

# Gravitation as a pressure force: a scalar ether theory

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**Mayeul Arminjon**

Laboratoire "Sols, Solides, Structures", Institut de Mécanique de Grenoble  
B.P. 53, 38041 Grenoble Cedex 9, France

## 1. Introduction and summary

The concept of an ether means primarily that empty space does not really exist. We may believe this, for instance, because electromagnetic waves, that go accross intergalactical space, ought to wave in some medium. We may also believe this, because quantum phenomena, such as the Casimir effect, suggest that "vacuum" actually has physical properties. It has been established by Builder [10-11], Jánossy [20-21], Prokhovnik [33-34], and others, that the concept of the ether as an inertial frame which should be the carrier of the electromagnetic waves (the Lorentz-Poincaré ether), is fully compatible with Special Relativity (SR). In connection with this, Zhang [41] has recently reestablished, against contrary statements, that the one-way velocity of light cannot be consistently measured – in the absence of any faster information carrier. As emphasized by Duffy [17], the Builder-Prokhovnik reconstruction of standard SR from the Lorentz-Poincaré ether concept may be criticized on the ground that this construction makes undetectable the absolute reference frame and its velocity, which are the physical entities with which the construction starts. It would not be an appropriate answer to recall that, after all, this is the way in which Lorentz, Larmor and Poincaré themselves derived the major part of SR: indeed, this methodological oddness – which is not a logical fault, however – contributed to bring discredit on the ether concept for a long time. Another possible answer would be to insist that, beyond physical concepts, one may still introduce metaphysical ones.

The answer that will be suggested here is, instead, that SR does not rule the whole of physics, for it does not involve gravitation. If the presence of a gravitational field breaks the symmetry expressed by the Lorentz group, then the Lorentz-Poincaré construction of SR is justified as the examination of a particular physical situation, the general situation (with gravitation) being one in which the "absolute motion" should be detectable. Although it is admittedly a risk to build a preferred-frame theory of gravitation, it will be shown that the risk is smaller than one would think, if one would blindly follow the standard arguments. Moreover, there are well-known physical reasons that may justify to search for a radical alternative to general relativity: e.g. the singularity occuring during gravitational collapse, the problem of dark matter, and the questions about the influence of the gauge condition.

The theory summarized here starts from a tentative interpretation of gravity as a pressure force, which would give a new theory already in a *Newtonian space-time* ; this will be recalled in **Section 2**. However, to become compatible with SR, this interpretation of gravity calls for a new version of Einstein's equivalence principle, which consists in admitting that our physical standards of space and time are affected by gravitation in much the same way as they are affected by a uniform motion (**Section 3**). Yet in the "pseudo-Riemannian space-time" which is thus obtained, motion is governed by an *extension of Newton's second law*, instead of assuming Einstein's motion along geodesic lines of the space-time metric (**Section 4**). The extended Newton law implies an energy balance for a test particle moving in a variable gravitational field; this balance equation may be translated so as to apply to a dust, of course (dust is a continuum made of non-interacting particles). The field equation of the theory allows to rewrite the balance equation for a dust as a *true conservation equation for the total energy*, including the gravitational energy; this conservation equation is then assumed valid for any kind of material or non-gravitational field, characterized by its energy-momentum tensor (**Section 5**). Using a different method, one may obtain the 4-component equation governing the dynamics of a dust continuum in terms of the energy-momentum tensor; this equation also may be assumed to be valid for

any continuum, it is the substitute for the classical equation  $T^{\mu\nu}{}_{;\nu} = 0$  of general relativity and other metric theories. The new equation implies that *mass conservation is obtained as a limit behaviour for a weak and slowly varying gravitational field and/or at low pressure (Section 6)*. The same method as in Sect. 6 may be used in the presence of a field of (non-gravitational) external force. When the external force is the Lorentz force, the 4-component dynamical equation thus derived gives the second group of the *gravitationally-modified Maxwell equations*, in the investigated theory. This new modification of Maxwell's equations is *consistent with the geometrical optics of the theory* as governed by the proposed extension of Newton's second law (Section 7). An essential question is *whether the theory agrees with experiment or not*, of course. The theory has the correct Newtonian limit; it predicts Schwarzschild's exterior metric of general relativity and geodesic motion in the static situation with spherical symmetry. Therefore, the question amounts principally to assessing the preferred-frame effects. To do this, one must develop a "post-Newtonian" approximation of this non-linear theory (Section 8). The first result is that *no preferred-frame effect occurs for photons at the (first) post-Newtonian approximation*. However, the absolute velocity of the solar system does influence the motion of mass points at this same approximation, thus it does have to be accounted for in celestial mechanics, according to the present theory. But it will be argued, contrary to well-known arguments, that *the existence of preferred-frame effects in celestial mechanics, comparable in magnitude with the "relativistic" effects, does not a priori invalidate a theory*.

