A THEORY OF GRAVITY AS A PRESSURE FORCE

I. NEWTONIAN SPACE AND TIME

par M. ARMINJON *

Gravity is interpreted as the pressure force exerted on matter at the scale of elementary particles by a perfect fluid, the rest frame of which defines the inertial frame. The first task is thus to extend newtonian mechanics so that it allows a deformation of the inertial frame. An application to the stability study of an expanding, rigid or contracting universe is given. The pressure of the inertial fluid is equivalent to a mass force, if the elementary particles have the same (average) mass density, depending only on the fluid pressure. Hence stable particles should be permanent flows in the fluid, such as vortices. Gravity should be only the macroscopic part of the pressure force. Newtonian gravity propagates with infinite speed and is interpreted as the incompressible case. In the compressible case, gravitational (pressure) waves are predicted as well as qualitatively correct modifications to planetary motion. In a forthcoming paper the theory is reinterpreted so as to describe the relativistic effects.

1. INTRODUCTION

Through the assumed existence of inertial frames, Newton's mechanics and Newton's gravitation theory need absolute space, without saying where it could come from. On the other hand, general relativity introduces an absolute *space-time* by its unique, covariantly defined metric [1], and seems to deny absolute *space*-but it needs Newton's theory: not only are Einstein's equations derived under the requirement that Newton's theory must be recovered for weak fields and low velocities, but also the solution of the simplest problem (the motion of a test particle in the gravitational field of a spherical massive body) is interpreted as a correction to the newtonian solution [2]—[4]. Consistently with general relativity, but differing from usual newtonian theory, the inertial frames are then described as purely local: they are "freely falling" [4]—[6]. Newton's first and second laws can indeed be formulated in a freely falling frame, but the actio-reactio principle requires a global inertial frame (the same for each pair of interacting particles), and the newtonian analysis of a motion with several bodies implies the validity of this principle for the gravity interaction. In the newtonian analysis of the solar system as a motion with several bodies the common inertial frame is thus not only the frame in which the mass center of this system falls freely with respect to the distant stars. However, the relativistic

* Fédération de Mécanique de Grenoble, Laboratoire «Sols, Solides, Structures», B.P. 53 X, F—38041 Grenoble Cedex, France

Rev. Roum. Sci. Techn. — Méc. Appl., Tome 38, N° 1, P. 3—24, Bucarest, 1993

calculation of the motion in a central gravitational field is used to refine this analysis. In summary, the nature of the inertial frames as they appear in newtonian theory is obscure in that theory, and still not clear in general relativity.

Another question which passes unsolved from classical to relativistic gravitation theory is the meaning of action at a distance, in so far as the field which is supposed to play the role of a propagator is not a physical substratum but an abstraction : in this regard, the change from classical to relativistic theory in essence is a substitution of a space-time tensor for a space vector. Of course, the modern theory is superior in that the interaction at least propagates with finite velocity. Recently, a theory of gravity with finite wave speed has been proposed by a mechanicist in a neo-classical framework [7]; as observed by Mazilu, a finite velocity of propagation is compatible with action at a distance only if the actio-reactio principle is modified so that the relation between action and reaction is affected by some inertia. With such a modification, however, it becomes desirable that newtonian mechanics should be modified accordingly.

A third reason may encourage mechanicists to attempt the construction of another gravitation theory, closer to the newtonian one : perhaps a theory starting from a more classical point of view could throw some new light on the question of gravitational collapse, which in general relativity leads to curious catastrophes — due to the inevitable singularities which appear there [4], [8]. Whereas in classical physics singularities only mark the limits of an approximate model, general relativity is forced to give to some of them the status of a physical reality, in some crucial cases the theory simply cannot remove them. Would any sensible theory of gravitation lead to such "physical singularities"?

In this spirit, a tentative new version of the old theory of ether semms to be permissible, along the following lines. If mechanics must be first formulated in the rest frame of a postulated fluid substratum, this will be a natural inertial frame ; if gravity is interpreted simply as Archimedes's thrust in the fluid substratum, there will be no action at a distance ; and if this can work, it is unlikely that endless implosion can occur in such a fluid, since *its* pressure will obviously *decrease* in the direction of attraction. A preliminary investigation will be to make newtonian mechanics compatible with the notion of a *fluid* inertial frame ; as an application, the stability of an expanding, rigid or contracting universe with nil gravity will be examined. Before doing so, since the idea of a fluid implies some deformability, some words need to be said on how to interpret the newtonian notion of space, independently of the substratum. Hereafter, a simple analysis will show that our postulated ether must be *constitutive* at the finest scale, i.e. the particles of matter themselves must be ether — perhaps permanent vortices in this perfect fluid, as was envisaged by von Helmholtz and Kelvin, and was more recently described by Romani [9] with some degree of detail on today's elementary particles. In his ambitious and inventive (though not very rigorous) work, Romani introduced the ether as a linearly compressible fluid whose "sound" velocity is that of light, c, but did not propose a definite theory of gravitation, while making some interesting suggestions: e.g. the gravity would be a density gradient in the ether, and light deflection, caused by an *increase* of ether density towards the massive body (the contrary of what we find), would obey the Fermat principle. As will be shown, with an incompressible constitutive ether (and thus without any density gradient), Newton's gravitation receives a satisfying interpretation in terms of pressure forces — more precisely those forces which come from the macroscopic part of the pressure gradient. With a compressible ether however, a new gravitation theory can already be obtained within purely classical concepts of space and time, and the corrections it makes to the newtonian theory for the static spherical problem, have a form which is similar to those of general relativity.

2. EUCLIDEAN OR RIEMANNIAN SPACE?

Essential to the newtonian theory is the notion of solid reference frame and thus of a *3-D* manifold M with a time-invariant metric $\mathfrak{g}^{\circ}$, but of less importance that such solids may exist only approximately as physical bodies (and rather small ones). The classical conception is also prepared, and this already since the work of Newton [10], to accept that the "common" distances and times, i.e. the measured ones, do not automatically coincide with the "true" ones, i.e. the distance evaluated with $\mathfrak{g}^{\circ}$ and the intervals of the absolute time t. As long as kinematics alone are discussed, only the absolute simultaneity is of importance, in the sense that t can be replaced by any increasing function of it. Let us note that relativistic cosmology must also introduce a "cosmic time" [4] without which, among other things, any speculation on the age of the universe would be a nonsense; and that in general relativity the cosmological assumptions are of importance also for "local" questions, since they determine the gross structure of the space-time curvature.

