On charge conservation in a gravitational field

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MAIN MOTIVATIONS FOR SCALAR ETHER THEORY (SET) (I)

Lorentz-Poincaré version of special relativity with an ether:

obtains Lorentz transfo. and "relativistic" effects as following from

- (i) "Absolute" effects of motion through that ether,
- (ii) Clock synchronization.

In it, "$v < c$" is not absolute, concerns mass particles.
SET extends it to situation with gravitation.

SET makes gravity thinkable as the pressure force of the ether: Archimedes' thrust on extended particles seen as organized flows in the ether.

Main motivations for Scalar Ether Theory (SET) (II)

Despite its successes, GR has problems:

- Unavoidable singularities (in gravitat' collapse & big bang).
- Interpretation of the necessary gauge condition.
- Coupling with quantum is problematic.
- Need for dark energy. Need for dark matter.

- SET has no singularity.
- No gauge condition.
- Avoids non-uniqueness problem of covariant Dirac theory.
- Predicts accelerated expansion. Preferred-frame effects more important at large scales.

MAIN EQUATIONS OF E.M. FIELD IN SET

NB. SET has a preferred reference frame $\mathcal{E}$. It has also a curved spacetime metric γ. The spatial metric in frame $\mathcal{E}$ is noted $\boldsymbol{g}$.

First Maxwell group unchanged. In terms of field tensor $\boldsymbol{F}$:

$$F_{\sigma\mu\,,\nu} + F_{\mu\nu,\sigma} + F_{\nu\sigma,\mu} = F_{\sigma\mu\,;\nu} + F_{\mu\nu;\sigma} + F_{\nu\sigma;\mu} = 0. \qquad (1)$$

2nd group: SET has an eqn for continuum dynamics. Apply it to *charged medium* subjected to *Lorentz force* and assume that:

(i) Total energy-momentum tensor $\mathbf{T} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}}$.

(ii) Total energy-momentum tensor $\mathbf{T}$ obeys the general equation for continuum dynamics, without any non-gravitational force.

$$\text{This gives } F^{\mu}{}_{\sigma}\, F^{\sigma\nu}{}_{;\nu} = \mu_0 \left[b^{\mu}\left(\mathbf{T}_{\text{field}}\right) - F^{\mu}{}_{\sigma}\, J^{\sigma} \right], \text{ where} \qquad (2)$$

$$b^0(\mathbf{T}) \equiv \frac{\gamma^{00}}{2}\, g_{ij,0}\, T^{ij}, \quad b^i(\mathbf{T}) \equiv \frac{1}{2}\, g^{ij} g_{jk,0}\, T^{0k}. \qquad (3)$$

CHARGE BALANCE: EXACT EQUATIONS

If $\det \boldsymbol{F} \neq 0$, where $\boldsymbol{F} \equiv (F^{\mu}{}_{\nu})$ (i.e. $\mathbf{E}.\mathbf{B} \neq 0$) we get from Eq. (2):

$$\hat{\rho} \equiv J^{\mu}_{;\mu} = \left(G^{\mu}{}_{\nu}\, b^{\nu}(\mathbf{T}_{\text{field}})\right)_{;\mu}, \quad (G^{\mu}{}_{\nu}) \equiv (F^{\mu}{}_{\nu})^{-1}. \tag{4}$$

Thus, charge conservation ($J^{\mu}_{;\mu} = 0$) is not true in general, according to Eq. (2). [MA, Open Physics 2016]

Let Ω be any "substantial" domain of the charged continuum. One can prove that the evolution rate of the charge contained in Ω is

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\int_{\Omega} \delta q\right) = \int_{\Omega} \hat{\rho}\,\sqrt{-\gamma}\;\mathrm{d}^3 x \qquad (\,\gamma \equiv \det(\gamma_{\mu\nu})\,) \tag{5}$$

in any coordinates x^{μ}. ($t \equiv x^0/c$.) Of course the domain Ω as well as its boundary depend on t in general spatial coordinates x^i.

Weak field approximation: I. Gravitational field

The *gravitational field* is assumed weak and slowly varying for the system of interest S (e.g. the Earth with some e.m. source on it).