## 2. Semi-heuristic considerations and a theory for Newtonian absolute space and time

Our starting point is that the concept of an ether should first be made compatible with *classical mechanics* [1]. This is not so easy: on the one hand, within classical mechanics, the only medium that could "fill the vacuum without us feeling it", would be a perfect fluid, which indeed would not brake any motion. But, on the other hand, we want that the ether defines an inertial frame, for otherwise we would have to postulate both an ether and an independent absolute space, and it seems that this would be too much of "absolute". (Moreover, the ether defines an inertial frame in the Lorentz-Poincaré interpretation of SR, although obviously this argument is less closely related to classical mechanics.) Hence, in the case of usual Newtonian mechanics, our perfect fluid should be perfectly rigid! The proposed answer is that *it is the average motion of the perfect fluid that defines the preferred inertial frame* [1-2] (the average should be taken at a very large scale, formally it is the asymptotic volume average [4]). It is worth to note that, even if one remains within classical mechanics, thus with our physical standards being assumed to measure Euclidean distance and absolute time, this preferred inertial frame does not actually need to be rigid. In other words, Newtonian mechanics may be extended to *deformable* inertial frames: the only restriction, imposed by the principle of inertia, is that the motion of the inertial frame with respect to a (any) rigid reference frame should be irrotational [1]. This generalization allows some simple cosmological considerations, in particular it gives a simple argument for an expanding universe [1]. An expanding preferred frame was already envisaged by Prokhovnik [33-34]. The motion (deformation) of the preferred inertial frame E, thus the average motion of the fluid ether, does not obey this "neo-classical" mechanics, since this average motion defines the frame of this mechanics. (It means that this mechanics does not say us whether E has a rigid motion, an expansion, or any other irrotational motion: we have to empirically determine this motion. Note that the "neo-classical" mechanics is the usual classical mechanics only in the case that the frame E has a rigid motion with respect to a (any) rigid reference frame.)

However, we may tentatively assume that the *microscopic* motion of the (micro-)ether *with respect to its mean rest frame* E or macro-ether, does obey mechanics, as well as the motion of material particles. The micro-ether being a perfect fluid, it exerts only surface forces due to its pressure, which may be reexpressed as volume forces depending on the gradient of the pressure of the micro-ether. Thus, any material, "elementary" particle, supposed to occupy a finite volume, is subjected to this

Archimedes thrust, just like a ball in the sea (in this tentative reasoning, we deliberately "forget" the fact that quantum, not classical mechanics, is usually admitted to apply at this scale). In order that this pressure force be a universal force depending only on the inertial mass of the material particles, it is sufficient to assume that the mass density inside any particle (averaged over the volume of the particle) is the same for all particles, say  $\rho_p$ . Then a field  $p_e$  of "ether pressure" would give the gravity acceleration  $\mathbf{g} = -\text{grad } p_e / \rho_p$  [1, 3]. (It turns out that this interpretation of a universal gravitation force was proposed by Euler in 1746! See Whittaker [39].) We then note that  $\rho_p$  may still depend on  $p_e$ , but on  $p_e$  only, and that this would seem miraculous unless the material particles themselves are made of the hypothetical fluid – as is also suggested by the unity of matter and fields, and by the observed transformations of particles into different ones. According to this notion of a "constitutive ether", material particles would be organized local flows in the ether, which seems a very promising heuristics as regards the realistic interpretation of these objects of microphysics – as shown in detail by Romani [35]. Hence, we equate  $\rho_p$  to  $\rho_e$ , the local "density" of the micro-ether, which must depend on  $p_e$  only, i.e. a barotropic fluid (this is necessarily the case for a perfectly continuous fluid, for which entropy and temperature do not make sense [35]). We thus assume that

$$\mathbf{g} = -\text{grad } p_e / \rho_e. \quad (1)$$

It is worth noting that Eq. (1) implies a *decrease* of the ether pressure  $p_e$  and the ether density  $\rho_e = \rho_e(p_e)$  in the direction of the gravitational attraction. Because the gravitation varies only over macroscopic distances,  $p_e$  must be the macroscopic pressure in the ether; more precisely,  $\text{grad } p_e$  must be the average of  $\text{grad } p'_e$  over a macroscopic volume, with  $p'_e$  the "true" microscopic pressure of the ether [1]. This leaves the possibility that the other interactions, which vary over much shorter length scales, could be explained as combined effects of the field  $p'_e$  on the matter particles, the latter being seen as organized flows. We see that gravitation can be envisaged as a *correction* defined by Eq. (1), even in the case where the "mass density inside a matter particle" would not be the same for all particles. In other words, at the scale of the elementary particles, the universality of the gravitation force might be a question of definition. Moreover, the sequel of the development of the theory (Sect. 3) leads to admit that the compressibility of the (micro-)ether is  $K = 1/c^2$  with  $c$  the velocity of light, thus an extremely low compressibility. In this situation, it is natural to expect that the mass density is indeed nearly the same for all particles.

