuum, in addition to the state equation. For instance, there are indeed four dynamical equations in classical continuum mechanics: Newton's second law plus the continuity equation. The same number applies also in GR, where the dynamical equations make the well-known 4-vector equation (24).

In order to get the required four scalar "equations of motion" for a continuous medium, we may again use the general principle of induction from a dust to a general behaviour, once the equation for dust has been expressed in terms of the energy-momentum tensor **T**. The most expedient way to operate this principle turns out to be passing through the expression of the *4-acceleration* [7]. The latter expression has been obtained for a free particle [5]. (The spatial components of the 4-acceleration and its time component were deduced from Newton's second law (19) and the energy equation (27), respectively, by using the relation between the Christoffel symbols of the spatial metric and those of the space-time metric.) It happens to be simpler in covector form [4]:

$$A_0 = \frac{1}{2}\, g_{jk,0}\, U^j U^k, \quad A_i = -\frac{1}{2}\, g_{ik,0}\, U^0 U^k. \tag{36}$$

(By the way, Eq. (36) shows at once that, in the present theory, Einstein's geodesic motion is recovered only for a gravitational field that is constant in the preferred frame: **A** = 0 is true for whatever 4-velocity **U** if and only if $g_{ij,0} = 0$.) For a dust, the **T** tensor has the form $T^{\mu\nu} = c^2\rho^* U^\mu U^\nu$ with ρ^* the proper rest-mass density, and, for any material continuum, the 4-acceleration may be expressed as $A^\mu = U^\nu U^\mu{}_{;\nu}$. Using this and the mass conservation, assumed valid by definition for a dust, one may rewrite Eq. (36), for a dust, as

$$T_\mu{}^\nu{}_{;\nu} \equiv T^\nu{}_{\mu;\nu} = b_\mu, \tag{37}$$

where b_μ is defined by

$$b_0(\mathrm{T}) \equiv \frac{1}{2} g_{jk,0}\, T^{jk}, \quad b_i(\mathrm{T}) \equiv -\frac{1}{2} g_{ik,0}\, T^{0k}. \tag{38}$$

Now the induction principle means simply that *Eq. (37) is the general equation for continuum dynamics* in the present theory. It thus plays the role played in GR by Eq. (24). The r.h.s. of Eq. (37), b_μ as defined by Eq. (38), is a 4-covector for transformations of the group (7), and so is also Eq. (37). The time component of Eq. (37) ($\mu = 0$) is equivalent to the energy balance equation (31), hence also to the energy conservation equation (34) [7]. Moreover, for a dust, i.e. the case $T^{\mu\nu} = c^2\rho^* U^\mu U^\nu$, Eq. (37) *implies* the mass conservation, i.e.

$$\left(\rho^* U^\nu\right)_{;\nu} = 0\,, \tag{39}$$

and (again for a dust), Eq. (37) *implies* also the expression (36) for the 4-acceleration, which is characteristic for free motion in the present theory, i.e. $\mathbf{F}_0 = 0$ in Newton's second law (19) [7]. Thus, Eq. (37) plus the relevant definition of the energy-momentum tensor as a function of the state variables (which, for dust, consist of the single variable ρ^*) characterize completely the dynamical behaviour of dust. This is an important consistency test.

A general continuum may thus be phenomenologically defined by the expression of tensor **T** as a function of some state variables, and Eq. (37) determines how the continuum couples to gravitation in the present theory (of course, Eq. (37) reduces to the equation valid in SR, i.e., $T^{\mu\nu}_{\ \ ,\nu} = 0$, if there is no gravitational field). But, for a general continuum, the *energy* is conserved, and this is in general *incompatible with the exact mass conservation:* in the case of a variable gravitational field, there are exchanges between the gravitational energy and the total energy of matter, so one may a priori expect that, in general, the rest-mass will not be conserved – except for the special case of a dust. One may already verify this for the simple case of a perfect isentropic fluid. The energy-momentum tensor of a perfect fluid is in general

$$T_{\text{fluid}}^{\ \mu\nu} = (\mu^* + p)\ U^\mu\, U^\nu - p\,\gamma^{\mu\nu}, \tag{40}$$

with p the pressure and μ^* the volume density of the rest-mass plus internal energy in the proper frame: in energy units,

$$\mu^* \equiv \rho^*(\,c^2 + \Pi), \tag{41}$$

where Π is the internal energy per unit rest mass. For a perfect fluid, the isentropy condition is simply

$$d\Pi + p\,d(1/\rho^*) = 0. \tag{42}$$

For instance, for a barotropic fluid, one assumes $\mu^* = \mu^*(p)$; then, ρ^* and Π also depend on p only, Π being given by [18]

$$\Pi(p) \equiv \int_0^p \frac{dq}{\rho^*(q)} - \frac{p}{\rho^*(p)}. \tag{43}$$

Due to Eq. (43), a barotropic fluid is automatically isentropic.

