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Point-Particle Limit in a Scalar Theory of Gravitation and the Weak Equivalence Principle

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1. *Basic Equations of the Investigated Theory*

2. *Asymptotic Post-Newtonian Approximation*

3. *Eqs of Motion for Mass Centers in the Scalar Theory*

4. *Point-Particle Limit & Reason for Conflict with WEP*

1. *Basic Equations of the Investigated Scalar Theory*

Bimetric theory, endows space-time with 2 Lorentzian metrics:

flat metric γ^0 ("background", or "prior-geometrical" metric)

curved metric γ ("physical" metric)

related with physical space and time measurements.

Admits a *preferred frame* E ("ether"): *equations are written in* E.

Space-time = $\mathbf{R} \times$ M

M = "space" manifold = {positions $\mathbf{x}$ in the preferred reference frame}

Metric γ *related with* γ^0 *through the scalar field* f:

$\mathrm{d}t_\mathbf{x} / \mathrm{d}T = \sqrt{f}$ $t_\mathbf{x}$: "local time" in E, measured by clock at $\mathbf{x}$

T : "absolute time"

(T = inertial time associated with γ^0 in frame E)

$$\mathrm{d}l = \begin{cases} \mathrm{d}l^0 \text{ if } (\mathrm{d}x^i) \perp \nabla f \\ \mathrm{d}l^0 / \sqrt{f} \text{ if } (\mathrm{d}x^i) \; // \; \nabla f \end{cases}$$

$\mathrm{d}l^0$: Euclidean distance, associated with γ^0 in frame E

$\mathrm{d}l$: distance associated with γ, also in frame E.

1. ***Basic Equations of the Investigated Scalar Theory*** **(continued)**

Dynamics deduced from relativistic form of *Newton's second law:* for a *test particle,*

$$\mathbf{F}_0 + m(v)\mathbf{g} = D\,[m(v)\mathbf{v}]/Dt_{\mathbf{x}}\,.$$

$\mathbf{F}_0$: non-gravitational (electromagnetic, ...) force;

velocity $\mathbf{v}$, modulus v evaluated with physical metric γ

$m(v) \equiv m(0).\gamma_v$: relativistic mass [γ_v: Lorentz factor]

$D\mathbf{w}/D\xi$: appropriate derivative of (spatial) vector $\mathbf{w}$ in space M endowed with time-dependent metric g

g = (space metric on M) = spatial part of γ in preferred frame E

$\mathbf{g} = -\frac{c^2}{2}\nabla f$: gravity acceleration

Static case: that dynamics *implies Einstein's geodesic motion.*

For a *dust continuum,* it implies the following equation:

$$T_{\mu\ ;\nu}^{\ \nu} = b_\mu(\,\mathbf{T}\,)$$

Here $\mathbf{T}$ = energy-momentum tensor and b_μ is defined by

$$b_0(\mathbf{T}) \equiv \frac{1}{2}\ g_{jk,0}\,T^{jk}\ , \qquad b_i(\mathbf{T}) \equiv -\frac{1}{2}g_{ik,0}\,T^{0k}$$

Universality of gravity ⇔ *same eqn for any material medium.*

Equation for the scalar field f (non-cosmological case):

$$\Delta f - \frac{1}{c^2}\frac{1}{f}\partial_T\left(\frac{1}{f}\partial_T f\right) = \frac{8\pi G}{c^2}\sigma$$

$\sigma \equiv T^{00}$ component in coordinates bound to E, $x^0 = cT$.

Δ : Laplacian defined with the Euclidean space metric $\mathbf{g}^0$ on M.

(The same is true for ∇ in $\mathbf{g} = -\frac{c^2}{2}\nabla f$ on previous slide.)

2. Asymptotic PN approximation for relativistic theories of gravitation

In such theories (including the investigated scalar theory),

natural boundary-value problem = *initial-value problem.*

In GR, asymptotic method introduced by Futamase & Schutz (1983).

Initial space metric was very special. *Cf.* also Rendall (1992).

They derived (not detailed) expansions of the *local equations*

but no "global" eqs i.e. for *mass centers of extended bodies.*

Scalar theory: detailed local *and global* eqs were derived.

A *family* (S_λ) of systems deduced from given system S (fluids):

A Newtonian exact ***similarity transformation*****:**

$$p^{(\lambda)}(\mathbf{x}, T) = \lambda^2 p^{(1)}(\mathbf{x}, \sqrt{\lambda}\,T), \qquad \rho^{(\lambda)}(\mathbf{x}, T) = \lambda \rho^{(1)}(\mathbf{x}, \sqrt{\lambda}\,T),$$

$$U_\mathrm{N}^{(\lambda)}(\mathbf{x}, T) = \lambda U_\mathrm{N}^{(1)}(\mathbf{x}, \sqrt{\lambda}\,T), \qquad \mathbf{u}^{(\lambda)}(\mathbf{x}, T) = \sqrt{\lambda}\,\mathbf{u}^{(1)}(\mathbf{x}, \sqrt{\lambda}\,T),$$

applied to the initial data for system **S**, *giving initial data for* S_λ

(p: pressure, ρ: density, U_N: Newt$^\mathrm{n}$ potential, $\mathbf{u}$: velocity)

[$P \equiv c^2(1 - f)/2$ substituted for U_N, where f = scalar gravity field

($f = \gamma_{00}$ component of space-time metric

in coordinates bound to the preferred reference frame of the theory)]

In units $[\mathrm{T}]_\lambda = [\mathrm{T}]/\sqrt{\lambda}$ and $[\mathrm{M}]_\lambda = \lambda[\mathrm{M}]$, the fields p^λ, ρ^λ, $\mathbf{u}^\lambda$, P^λ are ord(λ^0), and $\underline{\lambda \propto 1/c^2}$ ($\lambda = (c_0/c)^2$, c_0 = in the starting units).

