Testing a New Dark Matter Candidate that Emerges Naturally from a Scalar Theory of Gravitation

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ISGAC2023: International Summit on Gravitation, Astrophysics and Cosmology, Rome, March 6-7, 2023

Motivation

Formerly proposed an *alternative, relativistic theory of gravity with a preferred frame, based on a scalar field only:* "scalar ether theory" or **SET**: see (MA, Braz. J. Phys. **36**, 177 (2006)) and Refs. therein.

Initial aim of present research: to describe electromagnetism in the presence of gravity for that theory.

The eqs. of electrodynamics of general relativity (GR) rewrite those of special relativity (SR) by using the "comma goes to semicolon" rule: $_{,\nu} \rightarrow {}_{;\nu}$ (partial derivatives → covariant derivatives).

Not possible in SET, for the Dynamical Equation isn't generally $T^{\lambda\nu}_{;\nu} = 0$ (which rewrites $T^{\lambda\nu}_{,\nu} = 0$ valid in SR).

Electrodynamics in the presence of gravity in SET

In SET, first Maxwell group unchanged. Second group was got by applying the Dynamical Eqn of SET to a charged medium in the presence of Lorentz force, assuming that (as is the case in GR):

(A) Total energy tensor $\boldsymbol{T} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}}$.

The additivity (A) leads to a form of Maxwell's 2nd group in SET (MA, Open Phys. **14**, 395 (2016), or Proc. IARD 2016: J. Phys. Conf. Ser. **845**, 012014 (2017)).

But that form of Maxwell's 2nd group in SET predicts charge production/destruction at untenable rates ⇒ *discarded* (MA, Open Phys. **15**, 877 (2017)).

Necessity of the interaction tensor in SET

The additivity assumption (A) is contingent and may be abandoned.

Means introducing "interaction" energy tensor $\boldsymbol{T}_{\text{inter}}$ such that

$$\boldsymbol{T}_{\text{(total)}} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}} \underline{+\boldsymbol{T}_{\text{inter}}}\,. \tag{1}$$

One then has to constrain the form of $\boldsymbol{T}_{\text{inter}}$ and derive eqs for it.

Form of the interaction tensor

In SR, the additivity assumption (A) holds, thus $\boldsymbol{T}_{\text{inter}} = 0$.

In SET we may impose that $\boldsymbol{T}_{\text{inter}}$ should be Lorentz-invariant in the situation of SR, i.e. when the metric $\boldsymbol{\gamma}$ is Minkowski's metric $\boldsymbol{\gamma}^0$ ($\gamma^0_{\mu\nu} = \eta_{\mu\nu}$ in Cartesian coordinates).

This leads uniquely to the following definition:

$$T^{\mu}_{\text{inter}\ \ \nu} := p\,\delta^{\mu}_{\nu}, \qquad \text{or} \quad (T_{\text{inter}})^{\mu\nu} := p\,\gamma^{\mu\nu}, \tag{2}$$

with some scalar field p. (MA, J. Geom. Sym. Phys. **50**, 1–10 (2018); MA, Open Phys. **16**, 488 (2018))

INTERACTION ENERGY

Corresponding interaction energy: $E_{\text{inter}} := T^{00}_{\text{inter}} = p\gamma^{00}$.

The medium with energy tensor $(T_{\text{inter}})^{\mu\nu} := p\,\gamma^{\mu\nu}$ can be counted as "dark matter", because:

- it isn't localized inside usual matter: $p \neq 0$ at a generic point;
- it's gravitationally active: $T^{00} =$ source of grav. field in SET;
- it is not usual matter (e.g. no velocity can be defined).

Equation for the scalar field p (1)

With the interaction energy tensor (2) we have just one unknown more: the scalar field p. So we need just one scalar eqn more.

We may add *charge conservation* as the new scalar eqn. Then the system of eqs of electrodynamics of SET is again closed, and satisfies charge conservation.

Based on that closed system, eqs. were derived that, *in principle*, determine the field p in a given general EM field $(\mathbf{E}, \mathbf{B})$ and in a given weak gravitational field with Newtonian potential U (MA, Open Phys. **16**, 488 (2018)).

EQUATION FOR THE SCALAR FIELD p (2)

Main eqn is:

$$\mathrm{div}_4\left(\boldsymbol{G}.\nabla_4 p\right) := \left(G^{\mu\nu}\, p_{,\nu}\right)_{,\mu} = f. \tag{3}$$

$G^{\mu\nu}$: the components of antisymmetric spacetime tensor $\boldsymbol{G}$: inverse tensor of EM field tensor of the first approximation, that obeys the flat-spacetime Maxwell equations.
In addition, in Eq. (3), we have

$$f := \left(d^i \partial_T U\right)_{,i}. \tag{4}$$

d^i $(i = 1, 2, 3)$: components of a spatial vector d made with (E, B).

Maxwell model of the ISRF: Main ideas

To check if E_{inter} might build a "dark halo", we must have the Interstellar Radiation Field in a galaxy (ISRF) as a Maxwell field.

NB: Existing models of the ISRF don't do that.

Axial symmetry: relevant approximation for many galaxies.

Describe ISRF at galactic scale, not in the stars $\Rightarrow$ *source-free* Maxwell eqs.

Consider a *finite set of frequencies* (ω_j) $(j = 1, ..., N_\omega)$.

Found an *explicit representation* for totally-propagating axisymmetric free Maxwell fields (MA: Open Phys. **18**, 255 (2020)).