Equation (1), thus obtained semi-heuristically, is yet assumed to be an exact equation of the theory, which is a substitute for the Newtonian equation  $\mathbf{g} = \text{grad } U$ . In a phenomenological point of view, "a theory is defined by the set of its equations" (as Hertz said about Maxwell's electromagnetic theory). The next step is to state the equation relating the field of macroscopic ether pressure,  $p_e$ , to the mass density  $\rho$  (the latter is, of course, the usual density,  $\rho = \delta m / \delta V$ , with  $\delta m$  the sum of the masses of the material particles involved in some volume element  $\delta V$ ). To do this, we assume, as did Romani [35], that Newtonian gravitation (NG) corresponds to the limit case of an incompressible fluid. Indeed, gravitation propagates instantaneously in NG, whereas, if gravitation is a pressure force, it should propagate with the "sound" velocity in the fluid ether,

$$c_e(\rho_e) \equiv \sqrt{dp_e / d\rho_e}, \quad (2)$$

which is infinite only for an incompressible fluid. In order to recover Poisson's equation for the gravity acceleration  $\mathbf{g}$  (Eq. (1)), the field equation must be in the incompressible case:

$$\Delta p_e = 4\pi G \rho \rho_e. \quad (3)$$

In the static situation, the propagation speed plays no role; it is thus natural to assume that, in the static situation, this same equation still applies also in the case where the fluid indeed has a compressibility. However, this means already a new, non-linear theory of gravitation, implying e.g. a perihelion shift for the orbit of a test particle in a static field with spherical symmetry [1]. In assuming that the ether is conserved, that the wave motion of the ether with respect to its mean rest frame obeys Newton's second law, and by adapting standard arguments of acoustics, one then finds that the field equation of the general case is [1]

$$\Delta p_e - \frac{1}{c_e^2} \frac{\partial^2 p_e}{\partial t^2} = 4\pi G \rho \rho_e. \quad (4)$$

Thus, the concept of a fluid ether leads quite naturally to the notion that gravitation should propagate in essentially the same way as the most usual waves, i.e. acoustic and electromagnetic waves. Equation (4) is not Galileo-invariant, hence this theory with Newtonian space-time is a preferred-frame theory. NG and Galilean invariance are recovered as the limit case of an incompressible fluid.

### 3. Principle of equivalence between the metric effects of uniform motion and gravitation

The foregoing theory has to be adapted so as to account for the "relativistic" effects, here seen as resulting from the "real" Lorentz contraction [10-11, 20-21, 33-34]. Thus, these effects are essentially absolute metric effects of uniform motion. Einstein's principle of equivalence between inertial effects and effects of a true gravitational field is an extrapolation of a ("weak") principle of equivalence, valid in Newtonian theory as a simple consequence of the identity between inertial mass and passive gravitational mass. As it has been often discussed in the literature (see e.g. Fock [18]), these equivalence principles are valid only in an infinitesimal domain, such that the gravitational field, as well as the field of "fictitious" inertial forces due to the motion of the considered frame with respect to the inertial frame, may be considered as uniform. Now, in a relativistic theory, the infinitesimal domain must be in space-time, so that the uniformity should be true both in space and in time. But, in an infinitesimal domain of space-time, any motion may be considered as uniform, and a uniform motion can have none other effect as the metric effects due to the FitzGerald-Lorentz contraction of the measuring rods and the Larmor-Lorentz-Einstein dilation of the clock periods. Hence, we are lead to the conclusion that, if there can be an equivalence principle, this should give the correspondence between the metric effects of a uniform motion and certain metric effects that should appear as a consequence of a (locally) uniform gravitational field.

Under our assumption that a gravitational field is due to a variation in the ether pressure, or equivalently in the ether density, we indeed can see an analogy between a uniform motion and a gravitational field: due to the Lorentz contraction, the "apparent" density of the ether is modified by the uniform motion. More precisely, let us assume that an observer following the uniform motion, at constant velocity  $\mathbf{V}$  with respect to the macro-ether  $\mathbf{E}$ , can use the "true" simultaneity (of the frame  $\mathbf{E}$ ), which he indeed can use if he knows his absolute velocity  $\mathbf{V}$  – this, in turn, being *a priori* possible in a preferred-frame theory which will lead to definite effects of the absolute motion, at least in the presence of a gravitational field. Then, a given volume  $dV^0$  of ether (as evaluated with the rods of  $\mathbf{E}$ ) has, for this observer, a greater volume  $dV = dV^0/\beta_{\mathbf{V}}$ , with  $\beta_{\mathbf{V}} = (1 - V^2/c^2)^{1/2}$ , because his measuring rod is contracted in the ratio  $\beta_{\mathbf{V}}$  in the direction  $\mathbf{V}$ . The "mass" or rather the amount of ether is unchanged, for the mass increase with velocity concerns only material particles. Hence, due to the absolute motion, the

apparent ether density is lowered,  $\rho_e \mathbf{v} = \rho_e \cdot \beta \mathbf{v}$ . Now the metric effects of uniform motion depend only on  $\beta \mathbf{v}$ , thus they depend only on the ratio between the "apparent" density which might be "evaluated" by the moving observer, and the "true" density that he might evaluate if he were not in a motion:  $\beta \mathbf{v} = \rho_e \mathbf{v} / \rho_e < 1$ . Similarly, a gravitational field implies that the local ether density  $\rho_e$  is lower (see after Eq. (1)) than the ether density  $\rho_e^\infty$  in remote regions that are free from gravitation. We are thus naturally lead to postulate that [2-3]:

(A) *In a gravitational field, material objects are contracted, only in the direction of the field  $\mathbf{g}$  (with  $\mathbf{g} = -\text{grad } p_e / \rho_e$ ), in the ratio*

$$\beta = \rho_e / \rho_e^\infty < 1, \quad (5)$$

where  $\rho_e^\infty$  is the ether density at a remote point where no gravity is present, and the clock periods are dilated in the same ratio.

This statement is made for objects and clocks bound with  $\mathbf{E}$ ; if this is not so, one has to combine the metric effects due to motion and gravitation. Due to the contraction of measuring rods in the direction  $\mathbf{g}$ , the physical space metric  $\mathbf{g}$  in the frame  $\mathbf{E}$  becomes a Riemannian one. The contraction of the rods (hence the dilation of the measured distances) occurs with respect to a "background" Euclidean metric  $\mathbf{g}^0$ , which is assumed to be bound with the macro-ether  $\mathbf{E}$ . The latter assumption means that the components  $\mathbf{g}^0_{ij}$  are constant in coordinates bound to the preferred frame – i.e., space-time coordinates  $(x^\mu)_{\mu=0, \dots, 3}$ , such that each point bound to  $\mathbf{E}$  has constant space coordinates  $(x^i)_{i=1, 2, 3}$ .<sup>1</sup> In the same way, the dilation of the clock periods implies a contraction of the local time interval  $dt_{\mathbf{x}}$ , measured with a fixed clock (bound to  $\mathbf{E}$ ) in a gravitational field, as compared with the corresponding interval of the "absolute time",  $dT$ , thus

$$dt_{\mathbf{x}}/dT = \beta. \quad (6)$$

This assumption (A) gives a specific form to the space-time metric  $\boldsymbol{\gamma}$  [2]. In particular, the slowing down of clocks, as expressed by Eq. (6), implicitly assumes that the absolute time  $T$  is a globally synchronized time coordinate [23] in the frame  $\mathbf{E}$ , i.e., the components  $\gamma_{0i}$  are zero in coordinates  $(x^\mu)$  that are bound to the preferred frame and such that  $x^0 = cT$ , which means that a simultaneity is defined in the frame  $\mathbf{E}$  as a whole [3, 5]. The property " $\gamma_{0i} = 0$ " holds true after any coordinate transformation of the form

$$x'^0 = \phi(x^0), \quad x'^i = \psi^i(x^1, x^2, x^3). \quad (7)$$

These transformations, that both leave the frame unchanged and keep true the global synchronization [5], build a group which is important in the theory [7-8]. However, in order that the time-dilation (6) be univoquely defined, it is necessary to restrict oneself to the subgroup constituted by the merely spatial transformations [7-8]; this justifies the term "absolute time" for  $T$ . Note that  $T$  is the time which is measured by a clock bound to the preferred frame and far enough from massive bodies so that no gravitational field is felt.

The author recently learned that a somewhat similar equivalence principle between metric effects of uniform motion and gravitation had previously been assumed, in the special case of a static gravitational field, by Podlaha & Sjödin [30] (*cf.* also Podlaha [29] and Sjödin [36]). However, the assumption of these authors is different from assumption (A) hereabove: the ratio assumed by Podlaha & Sjödin is  $\beta' = (\rho_0/\rho)^{1/2}$ , where  $\rho_0$  is the ether density in an unaffected region and plays thus the same role as our  $\rho_e^\infty$ , so that one might write  $\beta = 1/\beta'^2$  if the "ether density"  $\rho_e$  (or  $\rho$  for Podlaha and Sjödin) were one and the same field in both approaches. Yet for Podlaha & Sjödin [30], the ether density *increases* in the direction of the gravitational attraction, this increase being approximately determined

<sup>1</sup> Latin indices vary from 1 to 3 (spatial indices), Greek indices from 0 to 3.

by the (opposite of the) Newtonian potential,  $-U = -GM/r$  for the static field produced by a spherical body with mass  $M$ : their equation (20) amounts to  $\rho/\rho_0 = (1 - U/c^2)^{-2}$ , or  $\beta' = 1 - U/c^2$ . For a weak gravitational field, our assumption (A) gives  $\beta \approx 1 - U/c^2$  [2], thus  $\beta \approx \beta'$  for a weak field. However, our assumption is stated for any gravitational field, and even for a weak field there are correction terms [6] (see Eq. (64) here). We have given a heuristic justification for assumption (A) (see also Refs. 2-3). Another difference is the way in which the spatial contraction is assumed to occur, i.e., whether it is isotropic or not: Podlaha & Sjödin do not precise this point, but Eq. (8) in Podlaha [29] seems to imply that the contraction would be locally isotropic. According to our hypothesis, the spatial contraction is anisotropic (just like the FitzGerald-Lorentz contraction), since it occurs in the direction of the  $\mathbf{g}$  vector only.