$\Rightarrow$ expansions are straightforward.

Only $1/c^2$ enters the field equations $\Rightarrow$ $1/c^2$, not $1/c$, turns out to be the effective small parameter.

All fields are λ^k-expanded, each with n+1 terms ($k = 0, ..., n$)

$\Rightarrow$ *each exact equation splits into* n+1 *exact equations*

(it is just coefficient identification for a polynomial in λ).

The theory admits consistent expansions in powers of λ (or $1/c^2$).

First term ($k = 0$) : Newtonian gravity $\Rightarrow$ *Newtonian limit OK*.

1st-order approximation: 1st PN approximation, linear in the PN fields.

Standard PNA (Fock-Chandras.): matter fields p, ρ, $\mathbf{u}$ *not* expanded, and $1/c^2$ is *formally* taken as small parameter.

3. PN eqs of motion of mass centers in the investigated theory

Mass centers (MC) defined as local barycenters

of the *rest-mass density* ρ_{exact}

which expands as $\rho_{\text{exact}} = \rho + \rho_1/c^2 + O(1/c^4) = \rho_{(1)} + O(1/c^4)$

To get the MC's PN eqs of motion: local PN eqs of motion

[= expansion of $T_{\mu\ ;\nu}^{\ \nu} = b_\mu(\mathbf{T})$] are integrated inside bodies.

Integration of *time* components of the local PN eqs of motion

in domain D_a occupied by body (a)

leads to define as the PN mass of body (a) :

$$M_a^{(1)} \equiv \int_{\mathrm{D}_a} \rho_{(1)}\ \mathrm{d}V \qquad (V \equiv \text{Euclid. volume measure on M})$$

and this integration gives just

$$M_a \equiv \int_{\mathrm{D}_a} \rho\ \mathrm{d}V = \text{Const.}, \quad M_a^{1} \equiv \int_{\mathrm{D}_a} \rho_1\ \mathrm{d}V = \text{Const.}$$

Thus, the PN mass center $\mathbf{a}_{(1)}$ of body (a) is defined by

$$M_a^{(1)}\,\mathbf{a}_{(1)} = \int_{\mathrm{D}_a} \mathbf{x}\left[\rho + \rho_1 / c^2\right] \mathrm{d}V(\mathbf{x}) \equiv M_a\,\mathbf{a} + M_a^1 \mathbf{a}_1 / c^2.$$

Integration of *space* components of local PN eqs of motion gives the sought eqs for motion of PN mass center $\mathbf{a}_{(1)}$:

$$M_a\, \ddot{a}^{\,i} = \int_{\mathrm{D}_a} \rho\, U_{,\,i}\, \mathrm{d}V \quad \text{(order 0, = Newton)},$$

$$M_a^{\,1}\, \ddot{a}_1^{\,i} = \int_{\mathrm{D}_a} f_1^{\;i}\, \mathrm{d}V \quad \text{(order 1)},$$

The "PN force" density $f_1^{\;i}$ depends in partic. on zero-order fields: 0-order pressure p, density ρ, velocity $\mathbf{u}$, Newtonian potential U.

⇒ ***internal structure of the bodies influences the motion already from the first PNA.***

Follows naturally from using the "asymptotic" method of PNA. Should hold true for GR if asymptotic PNA were used.

4. *Point-Particle Limit & Reason for Conflict with WEP*

Framework for point-particle limit: (size of *one* body) = $\xi \to 0$,

thus a family (S^{ξ}) of *1PN systems* is considered.

(The eqs of the asymptotic 1PN approx. make a closed exact system.)

Initial data is independent of ξ apart from the size of the small body.

Result: a *structure-dependent* part of the acceleration, $\mathbf{A}_S = \mathrm{ord}(\xi^0)$,

remains at the point-particle limit $\xi \to 0$.

Static spherical case: limit differs from *test particle* just by $\mathbf{A}_S$:

$\mathbf{A}_{\mathrm{lim}} = \mathbf{A}_S +$ (1PN acceler^{n} of a test particle in static spherical field).

¡ Violation of the weak equivalence principle !

$\mathbf{A}_S = A_{S\,\max}\,\mathbf{w}$, with $|\mathbf{w}|$ strongly orientation-dependent, and

$A_{S\,\max} \approx 2\times10^{-7} d$ m/s^2 on Earth (d: density in g/cm^3).

Reasons: *general:* non-linearity of the theory! + asymptotic PNA

specific: presence in PN metric of derivatives $U_{,i}$ of Newt potent.

But the $U_{,i}$ also there *in Schwarzschild metric* ("anisotropy")

(by the way, Schw. is the exact solution of scalar theory for SSS case),

so what about GR?? Should depend on the gauge condition.

Conclusions

- A scalar theory with a preferred reference frame was summarized.

- To test that theory in celestial mechanics, an *"asymptotic"* PN scheme was developed.

- The resulting eqs for a self-gravitating system of extended bodies include ***internal-structure*** effects.

- The internal-structure influence subsists at the point-particle-limit *(Violation of the weak equivalence principle).*

- Using the "asymptotic" approximation scheme, this might possibly occur in GR also, in a gauge where the PN space metric would not be "conformally Euclidean".

Present work (point-particle limit): `gr-qc/0301031`.

Work on this scalar theory in general:

`http://geo.hmg.inpg.fr/arminjon/INTRO.html`