Use an asymptotic *post-Newtonian* (PN) scheme. Associates with S a *family* (S_λ) of systems, depending on $\lambda \to 0$, $\lambda = 1/c^2$ in specific λ-dependent units. Writes Taylor expansions w.r.t. λ. E.g.

$$\beta \equiv \sqrt{\gamma_{00}} = 1 - U/c^2 + O(c^{-4}), \tag{6}$$

where U = Newtonian potential, obeys Poisson eqn.
Spatial metric assumed in the theory:

$$\boldsymbol{g} = \beta^{-2}\boldsymbol{g}^0 \tag{7}$$

with $\boldsymbol{g}^0$ = invariable Euclidean metric. We deduce from (6)–(7):

$$\frac{\partial g_{ij}}{\partial T} = 2c^{-2}\partial_T U \delta_{ij} + O(c^{-4}). \tag{8}$$

(We will take Cartesian coordinates for $\boldsymbol{g}^0$, i.e., $g^0_{ij} = \delta_{ij}$.)

Weak field approximation: II. E.M. field & current

Assume $\boldsymbol{F}$ and $\mathbf{J}$ depend smoothly on λ, hence they too admit Taylor expansions w.r.t. c^{-2} but the orders n and m not known:

$$\boldsymbol{F} = c^n \left(\overset{0}{\boldsymbol{F}} + c^{-2} \overset{1}{\boldsymbol{F}} + O(c^{-4}) \right) \tag{9}$$

and

$$\mathbf{J} = c^m \left(\overset{0}{\mathbf{J}} + c^{-2} \overset{1}{\mathbf{J}} + O(c^{-4}) \right) . \tag{10}$$

The integers n and m can be positive, negative, or zero.
Remind: $\lambda = 1/c^2$ = gravitational weak-field parameter.

Also, $\boldsymbol{F}$ not assumed slowly varying (nor weak). Means expansions (9)–(10) are *post-Minkowskian* (PM) expansions.

Expansion of the modified Maxwell 2nd group (I)

For the PM expansions (9)-(10), the time variable (such that the expansions are true at a fixed value of it) is $x^0 = cT$, not T as it is for PN expansions. (Not neutral since $c^2 = \lambda^{-1}$.)

Hence, in the modified 2nd group (2), the term $F^{\sigma\nu}{}_{;\nu}$ is of order c^n as is the term $F^{\mu}{}_{\sigma}$.

One thus finds that the r.h.s. of (2) is of order c^{2n}. The l.h.s. is of order c^{n+m+2}, for $\mu_0 = \mu_{00}c^2$ (from dimension and λ-dependent units).

Hence we must have

$$2n = n + m + 2, \qquad \text{i.e.} \quad m = n - 2. \tag{11}$$

Expansion of the modified Maxwell 2nd group (II)

Using the foregoing, one gets the lowest-order term in the weak-field expansion of (2) as

$$\overset{0}{F}{}^{\mu}{}_{\sigma}\,\overset{0}{F}{}^{\sigma\nu}{}_{,\nu} = -\mu_{00}\,\overset{0}{F}{}^{\mu}{}_{\sigma}\,\overset{0}{J}{}^{\sigma}. \tag{12}$$

Thus if $\overset{0}{\boldsymbol{F}} \equiv (\overset{0}{F}{}^{\sigma}{}_{\nu})$ is invertible, $\overset{0}{\boldsymbol{F}}$ is an exact solution of the flat-spacetime Maxwell equation, $\overset{0}{F}{}^{\sigma\nu}{}_{,\nu} = -\mu_{00}\,\overset{0}{J}{}^{\sigma}$.

EXPANSION OF THE CHARGE PRODUCTION RATE

Using (9) and (8) in (4) gives us

$$\hat{\rho} = c^{n-5} {\mu_{00}}^{-1} \left[\left(\overset{0}{G}{}^{\mu 0} \, \overset{0}{T}{}^{jj} - \overset{0}{G}{}^{\mu i} \, \overset{0}{T}{}^{0i} \right) \partial_T U \right]_{,\mu} + O(c^{n-7}), \quad (13)$$

where $\overset{0}{\boldsymbol{G}} \equiv (\overset{0}{G}{}^{\mu}{}_{\nu}) \equiv \overset{0}{\boldsymbol{F}}{}^{-1}$. Due to (9)-(10), $\overset{0}{\boldsymbol{F}}$, $\overset{0}{\boldsymbol{G}}$, $\overset{0}{\mathbf{T}}$ and $\overset{0}{\mathbf{J}}$ do not have the physical dimensions of the corresponding fields $\boldsymbol{F}, \boldsymbol{G}$, ...

