Testing a dark matter candidate emerging from the scalar ether theory

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MOTIVATION AND OUTLINE

According to an alternative, relativistic theory of gravity with a preferred frame, based on a scalar field only ("scalar ether theory" or **SET**, see (MA, Braz. J. Phys. **36**, 177 (2006))):

In the presence of both a gravitational field & an electromagnetic (EM) field, there must appear some exotic "interaction energy", which should be distributed in space, and be gravitationally active (MA, Open Phys. **16**, 488 (2018)).

That energy E_{inter} could thus possibly contribute to the dark matter. Depends on the EM field (E, B) and the gravity field.

$\Rightarrow$ To check if E_{inter} might indeed build a "dark halo" around a galaxy, we must have the Interstellar Radiation Field (hereafter **ISRF**) as a solution of the Maxwell equations.

Motivation and outline (continued)

The existing models for the ISRF focus on radiation transfer and do not produce an EM field $(\mathbf{E}, \mathbf{B})$, even less an exact solution of the Maxwell eqs — as is yet needed to check E_{inter} as a dark matter candidate.

Hence we built from scratch a model that does this. Based on axial symmetry (of the galaxy and the ISRF) as a relevant approximation (MA: Open Phys. **18**, 255 (2020) and **19**, 77 (2021)).

That model makes predictions for the Spectral Energy Density (SED) of the ISRF, that are close to those of the existing models (MA, Adv. in Astron. **2021**, 5524600 (2021)). Except that the new model predicts extremely high values of the SED on the galaxy's axis (MA, Preprint HAL 03341905 (2021)).

Motivation and outline (end)

Current work: Using the axisymmetric model of the ISRF in a galaxy as an exact Maxwell EM field, just mentioned, we seek to compute the interaction energy density field E_{inter} in the weak gravitational field of a galaxy.

Beyond the EM field, this depends also on the Newtonian potential U, more precisely on $\partial_t(\nabla U)$. The time derivative ∂_t is taken in the preferred frame ("ether") $\mathcal{E}$. Hence the field E_{inter} will depend on the velocity V of the center of the galaxy w.r.t. $\mathcal{E}$.

Electrodynamics in the presence of gravity in SET

The eqs. of electrodynamics of general relativity (GR) rewrite those of special relativity (SR) by using the "comma goes to semicolon" rule: $_{,\nu} \ \rightarrow \ _{;\nu}$

Not possible in SET, for the Dynamical Equation isn't generally $T^{\lambda\nu}_{\ \ ;\nu} = 0$ (which rewrites $T^{\lambda\nu}_{\ \ ,\nu} = 0$ valid in SR).

In SET, first Maxwell group unchanged. Second group was got by applying the Dynamical Eqn of SET to a charged medium in the presence of Lorentz force, assuming that (as is the case in GR):

(A) Total energy tensor $\boldsymbol{T} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}}$.

Necessity of the interaction tensor in SET

The additivity (A) leads to a form of Maxwell's 2nd group in SET (MA, Open Phys. **14** (2016), 395, or Proc. IARD 2016).
But that form of Maxwell's 2nd group in SET predicts charge production/destruction at untenable rates $\Rightarrow$ *discarded* (MA, Open Phys. **15** (2017), 877).

The additivity assumption (A) is contingent and may be abandoned. Means introducing "interaction" energy tensor $\boldsymbol{T}_{\text{inter}}$ such that

$$\boldsymbol{T}_{\text{(total)}} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}} \underline{+\boldsymbol{T}_{\text{inter}}}\,. \qquad (1)$$

One then has to constrain the form of $\boldsymbol{T}_{\text{inter}}$ and derive eqs for it.

Form of the interaction tensor (1)

In SR, the additivity assumption (A) holds, thus $\boldsymbol{T}_{\text{inter}} = 0$.

In SET we may impose that $\boldsymbol{T}_{\text{inter}}$ should be Lorentz-invariant in the situation of SR, i.e. when the metric $\boldsymbol{\gamma}$ is Minkowski's metric $\boldsymbol{\gamma}^0$ ($\gamma^0_{\mu\nu} = \eta_{\mu\nu}$ in Cartesian coordinates).