The space M can first be represented with only a differentiable structure, as a fictitious body which at any time contains all physical objects. Thus M may be replaced by a body of the same kind, the motion of which (relative to M) is given in the lagrangian description by a time-dependent diffeomorphism ψ_t of M onto itself. Mathematically, this second body is defined as the set M' of the trajectories, i.e. an element of M' is the mapping $\theta(X): t \rightarrow \psi_t(X)$ for a given X in M, and the manifold structure of M' is obtained by the transport of that of M by the one-to-one correspondence θ. This means that if we think of M as being a *deformable* body in a riemannian space (the euclidean space, for example ...), we can redress this inappropriate choice and take as the reference manifold a body which is rigid with respect to the metric. In other words, the physical space is supposed to be a manifold M with a natural metric $\mathfrak{g}^{\circ}$ and a large group G of isometries, but this natural metric might be difficult to catch at the global scale. On the other hand, it is assumed that locally and at a large distance from perturbations, the natural metric is the physical one: that one which is measured with light rays and atomic clocks. Now the metric $\mathfrak{g}^{\circ}$ can be qualified as absolute, but this is not at all true of the reference solid M, which may be replaced by M' as above, yet this time with $\psi_t \in G$. This is a very different situation from that of general relativity where the 4-D space-time manifold M^4 is equipped only with the physical space-time metric γ which can hardly be assumed to admit non-trivial isometries and which makes thus M^4 absolute (unique): in particular, when a cosmic time t may be defined, the 3-D surfaces $t = \text{const.}$ could be qualified as absolute, somehow reminding the ether. The fact that no isometry can be assumed in general relativity (except as an approximation introduced by a simplified model of our universe), is the cause of the serious difficulties in the formulation of conservation laws, as pointed out by Trautman [11]. This does not concern the mass conservation because the continuity equation may be derived for an arbitrary riemannian manifold, and moreover special relativity already abandons the general mass conservation of classical mechanics. It is interesting to note, however, that a continuity equation in the sense of distributions has been proved by Moreau [12] in the sparer mathematical context of two differentiable manifolds in relative motion with a time-independent measure (e.g. the mass) given on the "moving" one.

In what follows it is assumed that the reference manifold M (and hence any other possible reference M') is diffeomorphic to $\mathbb{R}^3$: this has the status of a pleasant working assumption, which seems rather difficult to invalidate (nevertheless, if this happened, the classical conception would remain tractable: riemannian geometry also is a bit simpler in three dimensions that in four). Thus M may be equipped with the euclidean metric and the euclidean displacement group (translations plus orthogonal transformations), and we suppose further that M could, and has indeed been chosen such that the euclidean metric coincides with the physical one in undisturbed regions, as explained above. The usual conventions of elementary mechanics are adopted: the position of a point in physical space is a point $P \in M$ or a vector $\mathbf{x} \in E$, E being the vector space to which M is identified once an origin $O \in M$ has been selected. A solid reference frame $\mathcal{R}$ (or a fluid one $\mathcal{F}$) is a time-dependent isometry η_t (or diffeomorphism ψ_t) of M onto itself.

3. NEWTONIAN MECHANICS WITH FLUID INERTIAL FRAMES

3.1. NEWTON'S SECOND LAW WITHOUT INERTIAL FRAME, AND THE EQUIVALENCE PRINCIPLE

Everyone knows that Newton's second law may be written in any solid reference frame $\mathcal{R}$, simply by adding the "fictitious" inertial forces, $\mathbf{f}_{i\mathcal{R}}$, per unit mass:

$$\mathbf{F}_{\mathcal{R}} = m\mathbf{a}_{\mathcal{R}}, \ \mathbf{F}_{\mathcal{R}} = \mathbf{F}_{\mathcal{R}0} + m\, \mathbf{f}_{i\mathcal{R}}, \ \mathbf{f}_{i\mathcal{R}} = \mathbf{a}_{\mathcal{R}} - \mathbf{a}_{\mathcal{R}0} \tag{1}$$

where m is the mass of the test particle, $\mathbf{a}_{\mathcal{R}}$ is the acceleration relative to the frame $\mathcal{R}$ and $\mathcal{R}_0$ is an inertial frame. Also, if forces are phenomenologically defined, i.e. by their effects, the inertial forces (e.g. the centrifugal force in a merry-go-round) have clearly the same reality as the others. They are only fictitious for an "inertial" observer, but this notion is obscure precisely from a rigorous phenomenological point of view. The quest of inertial frames leads naturally to enlarge the observation scale from the laboratory to the Earth, the Sun, stars, galaxies... [13] but if the universe is really expanding, at a still larger scale our sample of universe must have a high acceleration with respect to other, very distant samples : in that case what will people there think of our inertial frames ?

It is possible to start with a newtonian theory without assuming that inertial frames do exist, nor even that the inertial and (passive) gravitational masses $m^{(i)}$ and $m^{(p)}$ coincide, simply by modifying (1) so as to obtain the following covariant from of Newton's second law :

$$\mathbf{F}_{\mathcal{R}} = m^{(i)}\,\mathbf{a}_{\mathcal{R}},\ \mathbf{F}_{\mathcal{R}} = \mathbf{F}_0 + m^{(p)}\,\mathbf{f}_{\mathcal{R}}, \tag{2}$$

in which $\mathbf{F}_0$ is an invariant part of the force (in the macroscopic world of usual newtonian theory, this would always be a force of an electromagnetic origin) and $\mathbf{f}_{\mathcal{R}}$ is the "mass force field", a function only of the position $\mathbf{x}$ of any test particle *and* of its relative velocity $\mathbf{v}_{\mathcal{R}}$. The requirement that this relation holds true in any solid frame $\mathcal{R}$ with the same values $m^{(i)}$, $m^{(p)}$ and $\mathbf{F}_0$ is equivalent to ask that :

$$m^{(p)}(\mathbf{f}_{\mathcal{R}'} - \mathbf{f}_{\mathcal{R}}) = m^{(i)}(\mathbf{a}_{\mathcal{R}'} - \mathbf{a}_{\mathcal{R}})\ \ (\mathbf{f}_{\mathcal{R}'} = \mathbf{f}_{\mathcal{R}}'(\mathbf{x}, \mathbf{v}_{\mathcal{R}'}),\ \mathbf{f}_{\mathcal{R}} = \mathbf{f}_{\mathcal{R}}(\mathbf{x}, \mathbf{v}_{\mathcal{R}})). \tag{3}$$