Let $\boldsymbol{F}'$ and $\mathbf{J}'$ be solutions of the flat-spacetime Maxwell equation with the correct dimensions in the SI units:

$$F'^{\,\sigma\nu}{}_{,\nu} = -\mu_0 \, J'^{\,\sigma}. \quad (14)$$

Define the associated e.m. T-tensor $\mathbf{T}'$. Assume matrix $\boldsymbol{F}' \equiv (F'^{\,\sigma}{}_{\nu})$ is invertible. Define $\boldsymbol{G}' \equiv \boldsymbol{F}'^{\,-1}$. Eq. (13) rewrites as

$$\boxed{\hat{\rho} = c^{-3} \left[\left(G'^{\,\mu 0} \, T'^{\,jj} - G'^{\,\mu i} \, T'^{\,0i} \right) \partial_T U \right]_{,\mu} + O(c^{-5}).} \quad (15)$$

Explicit expression of charge production rate

To use (15) so as to assess the charge production: *conversely,* assume that to any solution $(\boldsymbol{F}', \mathbf{J}')$ of the full flat Maxwell, it corresponds a unique solution $(\boldsymbol{F}, \mathbf{J})$ of the first group (1) and the gravitationally-modified second group (2), such that $(\boldsymbol{F}', \mathbf{J}')$ be the main terms in the PM expansion of $(\boldsymbol{F}, \mathbf{J})$. Expectable from perturbative arguments.

Expressing $\boldsymbol{F}$ in terms of electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ we rewrite (15) as

$$\boxed{\hat{\rho} = c^{-3}\left(e^i \partial_T U\right)_{,i} + O(c^{-5}),} \tag{16}$$

$$e^i = \begin{pmatrix} \frac{{B_1}^3\, c^2 + B_1\, {B_2}^2\, c^2 + B_1\, {B_3}^2\, c^2 + B_1\, {E_1}^2 - B_1\, {E_2}^2 - B_1\, {E_3}^2 + 2\, B_2\, E_1\, E_2 + 2\, B_3\, E_1\, E_3}{2\, c\, \mu_0\, (B_1\, E_1 + B_2\, E_2 + B_3\, E_3)} \\ \frac{{B_1}^2\, B_2\, c^2 + 2\, B_1\, E_1\, E_2 + {B_2}^3\, c^2 + B_2\, {B_3}^2\, c^2 - B_2\, {E_1}^2 + B_2\, {E_2}^2 - B_2\, {E_3}^2 + 2\, B_3\, E_2\, E_3}{2\, c\, \mu_0\, (B_1\, E_1 + B_2\, E_2 + B_3\, E_3)} \\ \frac{{B_1}^2\, B_3\, c^2 + 2\, B_1\, E_1\, E_3 + {B_2}^2\, B_3\, c^2 + 2\, B_2\, E_2\, E_3 + {B_3}^3\, c^2 - B_3\, {E_1}^2 - B_3\, {E_2}^2 + B_3\, {E_3}^2}{2\, c\, \mu_0\, (B_1\, E_1 + B_2\, E_2 + B_3\, E_3)} \end{pmatrix}. \tag{17}$$

ASSESSING $\partial_T U$ AND $\partial_T(\nabla U)$ (I)

These time derivatives must be evaluated in the preferred reference frame $\mathcal{E}$ assumed by the theory.

The system of interest producing the e.m. field should move through $\mathcal{E}$: velocity field $\mathbf{v}$ with $|\mathbf{v}| \simeq 10 - 1000$ km/s?

We have $\mathrm{d}\, U/\mathrm{d}\, T \equiv \partial_T U + \mathbf{v}.\nabla U = 0$ exactly for self potential of a body with rigid motion (e.g. the Earth).

(For the Earth, the external potential due to the Sun is nearly constant also. The most important departure from $\mathrm{d}\, U/\mathrm{d}\, T = 0$ should come from the Moon.)

For a rigidly rotating *spherical* body, $\mathbf{v}.\nabla U = \mathbf{V}.\nabla U$, $\mathbf{V} \equiv \dot{\mathbf{a}}$.
$\mathbf{a}(T)$: body center.