This is true if *and only if* we have:

$$T_{\text{inter}\ \mu\nu} = p\,\gamma^0_{\mu\nu} \qquad \text{(situation of SR)}, \tag{2}$$

with some scalar field p. (MA, J. Geom. Sym. Phys. **50** (2018), 1–10)

Form of the interaction tensor (2)

Thus, demanding that $\boldsymbol{T}_{\text{inter}}$ be Lorentz-invariant in the situation of SR leads to the following definition:

$$T_{\text{inter}}{}^{\mu}{}_{\nu} := p\,\delta^{\mu}_{\nu}, \qquad \text{or} \qquad (T_{\text{inter}})_{\mu\nu} := p\,\gamma_{\mu\nu}, \tag{3}$$

which is generally-covariant and is hence adopted in general.

With the interaction energy tensor (3) we have just one unknown more: the scalar field p. So we need just one scalar eqn more.

We may add *charge conservation* as the new scalar eqn. Then the system of eqs of electrodynamics of SET is again closed, and satisfies charge conservation.

Equation for the scalar field p

Based on that closed system, eqs. were derived that determine the field p in a given general EM field $(\mathbf{E}, \mathbf{B})$ and in a given weak gravitational field with Newtonian potential U (MA, Open Phys. **16** (2018), 488). Main eqn is

$$\left(\frac{\mathrm{d}\, p}{\mathrm{d}\, t}\right)_{\mathbf{u}} := \frac{\partial p}{\partial t} + \mathbf{u}.\nabla p = S. \tag{4}$$

where the source field S and the vector field $\mathbf{u}$ are given. Thus p obtains by integrating S along the curves $\frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}\, t} = \mathbf{u}$.

The corresponding interaction energy $E_{\text{inter}} := T^{00}_{\text{inter}} = p\gamma^{00}$ can be counted as (macroscopic) "dark matter", for

- it isn't localized inside usual matter: $p \neq 0$ at a generic point;
- it's gravitationally active: $T^{00} =$ source of grav. field in SET;
- it is not usual matter (e.g. no velocity can be defined).

Maxwell model of the ISRF: Main assumptions

To check if E_{inter} might build a "dark halo", we must have the Interstellar Radiation Field in a galaxy (ISRF) as a Maxwell field.

Axial symmetry relevant approximation for many galaxies. (z axis)

Primary source of the ISRF: the stars. We want to describe ISRF at galactic scale, not in the stars or in their neighborhood
$\Rightarrow$ *source-free* Maxwell eqs.

Theorem: any time-harmonic axisymmetric source-free Maxwell field is the sum of two Maxwell fields:

- **1**) one deriving from vector potential A having just $A_z \neq 0$;
- **2**) one deduced from a field of the form (**1**) by EM duality

(MA, Open Physics **18** (2020), 255–263).

Maxwell model of the ISRF: Form of the model

We consider a finite set of frequencies (ω_j) $(j = 1, ..., N_\omega)$. Using the Theorem above, the EM field is generated by potentials A_{jz}, A'_{jz}: A_{jz} for a solution of the form (**1**), A'_{jz} for a dual solution (**2**).

In the relevant "totally propagating" case, the potential A_{jz} for frequency ω_j is given explicitly in terms of a spectrum function $S_j(k)$, with $-\frac{\omega_j}{c} \leq k \leq \frac{\omega_j}{c}$. Thus $A_{jz} = \psi_{\omega_j\, S_j}$, $A'_{jz} = \psi_{\omega_j\, S'_j}$.

Maxwell model of the ISRF: Model of a galaxy

Axisymmetric galaxy $\leftrightarrow$ Finite set $\{\mathrm{x}_i\ ;\ i = 1, ... i_{\mathrm{max}}\}$ of point-like "stars", the azimuthal distribution of which is uniform.

Obtained by pseudo-random generation of their cylindrical coordinates ρ, ϕ, z with specific probability laws, ensuring axisymmetry & representativity.

Maxwell model of the ISRF: Determining the potentials

For each frequency ω_j, we consider a sum of spherical potentials:

$$\Sigma_j := \sum_{i=1}^{i_{\text{max}}} \varphi_{\mathrm{x}_i\, \omega_j}. \tag{5}$$

Each potential $\varphi_{\mathrm{x}_i\, \omega_j}$ emanates from the star at point x_i.

We fit the sum Σ_j by the unknown potential $A_{jz} = \psi_{\omega_j\, S_j}$.