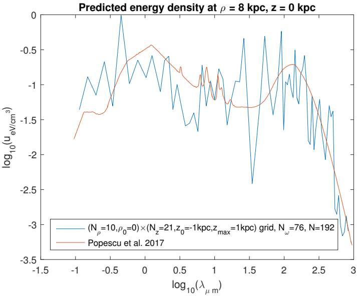
Maxwell model of the ISRF: Model of a galaxy

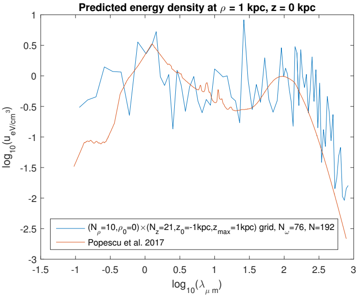
Axisymmetric galaxy $\leftrightarrow$ Finite set $\{\mathrm{x}_i\, ;\ i = 1, ... i_{\mathrm{max}}\}$ of point-like "stars", the azimuthal distribution of which is uniform.

Obtained by pseudo-random generation of their cylindrical coordinates ρ, ϕ, z with specific probability laws, ensuring axisymmetry & representativity.

The explicit representation is adjusted by fitting spherical scalar radiations emanating from each of those "stars". (MA: Open Phys. **19**, 77 (2021))

Then model adjusted so that SED (spectral energy density) at our local position ($\rho_{\mathrm{loc}} = 8\,\mathrm{kpc}$, $z_{\mathrm{loc}} = 0.02\,\mathrm{kpc}$) coincide with measured SED. (MA, Adv. in Astron. **2021**, 5524600 (2021))


Predicted energy density at ρ = 8 kpc, z = 0 kpc
$\log_{10}(u_{eV/cm^3})$
$\log_{10}(\lambda_{\mu m})$
(N_ρ=10,ρ_0=0)×(N_z=21,z_0=-1kpc,z_{max}=1kpc) grid, N_ω=76, N=192
Popescu et al. 2017


Predicted energy density at ρ = 1 kpc, z = 0 kpc
$\log_{10}(u_{eV/cm^3})$
$\log_{10}(\lambda_{\mu m})$
(N_ρ=10,ρ_0=0)×(N_z=21,z_0=-1kpc,z_{max}=1kpc) grid, N_ω=76, N=192
Popescu et al. 2017

NEED TO USE HOMOGENIZATION THEORY

Need to integrate *on a galactic scale* $r \sim 10^{19}\text{m}$ the PDE (3) for the scalar field p.

But: given fields $\boldsymbol{G}$ and f in (3) vary on scale $r \sim \lambda \simeq 10^{-6}\text{m}$ and $t \sim \lambda/c$, like E and B. No chance to succeed in the integration!

Situation typical of *homogenization theory*.

Aim of that theory: to get "homogenized" PDEs allowing one to describe at the macroscopic scale the medium, assumed periodic or quasi-periodic at a microscopic scale.

For Eq. (3), the "medium" is characterized by the pair of given heterogeneous fields $(\boldsymbol{G}, f)$.

A BIT MORE ON HOMOGENIZATION THEORY

Considers two spacetime variables related by a small parameter $\epsilon \ll 1$:

- slow variable $\mathrm{X} = (t, \mathrm{x})$: browses medium at macroscopic scale
- quick variable, $\mathrm{Y} = \mathrm{X}/\epsilon$: an $O(1)$ variation of it browses the quasi-period of the medium.

Fields are stated to be functions of X *and* $\frac{\mathrm{X}}{\epsilon}$, *periodic* or quasi-periodic w.r.t. $\frac{\mathrm{X}}{\epsilon}$.

Asymptotic expansions are stated, e.g.

$$p^{\epsilon}(\mathrm{X}) = p_0\,(\mathrm{X}, \mathrm{Y})\,\epsilon^0 + p_1\,(\mathrm{X}, \mathrm{Y})\,\epsilon + O(\epsilon^2), \qquad \mathrm{Y} = \frac{\mathrm{X}}{\epsilon} \tag{5}$$

Way to use homogenization theory

The PDE (3) for p has just the same form as the *stationary heat conduction equation for the temperature* θ, except that here we have 4-d *spacetime* instead of 3-d space $\Rightarrow$ May adapt known results: Caillerie, summer school Quiberon 2012.

Main result: homogenized PDE has same form as (3), replacing $\boldsymbol{G}$ by a “homogenized” tensor $\boldsymbol{G}^{\mathrm{H}}$. However, $\boldsymbol{G}^{\mathrm{H}}$ is *not* the local spacetime average of “microscopic” tensor $\boldsymbol{G}$:

$\boldsymbol{G}^{\mathrm{H}}$ obtained by solving a boundary value problem on a local microscopic cell for the linear first-order PDE

$$k^\mu \chi^\nu{}_{,\mu} = -k^\nu \quad (\nu = 0, ..., 3), \qquad k^\nu := G^{\mu\nu}{}_{,\mu}. \tag{6}$$

To be solved by finite element method.

Conclusion

In the alternative gravity theory "SET", electromagnetism in the presence of gravitation demands to introduce an additional energy tensor $\boldsymbol{T}_{\text{inter}}$, depending on a scalar field p.

This exotic energy tensor might contribute to dark matter.

To check this, we developed a model of the ISRF that provides it as an exact Maxwell field.

Then we may calculate the fields $\boldsymbol{G}$ (inverse EM tensor) and f that determine p and $\boldsymbol{T}_{\text{inter}}$ through the PDE (3).

But the very quick variation of $\boldsymbol{G}$ and f prevents integration of (3) on a galactic scale.

Need to use homogenization theory. The PDE stays unchanged but with a homogenized tensor $\boldsymbol{G}^{\text{H}}$.