The assumption $\mathbf{f}_{\mathcal{R}} = \mathbf{f}_{\mathcal{R}}(\mathbf{x}, \mathbf{v}_{\mathcal{R}})$ is consistent with (3) and the well-known transformation rule of acceleration, which depends on the same arguments :

$$\mathbf{a}_{\mathcal{R}}(\mathrm{P}) - \mathbf{a}_{\mathcal{R}'}(\mathrm{P}) = \mathbf{a}_{\mathcal{R}}(\mathrm{A}') + \dot{\boldsymbol{\omega}} \wedge \mathrm{A'P} + \boldsymbol{\omega} \wedge (\boldsymbol{\omega} \wedge \mathrm{A'P}) + 2\boldsymbol{\omega} \wedge \mathbf{v}_{\mathcal{R}'} \tag{4}$$

where A' is any point which is bound with $\mathcal{R}'$, $\boldsymbol{\omega}$ is the spin rate vector of $\mathcal{R}'$ with respect to $\mathcal{R}$ and the upper dot signifies time derivative. Thus in (3) and (4), $\mathbf{f}_{\mathcal{R}}' - \mathbf{f}_{\mathcal{R}}$ and $\mathbf{a}_{\mathcal{R}} - \mathbf{a}_{\mathcal{R}'}$ depend only on $\mathbf{x}$ and $\mathbf{v}_{\mathcal{R}}$ (or $\mathbf{x}$ and $\mathbf{v}_{\mathcal{R}}'$) for two given frames $\mathcal{R}$ and $\mathcal{R}'$. The consequence is that the ratio $m^{(i)}/m^{(p)}$ (which is of course assumed independent of $\mathbf{x}$ and $\mathbf{v}_{\mathcal{R}}$) is a constant, in particular it does not depend on the nature of the test particle : *writing Newton's second law in the form (2), assumed covariant, implies the identity between inertial and passive gravitational mass.* In other words this form expresses in a self-consistent way the principle of equivalence between gravitational and inertial forces — and this in a purely classical framework.

Now Eq. (2) (with $m^{(i)} = m^{(p)}$) has to be completed by assumptions regarding the form of the mass force $\mathbf{f}_{\mathcal{R}}$ and the field equations it is subjected to. From empirical observations such as those of Galileo and Foucault (and trying to forget Newton's theory !), it could seem natural to postulate the following Lorentz form :

$$\mathbf{f}_{\mathcal{R}}(\mathbf{x}, \mathbf{v}_{\mathcal{R}}) = \mathbf{g}_{\mathcal{R}}(\mathbf{x}) + \mathbf{v}_{\mathcal{R}} \wedge \boldsymbol{\beta}_{\mathcal{R}}(\mathbf{x}). \tag{5}$$

Since this form must hold in any solid reference frame $\mathcal{R}$, (3) and (4) imply that $\mathbf{g}$ and $\boldsymbol{\beta}$ must transform in this way :

$$\mathbf{g}_{\mathcal{R}'} - \mathbf{g}_{\mathcal{R}} = -(\mathbf{a}_{\mathcal{R}}(\mathrm{A}') + \dot{\boldsymbol{\omega}} \wedge \mathbf{x} + \boldsymbol{\omega} \wedge (\boldsymbol{\omega} \wedge \mathbf{x}) + \boldsymbol{\beta}_{\mathcal{R}} \wedge \mathbf{v}_{\mathcal{R}'/\mathcal{R}}(\mathbf{x}))$$

$$\boldsymbol{\beta}_{\mathcal{R}}' - \boldsymbol{\beta}_{\mathcal{R}} = 2\boldsymbol{\omega},\ (\mathbf{x} = \mathrm{A'P}) \tag{6}$$

Since ω is uniform, this transformation rule allows to set the *additional* assumption that $\beta_{\mathcal{R}}$ is *uniform*, $\beta_{\mathcal{R}} = \beta_{\mathcal{R}}(t)$. In that case there is a frame $\mathcal{R}_0$, whose spin $\omega_0 = -\beta_{\mathcal{R}}/2$ is uniquely defined with respect to any given frame $\mathcal{R}$, such that $\beta_{\mathcal{R}_0} = 0$, i.e. there is no Coriolis force in $\mathcal{R}_0$. However, $\mathcal{R}_0$ is still not a newtonian inertial frame, since it is only defined up to an arbitrary motion of one origin point bound with it, say A_0. If $\mathcal{R}'_0$ is another of these frames, the gravity acceleration fields $\mathbf{g}$ and $\mathbf{g}'$ in $\mathcal{R}_0$ and in $\mathcal{R}'_0$ differ by the (space-)uniform acceleration of $\mathcal{R}_0'$ relative to $\mathcal{R}_0 : \mathbf{g}' - \mathbf{g} = -\mathbf{a}(t)$.

To see the meaning of this residual arbitrariness independently of the particular form (5), let us consider an isolated system of two interacting particles P_1 and P_2 with mass m_1 and m_2 and analyse the forces in two frames $\mathcal{R}$ and $\mathcal{R}'$, the latter having the uniform acceleration $\mathbf{a}(t)$ with respect to the former. The interaction force of P_i on P_j is the total force over P_j and thus in $\mathcal{R}$ it decomposes in accordance with (2) as:

$$\mathbf{F}^{ij}_{\mathcal{R}} = \mathbf{F}^{ij}_0 + m_j \mathbf{f}^{ij}_{\mathcal{R}} = \mathbf{F}^{ij}_0 + m_j \mathbf{f}_{\mathcal{R}}(\mathbf{x}^j, \mathbf{v}^j_{\mathcal{R}}) \quad (i \neq j ; i,j = 1,2), \tag{7}$$

and the like in $\mathcal{R}'$. Now suppose that the actio-reactio principle holds in $\mathcal{R}$:

$$\mathbf{F}^{12}_{\mathcal{R}} + \mathbf{F}^{21}_{\mathcal{R}} = \mathbf{F}^{12}_0 + m_2 \mathbf{f}_{\mathcal{R}}(\mathbf{x}^2, \mathbf{v}^2_{\mathcal{R}'}) + \mathbf{F}^{21}_0 + m_1 \mathbf{f}_{\mathcal{R}}(\mathbf{x}^1, \mathbf{v}^1_{\mathcal{R}}) = 0. \tag{8}$$

From (3) and our assumption, we have $\mathbf{f}_{\mathcal{R}'}(\mathbf{x}^j, \mathbf{v}^j_{\mathcal{R}'}) - \mathbf{f}_{\mathcal{R}}(\mathbf{x}^j, \mathbf{v}^j_{\mathcal{R}}) = -\mathbf{a}(t)$ and since the $\mathbf{F}^{ij}_0$ are invariant, we obtain in $\mathcal{R}'$:

$$\mathbf{F}^{12}_{\mathcal{R}'} + \mathbf{F}^{21}_{\mathcal{R}'} = -(m_1 + m_2)\,\mathbf{a}. \tag{9}$$