This determines the spectrum function $S_j(k)$.
Done for $j = 1, ..., N_\omega$.

Note: the uniqueness of $\varphi_{\mathrm{x}_i\, \omega_j}$ leads us to assume $A'_{jz} = A_{jz}$.

EXPRESSIONS OF VECTOR FIELD u AND SOURCE S

Setting $P := \mathbf{E}.\mathbf{B}$, we get easily from (MA, Open Phys. **16** (2018), 488):

$$\mathbf{u} = \frac{\mathbf{E} \wedge \nabla P - (\partial_t P)\mathbf{B}}{\mathbf{B}.\nabla P}, \tag{6}$$

$$S = \frac{\mathrm{div}\,(\mathbf{e}\,\partial_t U)}{c^3 \mathbf{B}.\nabla \frac{1}{P}}, \tag{7}$$

where the spatial (3-)vector $\mathbf{e}$ is easily shown to be

$$\mathbf{e} = \frac{1}{c\mu_0}\left(\mathbf{E} + \frac{c^2\mathbf{B}^2 - \mathbf{E}^2}{2P}\mathbf{B}\right). \tag{8}$$

Since $A'_{jz} = A_{jz}$ for the model EM field, it follows that here $c^2\mathbf{B}^2 - \mathbf{E}^2 = 0$. Thus $\mathbf{e} = \mathbf{E}/(c\mu_0)$.

Free Maxwell eqs $\Rightarrow$ $\mathrm{div}\,\mathbf{E} = 0$, so numerator of (7) $\propto \mathbf{E}.\nabla(\partial_t U)$.

Time homogenization

u and S vary very quickly with time, like E and B. No chance to integrate on a galactic scale the PDE (4) for the scalar field p!

Use time-homogenization technique as proposed by Guennouni (Math. Model. Num. Anal. **22**, 417 (1988)).

Idea of the method: two "separated" time scales: a "quick" time τ and a "slow" time t, with $\tau = t/T$, where $T =$ typical period of the quick variation (here that of the EM radiation field).

Thus $T \ll t$ — here the galactic time scale: $t \simeq r/c$ with r a galactic distance $\Rightarrow t/T \simeq r/\lambda = O(10^{25})$...

Time homogenization (continued)

Formally, one assumes that the data u and S, as well as the boundary values for p, have the form $\mathrm{u} = \mathrm{u}(\mathrm{x},\ t,\ \tau) = \lambda(t,\tau)\mathrm{u}^*(\mathrm{x})$,..., where λ is τ-periodic of period 1 (since, when $\tau = t/T$, we have $\tau = 1$ for $t = T$).

Setting $\mathrm{u}^T(\mathrm{x}, t) := \mathrm{u}\left(\mathrm{x},\ t,\ \frac{t}{T}\right)$, ..., one has a boundary value problem Π^T for p, depending smoothly on T. Its solution field $p^T(\mathrm{x}, t)$ defines a field depending on two time variables:

$$p\left(\mathrm{x},\ t,\ \tau\right) := p^T(\mathrm{x}, t) \quad \text{with } T = \frac{t}{\tau}. \tag{9}$$

The total time derivative is

$$\frac{\mathrm{d}\, p^T}{\mathrm{d}\, t} = \frac{\partial}{\partial t}\left(p\left(\mathrm{x}, t, \frac{t}{T}\right)\right) = \frac{\partial p}{\partial t} + \frac{1}{T}\frac{\partial p}{\partial \tau}. \tag{10}$$

Time homogenization (end)

One states asymptotic expansions as $T \to 0$:

$$p^T(\mathrm{x}, t) = p_0(\mathrm{x}, t, \tau) + p_1(\mathrm{x}, t, \tau) T + O(T^2) \quad \left(\tau = \frac{t}{T}\right), \quad (11)$$

where p_0 and p_1 are τ-periodic of period 1. Using this and (10) in (4) and identifying powers, we get

$$\frac{\partial p_0}{\partial \tau} = 0 \ i.e. \ p_0 = p_0(\mathrm{x}, t), \quad \frac{\partial p_0}{\partial t} + \frac{\partial p_1}{\partial \tau} + \mathrm{u}.\nabla p_0 = S. \quad (12)$$

Averaging the last eqn over the period T:
$\bar{f}(\mathrm{x}, t) := \int_0^1 f(\mathrm{x}, t, \tau)\, \mathrm{d}\, \tau$, yields

$$\frac{\partial p_0}{\partial t} + \bar{\mathrm{u}}.\nabla p_0 = \bar{S}, \quad (13)$$

the sought-for time-averaged eqn. Note: from $(12)_1$, $p_0 = \bar{p}$.