As a result of the proposed equivalence principle, two different space metrics,  $\mathbf{g}$  and  $\mathbf{g}^0$ , as well as two different times (the local time  $t_{\mathbf{x}}$  and the absolute time  $T$ ), might be used by observers bound to the preferred frame. Combining the space and time evaluations into a space-time metric (which, in contrast, is valid for any observer in any reference frame), we may also say that we have two such metrics on the "space-time": an "abstract" one,  $\gamma^0$ , obtained by combining  $\mathbf{g}^0$  and  $T$ , and which is flat, and a "physical" one,  $\gamma$ , obtained by combining  $\mathbf{g}$  and  $t_{\mathbf{x}}$ , and which is curved. The question then arises as to which metric we should refer the equations for the field of ether pressure, Eqs. (1) and (4). We assume that, due to the local character of these two equations, and for yet another reason, the *physical* space and time metrics must be used [2-3]. This means (i) that the gradient operator in Eq. (1), as well as the Laplace operator in Eq. (4), are given by

$$(\text{grad } \phi)^i = (\text{grad}_{\mathbf{g}} \phi)^i = g^{ij} \frac{\partial \phi}{\partial x^j}, \quad (g^{ij}) \equiv (g_{ij})^{-1}, \quad (8)$$

$$\Delta \phi = \Delta_{\mathbf{g}} \phi = \text{div}_{\mathbf{g}} \text{grad}_{\mathbf{g}} \phi = \frac{1}{\sqrt{\mathbf{g}}} \frac{\partial}{\partial x^i} \left( \sqrt{\mathbf{g}} g^{ij} \frac{\partial \phi}{\partial x^j} \right), \quad \mathbf{g} \equiv \det (g_{ij}); \quad (9)$$

and (ii) that the time derivative in Eq. (4) must be understood as relative to the local time  $t_{\mathbf{x}}$ , that is

$$\frac{\partial \phi}{\partial t_{\mathbf{x}}} \equiv \frac{\rho_e^\infty}{\rho_e} \frac{\partial \phi}{\partial T} \equiv \frac{1}{\beta} \frac{\partial \phi}{\partial T}. \quad (10)$$

Now the "sound" velocity in the ether,  $c_e$ , depending on the local ether density  $p_e$  as given by Eq. (2), must be assumed to be equal (everywhere and at any time) to the constant velocity of light  $c$  [2]. This can be true only if  $p_e = c^2 \rho_e + a$ , and the constant  $a$  must be zero since the pressure must cancel for nil density, thus we get:  $p_e = c^2 \rho_e$ . The modification of Eq. (4) accounting for SR and the equivalence principle is hence written as

$$\Delta p_e - \frac{1}{c^2} \frac{\partial^2 p_e}{\partial t_{\mathbf{x}}^2} = \frac{4\pi G}{c^2} \rho p_e. \quad (11)$$

In this equation,  $\rho$  is now "the mass-energy density", a somewhat ambiguous expression to be precised in Section 5. Using assumption (A), one may rewrite Eq. (1) [now valid with the gradient defined in terms of the curved physical metric  $\mathbf{g}$ , Eq. (8)] and Eq. (11) in terms of the Euclidean metric  $\mathbf{g}^0$  and the absolute time  $T$  [4]:

$$\mathbf{g} = -\frac{c^2}{2} \text{grad}_{\mathbf{g}^0} f = -\frac{c^2}{2} \text{grad}_0 f, \quad f \equiv \beta^2 = (\gamma_{00})_{\text{E}}, \quad (x^0 = cT), \quad (12)$$

$$\Delta_0 f - \frac{1}{f} \left( \frac{f_{,0}}{f} \right)_{,0} = \frac{8\pi G}{c^2} \rho, \quad \Delta_0 \equiv \Delta_{\mathbf{g}^0} \equiv \text{div}_{\mathbf{g}^0} \text{grad}_{\mathbf{g}^0}, \quad (x^0 = cT). \quad (13)$$

However, Eq. (13) is equivalent to Eq. (11) only if the "reference ether pressure"  $p_e^\infty$  is time-independent, i.e., the universe is assumed "static at infinity", which may indeed be assumed, except for cosmological problems [4]. The simple form (12)-(13) of the main equations has important computational consequences.