The actio-reactio principle cannot hold in $\mathcal{R}'$ also, unless it has no acceleration relative to $\mathcal{R}$. The result is also true for any finite system of particles, provided that the field forces $\mathbf{F}^i_0$ and $\mathbf{f}^i_{\mathcal{R}}(\mathbf{x}^i, \mathbf{v}^i_{\mathcal{R}})$ generated by the individual particles $P^i (i = 1, N)$ obey $(2)_2$ and (3) separately — which is consistent with the newtonian superposition principle of the forces. Thus *the actio-reactio principle can hold only in one galileian class of solid reference frames*. Mechanics should have to restrict to very special problems if it could not use this principle, hence we come back to the question of a privileged class of reference frames. However, we have learned that Newton's second law can be written without using the notion of inertial frame (and integrating instead the equivalence principle). The covariant form (2)—(3) can be used also in *fluid* reference frames, provided we are able to define the acceleration with respect to a fluid. Then a natural application will be to extend the actio-reactio principle so that one may assume its validity in a particular, not necessarily rigid reference frame.

3.2. ACTIO-REACTIO PRINCIPLE IN A FLUID REFERENCE FRAME

The *velocity* of a moving point P with respect to a fluid reference frame, $\mathcal{F}$, is a well-known notion: it is the velocity of P with respect to the fluid "particle" Q (in the sense of continuum mechanics) which momentarily coincides with P; if the motion of $\mathscr{F}$ is given with respect to the reference solid $\mathscr{R}$ in the lagrangian description : $\mathbf{x} = \psi_t(\mathbf{X})$, and the position vector of P is $\mathbf{y}(t) = \mathbf{AP}$, one writes

$$\mathbf{v}_{\mathscr{F}}(P,t) = \left(\frac{\mathrm{d}\mathbf{QP}}{\mathrm{d}\tau}\right)_{\tau=t}, \ \mathbf{AQ} = \mathbf{x}(\tau) = \psi_\tau(\psi_t^{-1}(\mathbf{y}(t))), \qquad (10)$$

which leads to the addition formula

$$\mathbf{v}_{\mathscr{R}}(P,t) = \mathbf{v}_{\mathscr{F}}(P,t) + \mathbf{v}_{\mathscr{R}}(Q,t) = \mathbf{v}_{\mathscr{F}}(P,t) + \mathbf{v}_{\mathscr{F}/\mathscr{R}}(P(t),t). \qquad (11)$$

Defining the *acceleration* of P, relative to $\mathscr{F}$, is less obvious and more ambiguous, because acceleration is a metric notion (in order to compare vectors at different points of a manifold M, it is necessary that a linear connection be given on M : the simplest connections are metric connections). Hence the "acceleration with respect to the fluid", $\mathbf{a}_{\mathscr{F}}(P,t)$, depends on the considered metric. Possible, non-euclidean metrics g_{t_0} on the fluid manifold (the set of the trajectories of the fluid particles) are obtained by *convecting* the metric which is induced on the fluid at a given time t_0 by the euclidean one $\mathbf{g}^0$; if $\mathbf{u}$ and $\mathbf{v}$ are vectors attached to $\mathbf{x}$ at time t, their riemannian scalar product is then defined by :

$$[\mathbf{u}.\mathbf{v}] = g_{t_0}\cdot\mathbf{u}.\mathbf{v} = (D\Phi_{t\,t_0}(\mathbf{x}))(\mathbf{u}) \cdot (D\Phi_{t\,t_0}(\mathbf{x}))(\mathbf{v}), \ \Phi_{t\,t_0} = \psi_{t_0}\cdot(\psi_t)^{-1}, \qquad (12)$$

where the expression on the right of the second sign = is the euclidean scalar product of the vectors $\mathbf{u}_0$ and $\mathbf{v}_0$ attached to $\mathbf{x}_0 = \Phi_{tt_0}(\mathbf{x})$ at time t_0 ($Df(x)$ is the linear mapping tangent to f at x) : $\mathbf{u}$ and $\mathbf{v}$ are convected from $\mathbf{u}_0$ and $\mathbf{v}_0$. In order to define $\mathbf{a}_{\mathscr{F}}(P,t)$, there are only two natural choices : one takes either (i) the euclidean metric $\mathbf{g}^0$, or (ii) $\mathbf{g}_t$ which at any later time τ is convected from the metric induced on the fluid by the euclidean one at the current time t.

Although the foregoing may appear somewhat formal, it has a simple interpretation : choice (i) amounts to defining $\mathbf{a}_{\mathscr{F}}$ with reference to the *rigid* motion which is locally tangent to the fluid motion, whereas choice (ii) makes reference to the locally tangent *homogeneous fluid motion*. This second choice was made by Moreau [14], and it is imposed if one wants to define $\mathbf{a}_{\mathscr{F}}$ with reference to the fluid considered as an independent manifold M' with time-independent metric : the euclidean distance between two points bound with the fluid depends on time. As discussed in chapter 2, the notion of a metric that is insensitive to the physical objects and their motion, is essential in newtonian mechanics, so that the first definition will be adopted here ; one reason will appear more clearly in the next section. Thus $\mathbf{a}_{\mathscr{F}}$ is defined by :

$$\mathbf{a}_{\mathscr{F}}(P,t) = \left(\frac{\mathrm{d}\mathbf{v}_{\mathscr{F}}}{\mathrm{d}\tau}\right)_{\mathscr{R}_{\mathscr{F}}(P,t)} = \left(\frac{\mathrm{d}\mathbf{v}_{\mathscr{F}}}{\mathrm{d}\tau}\right)_{\mathscr{R}} - \boldsymbol{\omega}_{\mathscr{R}_{\mathscr{F}}(P,t)/\mathscr{R}} \wedge \mathbf{v}_{\mathscr{F}}, \qquad (13)$$

where $\mathscr{R}_{\mathscr{F}}(P,t)$ is the solid reference frame whose infinitesimal motion for $\tau \geqslant t$ is tangent to the local motion of the fluid, and $\boldsymbol{\omega}_{\mathscr{R}'/\mathscr{R}}$ is the spin rate of $\mathscr{R}'$ relative to $\mathscr{R}$. The frame $\mathscr{R}_{\mathscr{F}}(P, t)$ has a translation velocity $\mathbf{v}_{\mathscr{F}/\mathscr{R}}(P(t), t)$ and a spin rate $\boldsymbol{\omega} = (\text{rot } \mathbf{v}_{\mathscr{F}/\mathscr{R}}(P(t),t)/2$ relative to $\mathscr{R}$. From (11) and (13) it follows thus:

$$\mathbf{a}_{\mathscr{R}}(P, t) = \mathbf{a}_{\mathscr{F}}(P, t) + \left(\left(\frac{\mathrm{d}}{\mathrm{d}\tau}\right)_{\mathscr{R}} (\mathbf{v}_{\mathscr{F}/\mathscr{R}}(P(\tau), \tau))\right)_{\tau=t} + \frac{1}{2}(\text{rot } \mathbf{v}_{\mathscr{F}/\mathscr{R}}) \wedge \mathbf{v}_{\mathscr{F}}. \tag{14}$$