ASSESSING $\partial_T U$ AND $\partial_T(\nabla U)$ (II)

$\Rightarrow$ Main contribution to $\partial_T U$: translation motion of a nearly spherically symmetric body through $\mathcal{E}$:

$$\partial_T U \simeq -\mathbf{V}.\nabla U \simeq \frac{GM(r)}{r^2}\mathbf{V}.\mathbf{e}_r, \quad r \equiv |\mathbf{x}-\mathbf{a}(T)|, \quad \mathbf{e}_r \equiv (\mathbf{x}-\mathbf{a}(T))/r, \tag{18}$$

$M(r) \equiv 4\pi \int_0^r u^2 \rho(u)\, \mathrm{d}\, u;$ $\quad \rho(r)$: Newtonian density.

On the Earth's surface, this gives

$\boxed{\partial_T U \simeq g V_r \precsim 10 V \simeq 10^5 \text{ (MKSA)}}$ for $\boxed{V = 10}$ km/s.

If moreover the rotating spherical body is homogeneous, we have

$$\partial_T \nabla U = \frac{GM(r)}{r^3}\mathbf{V}. \tag{19}$$

On Earth:

$\boxed{\partial_T \nabla U \simeq g\mathbf{V}/R}$, $\quad |\partial_T \nabla U| \simeq 10^{-2}$ MKSA, $\; V = 10\,\text{km/s}.$

Case of a plane wave

A monochromatic plane e.m. wave in direction $\mathbf{i} \parallel Ox$:

$$E^1 = 0, \; E^i = E^i_0 \cos(kx - \omega T + \varphi_i) \; (i = 2, 3), \quad c\mathbf{B} = \mathbf{i} \wedge \mathbf{E}. \quad (20)$$

Then of course field matrix $\boldsymbol{F} \equiv (F^\mu{}_\nu)$ not invertible. But may add any constant e.m. field $(\mathbf{E}', \mathbf{B}')$. Then generically $\boldsymbol{F}$ is invertible.

Moreover, e^i [Eq. (16)] has $e^i_{,i} = 0$, for $e^1 = 0$ and $e^i = e^i(x^1)$.

Neglecting the term $c^{-3} e^i (\partial_T U)_{,i}$ in view of (19), we get that

$$\boxed{\hat{\rho} = 0 \qquad \text{(Plane wave, } c^{-3} e^i (\partial_T U)_{,i} \text{ neglected).}} \quad (21)$$

However, depending on the constant e.m. field, the neglected term may give high values of $\hat{\rho}$. (Check the case without the wave part.)

THE CASE WITH HERTZIAN DIPOLES

Hertz's oscillating dipole: the charge *distribution*

$$\rho = T_{\mathbf{d},\mathbf{b},\omega} \equiv -e^{-i\omega t}\, \mathbf{d}.\nabla\delta_{\mathbf{b}} \tag{22}$$

($\mathbf{b}$ = dipole position, $\mathbf{d}$ = dipole vector). Associated 3-current:

$$\mathbf{j} = -i\omega\mathbf{d}\, e^{-i\omega t}\, \delta_{\mathbf{b}}. \tag{23}$$

Exact solution of the flat Maxwell eqs in distributional sense:

$$\mathbf{E} = \alpha\left\{\frac{k^2}{r}(\mathbf{d} - (\mathbf{n}.\mathbf{d})\mathbf{n})\cos\varphi + [3(\mathbf{n}.\mathbf{d})\mathbf{n} - \mathbf{d}]\left(\frac{\cos\varphi}{r^3} + \frac{k\sin\varphi}{r^2}\right)\right\}, \tag{24}$$

$$\mathbf{B} = \beta k^2(\mathbf{n}\wedge\mathbf{d})\left(\frac{\cos\varphi}{r} - \frac{\sin\varphi}{kr^2}\right), \quad k = \frac{\omega}{c}, \quad \varphi \equiv kr - \omega t. \tag{25}$$

($\alpha \equiv \frac{1}{4\pi\epsilon_0} = 9\times 10^9$, $\beta \equiv \frac{c}{4\pi} \simeq 2.39\times 10^7$ (MKSA).)

Has $\mathbf{E}.\mathbf{B} = 0$. Adding dipoles with different $\mathbf{b}$ and $\mathbf{d}$ gives $\mathbf{E}.\mathbf{B} \neq 0$.

CASE OF A GROUP OF HERTZIAN DIPOLES

♢ The dipoles are at rest in a common frame moving at $\mathbf{V}$ w.r.t. $\mathcal{E}$.