#### 4. The motion of a test particle as defined by an extension of Newton's second law

In the investigated theory, the motion of mass points and light-like particles, as well as the motion of any continuous medium (Sections 6 and 7), is governed by an extension of Newton's second law to curved space-time. The main reason to seek after such an extension was that Einstein's assumption, according to which free test particles follow space-time geodesics, gives a physical status to space-time. That the primary object of physics should be such mixture of space and time, is very difficult to accept for the "common sense", and has puzzling consequences such as the possibility of a travel back in time, with its well-known paradoxes. In contrast, if the Lorentz-Poincaré interpretation of SR is adopted, the space-time may be envisaged simply as a (very convenient and clever) mathematical tool. This is indeed the way in which Poincaré [31] introduced in 1905 the concept of the space-time as a 4-dimensional space with coordinates  $x, y, z, ict$ . According to the view adopted here, the "relativistic space-time couplings" are merely due to physical effects, on clocks and meters, of absolute motion and gravitation. Einstein's geodesic assumption could be compatible with this view only in the case that this assumption could be deduced from another one, directly compatible with the notion of distinct space and time. Such is Newton's assumption "Force = time-derivative of momentum".

In classical mechanics, it is indeed possible to rewrite Newton's second law, in a force field deriving from a *constant* potential, as the geodesic equation for a certain artificial space-time metric, and this *exactly* (cf. Mazilu [25]). In relativistic gravitation theory, it is known to be possible, conversely, to rewrite the spatial part of the geodesic equation as Newton's second law, again in the case of a *constant* gravitational field, i.e. for a time-independent space-time metric [23]. However, Landau & Lifchitz [23] did not actually prove that the obtained Newton law, thus three scalar equations, is equivalent to the geodesic equation, which involves four scalar equations. In the case of a variable metric, the various attempts to rewrite the geodesic equation with frame-dependent gravitational forces have been reviewed by Jantzen *et al.* [22]. As it appears from their review, and as it is also discussed in Ref. 5, none of these attempts is fully convincing. The reason is, in the author's opinion, that most of these researchers did not try to play the game "Newton's second law" with an *equivalent* equipment to that in classical mechanics, thus one reference frame (of which the possible changes may be discussed, if desired, in a *subsequent* step), with one spatial metric and one measure of physical time (Møller [27] did try this, but his pioneering proposal has one serious shortcoming, to be recalled hereafter). Instead, most authors retain the whole of the space-time metric with its ten independent components – in contrast with the seven scalars involved in the spatial metric (six) plus the local time (one).

To get a real, compelling equivalent of Newton's second law in a given reference frame, in the "relativistic" situation where the spatial metric  $\mathbf{g}$  is curved by gravitation and *evolves with time*, and where clocks go differently at different places and times, is the objective that, the author believes, has been reached by the following proposal [4-5].

i) The precise expression of the *force*  $\mathbf{F}$  will depend, of course, on each particular theory, and will be decomposed into a non-gravitational (non-universal) force  $\mathbf{F}_0$ , whose expression should be adapted from SR, and a gravitational force  $\mathbf{F}_g$ . The universal character of the latter, plus the assumption that SR

must hold true locally, impose that it has the form  $\mathbf{F}_g = m(v) \mathbf{g}$ , where  $m(v)$  is the relativistic inertial mass,

$$m(v) \equiv m(v=0) \cdot \gamma_v \equiv m(0) \cdot (1-v^2/c^2)^{-1/2}, \quad (14)$$

the velocity vector  $\mathbf{v}$  and its modulus  $v$  being evaluated in terms of the local physical standards of space and time,

$$v^i \equiv dx^i/dt_{\mathbf{x}}, \quad v \equiv [\mathbf{g}(\mathbf{v}, \mathbf{v})]^{1/2} = (\mathbf{g}_{ij} v^i v^j)^{1/2}, \quad (15)$$

and where  $\mathbf{g}$  is a space vector that should depend only on the current position and velocity of the test particle. The dependence on the velocity may be surprising at first if one remembers Newtonian gravity, but it means simply that the gravity acceleration  $\mathbf{g}$  may contain an inertial part – as is to be expected if a general reference frame is considered. Even in a general reference frame, the data of the space-time metric determines uniquely the increment of the local time,  $dt_{\mathbf{x}}$  ("synchronized" along the trajectory of the test particle, *cf.* Cattaneo [13] and Landau & Lifchitz [23]). This means that the ratio  $dt_{\mathbf{x}}/d\chi$ , with  $\chi$  the parameter used to run along the trajectory, is unambiguously defined once a fixed reference frame is considered [5]. In the present theory, the preferred frame  $\mathbf{E}$  is a globally synchronized reference frame (i.e., the  $\gamma_{0i}$  components are zero, e.g. with the choice  $x^0 = cT$  for the time coordinate). This implies that the trajectory may always be parametrized with the absolute time  $T$  (which is also true for any other globally synchronized coordinate time,  $T' = \phi(T)$ ), and that the relevant ratio is very simple, Eq. (6).