The particle derivative (with right d) expresses through the chain rule as

$$[\text{grad } \mathbf{v}_{\mathscr{F}/\mathscr{R}}(P(t), t)] \cdot \mathbf{v}_{\mathscr{R}}(P, t) + \frac{\partial \mathbf{v}_{\mathscr{F}/\mathscr{R}}}{\partial t}(P(t), t \tag{15}$$

which is identical to the classical formula giving the acceleration of the fluid particle, $\mathbf{a}_{\mathscr{F}/\mathscr{R}}(P(t),t)$, except for the fact that here the gradient tensor applies to $\mathbf{v}_{\mathscr{R}}(P, t)$ instead of $\mathbf{v}_{\mathscr{F}/\mathscr{R}}(P(t),t)$. The acceleration [transformation rule for a moving point rewrites thus:

$$\begin{aligned} \mathbf{a}_{\mathscr{R}} &= \mathbf{a}_{\mathscr{F}} + \mathbf{a}_{\mathscr{F}/\mathscr{R}} + (\text{grad } \mathbf{v}_{\mathscr{F}/\mathscr{R}}) \cdot \mathbf{v}_{\mathscr{F}} + \frac{1}{2}(\text{rot } \mathbf{v}_{\mathscr{F}/\mathscr{R}}) \wedge \mathbf{v}_{\mathscr{F}} \\ &= \mathbf{a}_{\mathscr{F}'} + \mathbf{a}_{\mathscr{F}/\mathscr{R}} + \mathbf{D}_{\mathscr{F}/\mathscr{R}} \cdot \mathbf{v}_{\mathscr{F}} + (\text{rot } \mathbf{v}_{\mathscr{F}/\mathscr{R}}) \wedge \mathbf{v}_{\mathscr{F}}. \end{aligned} \tag{16}$$

It differs from the Coriolis formula for two solids by the additional term $\mathbf{D}_{\mathscr{F}/\mathscr{R}} \cdot \mathbf{v}_{\mathscr{F}}$ (where the strain-rate $\mathbf{D}$ is the symmetric part of the velocity gradient). Consistently with his different definition, Moreau [14] has this term multiplied by 2. The transformation rule for two fluid reference frames $\mathscr{F}$ and $\mathscr{F}'$ is obtained by the same way of reasoning and it is exactly the same:

$$\begin{aligned} \mathbf{a}_{\mathscr{F}} &= \mathbf{a}_{\mathscr{F}'} + \mathbf{a}_{\mathscr{F}'/\mathscr{F}} + (\text{grad } \mathbf{v}_{\mathscr{F}'/\mathscr{F}}) \cdot \mathbf{v}_{\mathscr{F}'} + \frac{1}{2}(\text{rot } \mathbf{v}_{\mathscr{F}'/\mathscr{F}}) \wedge \mathbf{v}_{\mathscr{F}'} \\ &= \mathbf{a}_{\mathscr{F}'} + \mathbf{a}_{\mathscr{F}'/\mathscr{F}} + \mathbf{D}_{\mathscr{F}'/\mathscr{F}} \cdot \mathbf{v}_{\mathscr{F}'} + (\text{rot } \mathbf{v}_{\mathscr{F}'/\mathscr{F}}) \wedge \mathbf{v}_{\mathscr{F}'}. \end{aligned} \tag{17}$$

Now Newton's second law can be written in the covariant form (2)—(3) for fluid reference frames also. It does not change the physical content of this formulation which is to unify gravitational and inertial forces under the common definition of mass forces, but it takes into account the empirical difficulty in defining exact solid reference frames. Moreover, it makes sense now to postulate the existence of one particular, possibly *fluid* reference frame $\mathscr{E}$, in which the actio-reactio principle is valid — and this does change mechanics, because the new term in (16) shows that in (approximate) solids, it would then exist a new kind of inertial forces, which would systematically *work* against or with the relative motion. The discussion of this effect seems to be important in connection with the assumption of an expanding universe, because certainly an expanding privileged frame $\mathscr{E}(\mathbf{D}_{\mathscr{E}/\mathscr{R}} = a(t)\ \mathbf{I}$, $\mathbf{I}$ the identity tensor) would be a better description of this situation than a rigid one; an expanding ether was already considered by Prokhovnik [15]. Such a discussion is beginning at the end of the next section.

3.3. THE PRINCIPLE OF INERTIA AND THE FLUID VORTICITY. APPLICATION TO AN EXPANDING UNIVERSE

In its usual form, the principle of inertia states that a particle which is "subjected to no force" keeps a constant velocity with respect to some special frames (which form thus a galileian class). Since in physical world this situation never happens, such a definition is unpractical: it should introduce the inertial frames, but their actual definition is that Newton's second law *and* the actio-reactio principle apply in them. In pure newtonian theory an inertial frame $\mathcal{E}$ (in the latter sense) is supposed to be a solid. Then it is immediate that any solid $\mathcal{E}'$ having uniform and constant velocity with respect to $\mathcal{E}$ is also an inertial frame — in the sense that assuming that the force is unaltered when passing from $\mathcal{E}$ to $\mathcal{E}'$, is consistent with the invariance of the acceleration. With a fluid inertial frame, however, this is no longer obvious. We thus postulate, and this seems to be a correct formulation of the principle of inertia, that $\mathcal{E}$ is such that there are other fluid reference frames $\mathcal{E}'$, depending on an arbitrary vector $\mathbf{u}$ (in the case where $\mathcal{E}$ is a solid, this is the uniform and constant velocity $\mathbf{v}_{\mathcal{E}'/\mathcal{E}}$), in which the acceleration of any moving point is the same as in $\mathcal{E}$. Let us examine the restrictions to be imposed to the velocity field $\mathbf{v}$ of $\mathcal{E}$ with respect to some solid reference frame $\mathcal{R}$, in order that this is true. In (17) where we substitute $\mathcal{E}$ and $\mathcal{E}'$ for F and F', the position $\mathbf{x}$ of the moving point and its velocity $\mathbf{v}_{\mathcal{E}'}$ are independent variables. The requirement that $\mathbf{a}_{\mathcal{E}'} = \mathbf{a}_{\mathcal{E}}$ holds for any moving point is hence equivalent to