For any isentropic fluid, Eq. (37) leads to the following *equation for mass creation/destruction:*

$$\left(\rho^* U^\nu\right)_{;\nu}\left(1 + \frac{\Pi + p/\rho^*}{c^2}\right) = -\frac{pU^0}{2c^2}\frac{f_{,0}}{f}, \tag{44}$$

which indeed shows that, except for the limit case of a dust ($p = 0$) and for the limit case of a constant gravitational field ($f_{,0} = 0$), the rest-mass is not conserved – according to the present theory [7]. In contrast, GR and other relativistic theories are based on Eq. (24), which, for an isentropic fluid, implies the conservation of the rest-mass, Eq. (39) (*cf.* Chandrasekhar [15]). On the contrary, the present theory predicts that matter may really be produced or destroyed, due to the variation of the gravitational field. Prigogine *et al.* [32] consider that matter should be produced by a such exchange (albeit in an irreversible way, excluding matter *destruction*), and this exchange would indeed seem *a priori* natural in a theory with conserved energy, due to the mass-energy equivalence. However, matter production can hardly happen so in GR: we insist that, due to the equation for continuum dynamics that goes with geodesic motion, i.e., $T^{\mu\nu}{}_{;\nu} = 0$, matter can only be produced if one phenomenologically inserts an additional term, which is *not* determined by the set of the state equation plus the Hilbert-Einstein equations [12, 19, 32]. Roughly speaking, this "creation term" appears thus as an *ad hoc*, adjustable source term, which is used to allow production of matter in some cosmological models. It seems interesting to investigate the possibility that matter might be produced (*or destroyed*) by an exchange with the gravitational field (a more complete discussion of this question is given in Ref. 7, that includes in particular a discussion of the thermodynamical constraints). Yet in our opinion, such exchange should not be considered merely in a cosmological context, but actually for any gravitational field.

Precisely, the way in which matter production occurs in any variable gravitational field, as implied by Eq. (44), may seem dangerous for the present theory, because it would mean that matter is continuously produced or wasted away under our eyes. However, the rates would be extremely small and often the mean gain would be rather close to zero [7]. If the absolute velocity **V** of the solar system is of the order 300 km/s, the main contribution in Eq. (44) would come from that variation of the gravitational field which is due to the translation of any celestial body through the ether, giving a creation rate (the amount produced per unit time in some material domain, divided by the mass of that domain)

$$C \equiv \frac{\hat{\rho}}{\rho} \approx \frac{p}{c^2 \rho} \frac{GM}{c^2 R} \frac{r}{R} \frac{V_r}{R} \qquad (V_r \equiv \mathbf{V}.\, \mathbf{e}_r), \tag{45}$$

with M and R the mass and radius of the spherical celestial body, whose attraction g ($g = GM/r^2$ outside the body) dominates in its near environment ($\mathbf{e}_r$ is the unit radial vector). At a fixed point on the surface of a body in self-rotation (as it is indeed the case for the planets), the corresponding contribution would be exactly cyclic at the equator, and instead would constantly accumulate production of matter (resp. destruction) at one pole (resp. at the other pole). Near the surface of the Earth, for instance, the ratio $p/(c^2\rho)$ can hardly take values much higher than 10^{-12} (which is its value in the air at the atmospherical pressure). A ratio equal to 10^{-12} leads to a maximum value of the creation rate $C_{\max} \approx 3.\ 10^{-23}\ \mathrm{s}^{-1}$ (obtained for $\mathbf{e}_r$ parallel to $\mathbf{V}$, and with $V \approx 300$ km/s), which seems very difficult to detect [7]. Hence, it might be the case that this new form of energy exchange be a real phenomenon. Needless to emphasize, this would be interesting.