ii) There is no choice for the expression of the *momentum*, i.e. the product of the inertial mass and the velocity:

$$\mathbf{P} \equiv m(v) \mathbf{v}. \quad (16)$$

The difficult point is definition of the *time-derivative* of the momentum along the trajectory. Indeed, to define this consistently, we have to define, more generally, the derivative of any vector  $\mathbf{w}(\chi)$  on the trajectory (defined by the dependence  $x^i = x^i(\chi)$ ), and this trajectory is in a space (3-dimensional manifold) equipped with a *Riemannian* metric  $\mathbf{g}$  that *varies* with the parameter  $\chi$  along the trajectory. The variation referred to is not merely the spatial variation in  $\mathbf{g}$  (or in its components  $\mathbf{g}_{ij}$ ) as the point  $\mathbf{x} = (x^i)$  is changed (this variation is usual for a Riemannian manifold). It is also the time variation, precisely it is the fact that the spatial *field*  $\mathbf{g}$  (the correspondence between *any* spatial point  $\mathbf{x}'$  and the local metric  $\mathbf{g}(\mathbf{x}')$ ), is not the same for different values of the parameter  $\chi$  (which indeed plays the role of a time).

The first difficulty, i.e. the spatial variation of the metric, has been solved since a long time, and it is for this reason that the case of a constant gravitational field has been solved also: the correct time-derivative of a vector  $\mathbf{w}(\chi)$  along a trajectory in a space equipped with a *fixed* Riemannian metric, is what is usually called the *absolute derivative*  $D_0\mathbf{w}/D\chi$  (relative, however, to the considered metric  $\mathbf{g}$  and to the considered parameter  $\chi$ ; of course, if one changes  $\chi$  to  $\xi = \xi(\chi)$ , one gets  $D_0\mathbf{w}/D\xi = (d\chi/d\xi)D_0\mathbf{w}/D\chi$ ). The absolute derivative is usually introduced as a by-product of the "covariant derivative", the latter being valid for fields defined in volume domains of the manifold (e.g. Brillouin [9], Lichnerowicz [24]). However, the absolute derivative may be introduced specifically as the only consistent time-derivative of a vector along a trajectory, in the case of a fixed metric [2]. (Of course, the absolute derivative may be defined for manifolds with an arbitrary number of dimensions: e.g. in the case of the 4-dimensional space-time, the absolute derivative with respect to the space-time metric  $\gamma$  and to the parameter  $s = c\tau$  with  $\tau$  the proper time,  $\Delta\mathbf{W}/\Delta s$ , allows definition of the 4-acceleration of a test particle,  $\mathbf{A} \equiv \Delta\mathbf{U}/\Delta s$  with  $\mathbf{U} \equiv d\mathbf{x}/ds$  the 4-velocity.)

In order to define the time-derivative  $D\mathbf{w}/D\chi$  of a vector  $\mathbf{w}(\chi)$  along a trajectory  $\mathbf{x}(\chi)$ , in the case of a *variable* metric field  $\mathbf{g} = \mathbf{g}_\chi$ , we impose the following essential requirements [5]: 1) it must be a space vector depending linearly on  $\mathbf{w}$ ; 2) it must reduce to the absolute derivative if  $\mathbf{g}$  does not depend on  $\chi$ ; 3) it must be expressed in terms of the space metric  $\mathbf{g}$  and its derivatives; and 4) it must obey Leibniz' derivation rule for a scalar product, i.e.

$$\frac{d}{d\chi}(\mathbf{g}(\mathbf{w}, \mathbf{z})) = \mathbf{g}\left(\mathbf{w}, \frac{D\mathbf{z}}{D\chi}\right) + \mathbf{g}\left(\frac{D\mathbf{w}}{D\chi}, \mathbf{z}\right). \quad (17)$$

It turns out that these requirements allow to define one and only one time-derivative, given by [5]:

$$D\mathbf{w}/D\chi \equiv D_0\mathbf{w}/D\chi + (1/2) \mathbf{t} \cdot \mathbf{w}, \quad \mathbf{t} \equiv \mathbf{g}_\chi^{-1} \cdot \frac{\partial \mathbf{g}_\chi}{\partial \chi}. \quad (18)$$

In particular, Leibniz' rule (17) is responsible for the unique possible value of the coefficient multiplying the vector  $\mathbf{t} \cdot \mathbf{w} = (t^i_j w^j)$ , i.e.  $\lambda = 1/2$ . In contrast, Møller's definition of Newton's second law [27] amounts to using the value  $\lambda = 0$ , i.e. to take the absolute time-derivative relative to the *frozen* space metric of the "time"  $\chi_0$  where the derivative is to be calculated; thus, this time-derivative does *not* obey Leibniz' rule with the actual, variable metric, Eq. (17). Leibniz' rule is very important to obtain correct energy equations [4-5].