$$\forall \mathbf{x} \ \ \forall t \ \ \mathbf{a}_{\mathcal{E}'/\mathcal{E}}(\mathbf{x}, t) = 0, \ \ \mathbf{grad}\ \mathbf{v}_{\mathcal{E}'/\mathcal{E}}(\mathbf{x}, t) = 0, \tag{18}$$

the second of which means that $\mathbf{v}_{\mathcal{E}'/\mathcal{E}}$ is a uniform field $\mathbf{u}(t)$. The first one writes thus:

$$\mathbf{a}_{\mathcal{E}'/\mathcal{E}}(\mathbf{x}, t) = \left(\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}\right)_{\mathcal{R}_{\mathcal{E}(\mathbf{x},t)}} = \left(\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}\right)_{\mathcal{R}} - \frac{1}{2}(\mathrm{rot}\ \mathbf{v}(\mathbf{x}, t)) \wedge \mathbf{u} = 0. \tag{19}$$

If the field $\mathbf{v}(\mathbf{x}, t)$ is given, this is a differential equation in $\mathbf{u}$, which must be solved as a differential system involving the other vector equation $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{u}(t)$. In general, the solution will depend on the initial value $\mathbf{x}(t_0)$ as well as on $\mathbf{u}(t_0)$, which is unwanted. At a given time t, (19) may be conversely regarded as an equation with $\mathbf{w} = \mathrm{rot}\ \mathbf{v}$ as unknown: it has only a solution if $2\mathbf{u} \cdot \mathrm{d}\mathbf{u}/\mathrm{d}t = \mathrm{d}(\mathbf{u}^2)/\mathrm{d}t = 0$, hence the relative velocity $\mathbf{u}$ is constant in norm. Moreover, the general solution is then

$$\mathbf{w} = \mathrm{rot}\ \mathbf{v} = 2\left(\mathbf{u} \wedge \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}\right) / \mathbf{u}^2 + \alpha(\mathbf{x}, t)\, \mathbf{u}, \tag{20}$$

(where the unknown function α is restricted by the requirement that $\mathbf{w} = \mathrm{rot}\ \mathbf{v}$, i.e.

$$\mathrm{div}(\mathrm{rot}\ \mathbf{v}) = \mathbf{u} \cdot \mathbf{grad}\ \alpha = 0). \tag{21}$$

So, the spatial variation of the vector field $\mathbf{w}$ is along $\mathbf{u}$, the relative velocity of $\mathcal{E}'$ with respect to $\mathcal{E}$. Since we want that $\mathbf{u}$ is arbitrary, $\mathbf{w} = \text{rot } \mathbf{v}$ *must be a uniform field*:

$$\text{rot } \mathbf{v} = \mathbf{w}(t). \tag{22}$$

If rot $\mathbf{v}$ is uniform, Eq. (19) for the evolution of the uniform velocity $\mathbf{u}$ has always a unique solution, constant in norm: (22) is the searched restriction. Thus there is a solid reference frame (and those having time-dependent translation with respect to it, which form a class $\mathcal{C}$) in which rot $\mathbf{v} = 0$. *We have thereby identified the restriction imposed by the principle of inertia to the velocity field* $\mathbf{v}$ *of a fluid inertial frame.* Romani [9] who did not regard his ether as an inertial frame (but rather left newtonian mechanics as such, thus with rigid inertial frames) observed that it should have a curl-free velocity field, based on the classical argument of "isentropic" flow of a perfect fluid: for a barotropic perfect fluid, the "isentropy" holds for any flow since no entropy intervenes; however, this argument does not apply to the motion of the inertial frame itself, but only to the motion of any fluid with respect to this (fluid) inertial frame. It will be used in sect. (4.2).

Let us select one solid reference frame in the class $\mathcal{C}$ in which rot $\mathbf{v} = 0$. Then Eq. (19) means that the fluid reference frames $\mathcal{E}_\mathbf{u}$ in which the acceleration is the same than in $\mathcal{E}$, have a uniform and constant velocity $\mathbf{u}$ with respect to $\mathcal{E}$, as in usual newtonian theory. From the condition of covariance (3), it comes that the mass force fields $\mathbf{f}$ and $\mathbf{f}_\mathbf{u}$ in $\mathcal{E}$ and in $\mathcal{E}_\mathbf{u}$, verify:

$$\forall \mathbf{u}\ \forall \mathbf{x}\ \ \forall \mathbf{v}_\mathcal{E}\ \ \mathbf{f}_\mathbf{u}(\mathbf{x}, \mathbf{v}_\mathcal{E} - \mathbf{u}) = \mathbf{f}(\mathbf{x}, \mathbf{v}_\mathcal{E}). \tag{23}$$

Note that the galileian principle of relativity (the mechanical equivalence of all inertial frames) would demand that $\mathbf{f}(\mathbf{x}, \mathbf{v}') = \mathbf{f}^\mathbf{u}(\mathbf{x}, \mathbf{v}')$ for all $\mathbf{x}$ and $\mathbf{v}'$, which with (23) would imply that $\mathbf{f}$ does not depend on the velocity $\mathbf{v}_\mathcal{E}$ of the test particle. Instead, we postulate on physical grounds which will be given in sect. (4.1), that the mass force field $\mathbf{f}$ in one particular inertial frame $\mathcal{E}$ (the ether) is indeed independent of $\mathbf{v}_\mathcal{E}$, and we note $\mathbf{f} = \mathbf{g}(\mathbf{x})$ in accordance with (5). Then (23) implies that $\mathbf{f}_\mathbf{u}$ also is independent of $\mathbf{v}_{\mathcal{E}_\mathbf{u}} = \mathbf{v}_\mathcal{E} - \mathbf{u}$ and thus is equal to $\mathbf{g}$, i.e. *the galileian equivalence is obtained as a consequence of an "absolute" assumption.* Exactly the same occurs in special relativity [15]. Now the mass force field in any solid reference frame $\mathcal{R}$ follows from (3) and the acceleration transformation rule (17) where we set $\mathcal{F}' = \mathcal{R}$ and $\mathcal{F} = \mathcal{E}$:

$$\mathbf{f}_\mathcal{R}(\mathbf{x}, \mathbf{v}_\mathcal{R}) = \mathbf{g}(\mathbf{x}) + (\mathbf{a}_\mathcal{R} - \mathbf{a}_\mathcal{E}) =$$

$$= \mathbf{g}(\mathbf{x}) - (\mathbf{a}_{\mathcal{R}/\mathcal{E}}(\mathbf{x}) + \mathbf{D}_{\mathcal{R}/\mathcal{E}}(\mathbf{x}) \cdot \mathbf{v}_\mathcal{R} + (\text{rot } \mathbf{v}_{\mathcal{R}/\mathcal{E}}) \wedge \mathbf{v}_\mathcal{R}). \tag{24}$$