TIME-AVERAGING u AND S

The Maxwell model of the ISRF provides E and B in the form

$$F^{(q)}(t,\mathrm{x}) = \mathcal{R}e\left(\sum_{j=1}^{N_\omega} C_j^{(q)}(\mathrm{x})e^{-\,\mathrm{i}\,\omega_j t}\right) \qquad (q=1,...,6). \qquad (14)$$

Then, $P := \mathrm{E.B}$ expands on the $e^{-\,\mathrm{i}(\omega_j+\omega_k)t}$'s and $e^{-\,\mathrm{i}(\omega_j-\omega_k)t}$'s. Hence, $\mathrm{E}\wedge\nabla P$, $(\partial_t P)\mathrm{B}$, and $\mathrm{B}.\nabla P$, which enter (6) and (7), expand on the $e^{-\,\mathrm{i}\,\Omega_\kappa t}$'s and $e^{-\,\mathrm{i}\,\Psi_\kappa t}$'s, with
$\kappa = (j,k,m), \quad \Omega_\kappa = \omega_j+\omega_k+\omega_m, \quad \Psi_\kappa = \omega_j+\omega_k-\omega_m.$

Those 3 fields time-average to zero ($\omega_j+\omega_k-\omega_m \neq 0$). But Eqs. (6) and (7) for u and S involve *ratios* of these fields. How to compute the time averages of such ratios?

TIME-AVERAGING A RATIO OF TRIGONOMETRIC POLYNOMIALS

Consider

$$Q(t) = \frac{\mathcal{R}e\left(\sum_j C_j e^{-\mathrm{i}\,\omega_j t}\right)}{\mathcal{R}e\left(\sum_k D_k e^{-\mathrm{i}\,\omega_k t}\right)} = \frac{\sum_j C_j e^{-\mathrm{i}\,\omega_j t} + C_j^\star e^{\mathrm{i}\,\omega_j t}}{\sum_k D_k e^{-\mathrm{i}\,\omega_k t} + D_k^\star e^{\mathrm{i}\,\omega_k t}}$$

$$= \sum_j \frac{1}{\sum_k \frac{D_k}{C_j} e^{-\mathrm{i}(\omega_k - \omega_j)t} + \frac{D_k^\star}{C_j} e^{\mathrm{i}(\omega_k + \omega_j)t}} + \frac{1}{\sum_k \frac{D_k}{C_j^\star} e^{-\mathrm{i}(\omega_k + \omega_j)t} + \frac{D_k^\star}{C_j^\star} e^{\mathrm{i}(\omega_k - \omega_j)t}}. \tag{15}$$

We compute that, to an often quite good approximation,

$$\left\langle \frac{1}{a + \sum_\kappa b_\kappa e^{-\mathrm{i}\,\Omega_\kappa t}} \right\rangle \simeq \frac{1}{a}. \tag{16}$$

With this approximation, we get

$$\overline{Q} \simeq \sum_j \frac{1}{\frac{D_j}{C_j}} + \frac{1}{\frac{D_j^\star}{C_j^\star}} = 2 \sum_j \mathcal{R}e\left(\frac{C_j}{D_j}\right). \tag{17}$$

Conclusion

In the alternative gravity theory "SET", electromagnetism in the presence of gravitation demands to introduce an additional energy tensor $\boldsymbol{T}_{\text{inter}}$, depending on a scalar field p.

This exotic energy tensor might contribute to dark matter.

To check this, we developed a model of the ISRF that provides it as an exact Maxwell field.

Then we may calculate the fields u and S that determine p and $\boldsymbol{T}_{\text{inter}}$ through the PDE (4). But the very quick variation of u and S prevents from integrating (4) on a galactic scale. We use a time-homogenization technique. The PDE stays unchanged but with time-averaged fields.

Currently computing the time-averaged fields $\bar{\mathrm{u}}$ and $\bar{S}$.