Now, to write Newton's second law in terms of local standards, the parameter  $\chi$  must be the local time  $t_{\mathbf{x}}$ , "synchronized" along the trajectory. Moreover, one defines naturally for a mass particle the "purely material" energy (i.e., not accounting for the potential energy in the gravitational field [4]) as  $E \equiv m(\nu)c^2$ , so we finally write:

$$\mathbf{F}_0 + (E/c^2) \mathbf{g} = D\mathbf{P}/Dt_{\mathbf{x}}. \quad (19)$$

iii) For a light-like particle (photon, neutrino?), the energy  $E$  of the particle (or rather the ratio  $E/c^2$  i.e. the mass equivalent of the energy) plays the role of the inertial mass [13], so we define

$$\mathbf{P} \equiv (E/c^2) \mathbf{v}, \quad (20)$$

it being understood that  $E$  is the usual, "purely material" energy, related to the frequency  $\nu$  by Planck's relation

$$E = h\nu \quad (21)$$

(here  $\nu$  is the frequency as measured with the local time, i.e. by the momentarily coincident clock). Equations (19) and (20) define one and the same Newton law for both mass particles and light-like test particles. Furthermore, the frequency  $\nu$  deduced from  $E$  by Eq. (21) is none other than de Broglie's frequency [30].

iv) Having thus defined a unique extension of Newton's second law, Eq. (19), to any space-time curved by gravitation, a question naturally arises: is this extended Newton law compatible with geodesic motion? Since geodesic motion applies only to *free* particles, we have to investigate the case  $\mathbf{F}_0 = 0$  in Eq. (19); and since the gravity acceleration  $\mathbf{g}$  has not been defined in the general statement of Newton's second law (although it is defined, in our theory, by Eq. (1) or equivalently by Eq. (12)), it is clear that the question can be reformulated thus: *which form of the gravity acceleration  $\mathbf{g}$  is compatible with*

*Einstein's geodesic motion?* Recall that Eq. (19) can be written in any reference frame (although the transformation law of  $\mathbf{g}$  from one frame to another one has not been given, and does not seem to be easy to get:  $\mathbf{g}$  is only assigned to be a spatial vector in a given reference frame). As in Newtonian theory, one may expect that the form of  $\mathbf{g}$  should be simpler in some special class of reference frames. It turns out that the "convenient" frames are the "globally synchronized" ones, i.e. frames such that, in certain coordinates bound to the frame, the  $\gamma_{0i}$  components are zero. This is not surprising in this "neo-Newtonian" approach: in general frames, the time can be synchronized only on a trajectory, whereas the notion of a global simultaneity is essential (perhaps the most essential notion at all) in classical mechanics. According to Landau & Lifchitz [23], the very existence of globally synchronized frames is a property valid for generic space-times. However, there are "pathological" space-times in which it is not valid. The answer to the above question has been found [5] for such generic space-times: *In order that free particles follow space-time geodesics, it is necessary and sufficient that, in any globally synchronized reference frame, the gravity acceleration have the following expression:*

$$\mathbf{g}_{\text{geod}} = -c^2 \frac{\text{grad}_{\mathbf{g}} \beta}{\beta} - \frac{1}{2} \mathbf{g}^{-1} \cdot \frac{\partial \mathbf{g}}{\partial t_{\mathbf{x}}} \cdot \mathbf{v}, \quad \beta \equiv \sqrt{\gamma_{00}}. \quad (22)$$

**v)** It is seen that the "Einsteinian" gravity acceleration, Eq. (22), *differs* from the gravity acceleration assumed in the present theory by the second, velocity-dependent term in the right-hand side: this term is absent from Eq. (1), whereas, due to the equation  $p_e = c^2 \rho_e$  (see after Eq. 10), the first term in the r.h.s. of Eq. (22) is just the r.h.s. of Eq. (1). Note that the velocity-dependent term cancels for a gravitational field that is constant in the considered frame. Therefore, another question arises: *how to phenomenologically characterize the assumed gravity acceleration, Eq. (1) with  $p_e = c^2 \rho_e$ ?* The answer is [5]:

*Assume that in some "globally synchronized" reference frame F ( $\gamma_{0i} = 0$ ), the gravity acceleration be a space vector  $\mathbf{g}$  depending only on the metric field  $\boldsymbol{\gamma}$ . More precisely, assume that  $\mathbf{g}$  does not depend on the time variation of  $\boldsymbol{\gamma}$  and is linear with respect to the space variation of  $\boldsymbol{\gamma}$ . In order that free particles follow space-time geodesics in the static case ( $\gamma_{\mu\nu,0} = 0$ ), it is necessary and sufficient that the general expression of vector  $\mathbf{g}$  in the frame F be*

$$\mathbf{g} = -c^2 \frac{\text{grad}_{\mathbf{g}} \beta}{\beta} = -\frac{c^2}{2} \frac{\text{grad}_{\mathbf{g}} f}{f}, \quad f \equiv \gamma_{00} \equiv \beta^2, \quad (23)$$

*with  $\mathbf{g}$  the space metric in F.*

This result characterizes the assumed field  $\mathbf{g}$  independently of any heuristic consideration on the ether. The assumption that the metric field  $\boldsymbol{\gamma}$  is a "spatial potential" for the field  $\mathbf{g}$  is natural if one wants to account for the equivalence principle without going too far from Newtonian theory. Thus, Einstein's geodesic motion would be valid only in a static gravitational field.