We know from (22) that rot $\mathbf{v}_{\mathcal{R}/\mathcal{E}} = -\text{rot } \mathbf{v}_{\mathcal{E}/\mathcal{R}} = \boldsymbol{\beta}(t)$ is uniform, hence (24) differs from the newtonian form, i.e. the Lorentz one (5) with $\boldsymbol{\beta} = \boldsymbol{\beta}(t)$, only by the third term. Thus *the Lorentz form (5) of the mass force field is obtained only with rigid inertial frames, but also in the general case the "magnetic" part is uniform, i.e. corresponds to a Coriolis force.* This result is obtained under the mere assumption that Newton's three laws hold in the proposed form, which is more general than the usual one since it allows fluid inertial frames (the assumption that the mass force field in $\mathscr{E}$ has the form $\mathbf{f} = \mathbf{g}(\mathbf{x})$ playing the role of the galileian relativity principle which is a part of newtonian mechanics). In particular, the gravitation theory, understood as a model to get equations for the field $\mathbf{g}$, is still completely free.

The simplest generalization of newtonian *mechanics* is thus to allow a uniform expansion of the inertial frame $\mathscr{E}$, i.e. $\mathbf{D}_{\mathscr{E}/\mathscr{R}} = -\mathbf{D}_{\mathscr{R}/\mathscr{E}} = \mathbf{D}_e(t)$ and moreover $\mathbf{D}_e(t) = a(t)\,\mathbf{I}$. In any solid frame of the class $\mathscr{C}$, where rot $\mathbf{v} = 0$, this gives

$$\mathbf{v} = \mathbf{v}_0(t) + \mathbf{D}_e(t)\cdot\mathbf{x}, \tag{25}$$

and in one frame $\mathscr{R}_0$, $\mathbf{v}_0$ cancels : $\mathbf{v}_0(t) = \mathbf{0}$. Let us assume that the field $\mathbf{g}$ is nil (at a large scale where the universe would be homogeneous ; see sect. (4.2)), whence $\mathbf{a}_{\mathscr{E}} = 0$. The motion of a test particle (perhaps a galaxy ... this is common practice in cosmology [4]) is then given in the frame $\mathscr{R}_0$, owing to (16) (with $\mathscr{E}$ in the place of $\mathscr{F}$) and (25), by

$$\begin{gathered}\ddot{\mathbf{x}} = \mathbf{a} = \left(\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{grad}\ \mathbf{v})\cdot\mathbf{v}\right) + \mathbf{D}(t)\cdot(\dot{\mathbf{x}} - \mathbf{v}),\\ \ddot{\mathbf{x}} - (\mathbf{D}_e\cdot\dot{\mathbf{x}} + \dot{\mathbf{D}}_e\cdot\mathbf{x}) = 0.\end{gathered} \tag{26}$$

With $\mathbf{D}_e = a(t)\,\mathbf{I}$, the solution of (26) with initial value $\mathbf{x}(t_0) = \mathbf{x}_0$ and $(\mathrm{d}\mathbf{x}/\mathrm{d}t - \mathbf{D}_e\cdot\mathbf{x})(t_0) = \mathbf{V}_0$, is :

$$\mathbf{x}(t) = \mathbf{x}_0\cdot\exp A(t) + \mathbf{V}_0\int_{t_0}^{t}\exp\left(A(t) - A(s)\right)\mathrm{d}s,\ A(t) = \int_{t_0}^{t}a(s)\,\mathrm{d}s. \tag{27}$$

If the test particle, subjected only to the mass force (nil in $\mathscr{E}$), is initially at rest in the expanding inertial frame $\mathscr{E}(\mathbf{V}_0 = 0)$, it remains so at any later time — this was obvious and shows only that our acceleration transformation rule is correct. The stability of the equilibrium in $\mathscr{E}$, i.e. of the global expansion, should be characterized by the relative difference $\mathbf{r} = (\mathbf{x} - \mathbf{y})/x$ between the position $\mathbf{x}(t)$ of a particle having a perturbed initial value $\mathbf{V}_0 \neq 0$ and the position $\mathbf{y}(t)$ of particles bound with $\mathscr{E}$: one has to determine if there is a fluid particle $\mathbf{y}$ for which $\mathbf{r}$ cancels at large t. The absolute differences are not relevant : for true expansion (contraction), the distance between any two particles bound with $\mathscr{E}$ tends towards $+\infty$ (evanesces) as t increases. Thus :

$$\mathbf{u}(t) = \mathbf{x}(t) - \mathbf{y}(t) = \left(\mathbf{x}_0 - \mathbf{y}_0 + \mathbf{V}_0\int_{t_0}^{t}\exp(-A(s))\,\mathrm{d}s\right)\exp A(t). \tag{28}$$

Consider first the case $a(t) \geqslant \alpha > 0$ (expansion). Then the inegral in (28) has a finite limit I as $t \to \infty$. Hence, with $\mathbf{y}_0 = \mathbf{x}_0 + \mathbf{V}_0\,\mathbf{I}$, $\mathbf{r}$ evanesces at large t. Consider now the case $a(t) \leqslant \alpha < 0$ (contraction), and set

$g(t) = \exp(-A(t))$ which tends towards ∞. We have $g'(t)/g(t) = -a(t)$, which is > 0 and $\gg 1/t$ as $t \to \infty$. *If* $a'(t) \ll a(t)^2$, the integral in (28) diverges and is equivalent to $g(t)^2/g'(t) = -g(t)/a(t)$ at large t [16, p. 96]. Thus $\mathbf{u}(t) \to -\mathbf{V}_0/a(t)$ as $t \to \infty$, independently of $\mathbf{y}_0$, exactly like $\mathbf{x}$: the perturbed particle "stays apart from the contraction", and r is equivalent to 1, even if $a(t) \to -\infty$.

Of course $a = 0$ is unstable (if a small velocity is given to a particle at rest in a rigid inertial frame, it will got to infinity). We conclude that *an expanding universe with nil gravity would be stable, a contracting or rigid one unstable.* This goes in the same direction with relativistic models [4], though with different (and simpler) arguments.