♢ Their e.m. field is Lorentz-transformed to $\mathcal{E}$.

♢ In view of (16), compute

$$\hat{\rho}(T, \mathbf{x}) = c^{-3} \left(e^i \partial_T U\right)_{,i} \approx c^{-3} \int_{\partial \mathrm{C}} e^i n_i \, \partial_T U \; \mathrm{d}\, S / v(\mathrm{C}), \qquad (26)$$

with C a small cube moving at $\mathbf{V}$, centered at calculation point $\mathbf{x}$.

♢ For three dipoles with $d = 100\,\mathrm{nC.m}$, $\nu = 100\,\mathrm{MHz}$ ($\sigma = 3\,\mathrm{m}$), situated at $\lessapprox \sigma$ from one another, get fields $E \lessapprox$ a few $10^5\,\mathrm{V/m}$, $B \lessapprox 15\,\mathrm{T}$.

♢ with $V = 10\,\mathrm{km/s}$, $\hat{\rho}(T, \mathbf{x})$ has peaks at $\approx \pm 2\mathrm{x}10^8\, e/\mathrm{m}^3/\mathrm{period}$. Seems untenable!

$\Rightarrow$ This version of the gravitationally-modified Maxwell equations looks like being discarded.

WHY WERE THESE NOT THE RIGHT MAXWELL EQS OF THE THEORY?

Dynamical eqn in SET for general continuous medium (velocity field $\mathbf{v}$) subjected to external force density field $\mathbf{f}$:

$$T^{0\nu}_{\text{medium}\ ;\nu} = b^0(\mathbf{T}_{\text{medium}}) + \frac{\mathbf{f.v}}{c\beta}, \quad T^{i\nu}_{\text{medium}\ ;\nu} = b^i(\mathbf{T}_{\text{medium}}) + f^i. \tag{27}$$

Assumption (i): total $\mathbf{T} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}}$.

Assumption (ii): $T^{0\nu}_{\ ;\nu} = b^0(\mathbf{T}), \qquad T^{i\nu}_{\ ;\nu} = b^i(\mathbf{T})$.

(i) + (ii) + (27) with "medium" = "charged medium" gives:

$$T^{0\nu}_{\text{field}\ ;\nu} = b^0(\mathbf{T}_{\text{field}}) - \frac{\mathbf{f.v}}{c\beta}, \qquad T^{i\nu}_{\text{field}\ ;\nu} = b^i(\mathbf{T}_{\text{field}}) - f^i. \tag{28}$$

This has the form (27), with $f^i_{\text{field}} = -f^i_{\text{charged medium}}$ *and* $\underline{\mathbf{v}_{\text{field}} = \mathbf{v}_{\text{charged medium}} \equiv \mathbf{v}}$. But $\mathbf{v}_{\text{field}} \neq \mathbf{v}_{\text{charged medium}}$!

WHAT ARE THE RIGHT MAXWELL EQS OF THE THEORY?

The assumption to be relaxed is (i): the problem with $\mathbf{v}$ is solved if there is an interaction energy-momentum tensor $\mathbf{T}_{\text{interact}}$ such that

$$\text{total}\,\mathbf{T} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}} \underline{+\mathbf{T}_{\text{interact}}}\,. \qquad (29)$$

With (29), Assumption (ii) and (27) do not determine the 2nd group any more.

May postulate the *standard* gravitationally-modified second group:

$$F^{\sigma\nu}{}_{;\nu} = -\mu_0\, J^{\sigma}\,, \qquad (30)$$

which, one may show, is writing almost the usual (3-vector-form) 2nd group in terms of the local time and the space metric in frame $\mathcal{E}$.

Conclusion

Maxwell eqs for the "scalar ether theory" of gravity (SET) were proposed. Predict charge non-conservation in a variable gravitational field.

This occurs already for a translation through SET's "ether".

Using asymptotic PN (respectively PM) expansions for the gravitational field (resp. the e.m. field), an explicit expression for the charge production rate $\hat{\rho}$ was obtained.

For a group of Hertz dipoles producing moderate e.m. field (& with a moderate translation velocity $V = 10\,\mathrm{km/s}$), $\hat{\rho}$ seems unrealistically high.

Actually: those Maxwell eqs are not consistent with continuum dynamics of SET applied to *the e.m. field itself*. Must assume an additional, "interaction", energy tensor. Then the standard gravitationally-modified Maxwell eqs are consistent with SET.