4. ETHER PRESSURE AS THE CAUSE OF GRAVITY

4.1 PRESSURE FORCES IN A PERFECTLY FLUID CONSTITUTIVE SUBSTRATUM (OR ETHER)

It is desired now to give some physical existence to the preferred fluid inertial frame $\mathcal{E}$, and to relate the gravity acceleration $\mathbf{g}$ in $\mathcal{E}$ to the local state of this physical substratum : this seems to be the only way to understand the nature of global inertial frames as well as to avoid the puzzling idea of an action at a distance. It has long been recognized that Poisson's equation for the field $\mathbf{g}$ in newtonian theory gives to its "propagation" the character of a contact action [10] : in an inertial frame,

$$\int_{\partial\Omega} \mathbf{g} \cdot \mathbf{n} \, \mathrm{d}S = \int_{\Omega} \mathrm{div}\, \mathbf{g} \, \mathrm{d}V = -4\pi G\, m(\Omega), \quad m(\Omega) = \int_{\Omega} \rho \, \mathrm{d}V, \tag{29}$$

where ρ is the mass density and G is Newton's gravitation constant. In newtonian theory the propagation is in fact instantaneous, and we have seen that newtonian mechanics may be conciled with fluid inertial frames. This leads to the idea that newtonian gravity could be a pressure action in an incompressible fluid, and that a compressible fluid could lead to a gravity with finite wave speed. Since this imagined fluid does not brake our motion, it should have no viscosity. Then the fluid would exert, over a body Ω, the force

$$\mathbf{F} = \int_{\partial\Omega} - p_e \mathbf{n} \, \mathrm{d}S = -\int_{\Omega} \mathrm{grad}\, p_e \, \mathrm{d}V = \int_{\Omega} \rho \mathbf{g} \, \mathrm{d}V, \tag{30}$$

where for the moment the last equality is merely wished. If we think of Ω as being a macroscopic body, this equality cannot hold, since $\mathbf{g}$ and grad p_e should not depend on the kind of matter which constitutes the body, while ρ of course does. However, the macroscopic matter is made of particles which are already subjected to the gravitation, hence the fluid pressure would have to act only on a small part of the macroscopic volume Ω : the union of the volumes $\omega_i (i = 1, N(\Omega))$ occupied by the constitutive particles. In other words, the fluid would fill the place left by the particles. Let ρ_i be the mass density of particle (i). In order that (30) hold with $\Omega = \omega_i$ and $\rho = \rho_i$, it is necessary that ρ_i be independent of the particle (i). Then in a domain where $\mathfrak{g}$ may be considered uniform (and for the real gravity this is true even with rather large domains), the following would be true for any kind and state of macroscopic matter, provided that grad p_e also be uniform :

$$\mathbf{F} = -\Sigma\, V(\omega_i) \text{ grad } p_e = \Sigma m_i\, \mathfrak{g} = \mathfrak{g}\, \Sigma\, \rho_i\, V(\omega_i). \tag{31}$$

We arrive thus at the conclusion that one should have :

$$\mathfrak{g} = -\text{grad}\, p_e/\rho_p, \tag{32}$$

with ρ_p the assumed common density of the particles — which could still depend on the pressure p_e in the thought ether. Now the identity between inertial and passive gravitational mass does not seem to be known with the same precision for elementary particles than for macroscopic matter. Thus the density in the particles might be allowed to vary from one particle to another, and ρ_p will be the *average* mass density in the different particles of a macroscopic domain $\Omega : \rho_p = \Sigma\, \rho_i V(\omega_i)/\Sigma V(\omega_i)$. But in order that $\mathfrak{g}$ in (32) remains independent of the kind of present matter, it is necessary that ρ_p be a function of p_e only. This would be somewhat miraculous, unless *the particles themselves are made of ether and the ether is a barotropic fluid*, i.e. its density ρ_e depends only on its pressure p_e. The assumption of a constitutive ether was made by Romani [9] on a more philosophical basis, and its barotropic character was justified by the fact that no temperature can be defined for such a true continuum (since the temperature is known to be the kinetic energy, averaged over the different particles). Romani has explained with a great amount of details (including numerical ones) how the known elementary particles could be identified with stable vortices in the perfectly fluid ether, or with (stable or unstable) complexes of several vortices : e.g. the electron would be a simple ring-vortex and its antiparticle the positron would be the same vortex with opposite helicity ; the photon would be a pair of an electron and a positron having common central axe, mutually accelerating in what is called a "leap-frog" in the classical mechanics of vortices, so as to rapidly reach the limit, light velocity ; and so forth. In that way, the appearance of quantum numbers is geometrically explained [9, vol. 2]. Discussing this theory of matter is beyond the scope of this work. It will be assumed that all material particles are made of ether, that the average ρ_p of their individual densities in a macroscopic volume equals the macroscopic density ρ_e of the ether "minus the particles", and that the gravity acceleration is a function only of the macroscopic pressure p_e, given by

$$\mathfrak{g} = -\frac{1}{\rho_e(p_e)} \text{ grad } p_e\,. \tag{33}$$

Thus, under the assumption $\rho_p = \rho_e$, the gravitation force can indeed be interpreted as a pressure force due to the *macroscopic part* p_e of the pressure. Yet the local (or "true") ether pressure p'_e contains also a short-range spatial fluctuation q_e around 0, assumed such that:

$$p'_e = p_e + q_e, \quad \frac{1}{v(\Omega)} \int_{\partial\Omega} |q_e| \, dS \ll |\mathrm{grad}\, p_e|, \tag{34}$$

independently of the spatial position of the sufficiently large (and simple-form, e.g. cubic) domain Ω. This condition ensures that p_e and grad p_e are well-defined, the latter being the macro-volume average $\overline{\mathrm{grad}\, p'_e}$ of grad p'_e (not precluding that q_e can have macroscopic effects, but $\overline{\mathbf{z} \cdot \mathrm{grad}\, q_e} \ll \overline{\mathbf{z}} \cdot \mathrm{grad}\, p_e$ if $\mathrm{div}\, \mathbf{z} = 0$) [17]–[18]. In partial agreement with Romani's conceptions, the fluctuation q_e is thought to be responsible for the shorter range physical interactions between material particles: electromagnetic, strong and weak nuclear interactions; in view of the preceding remark, this is compatible with the macroscopic effects of the electromagnetic interaction. It is clear also that the ether pressure, either p_e or p'_e, is completely different from the usual pressure p in material bodies, which is a kinetic energy of material particles, per unit volume [9]. In particular, it results from our Eq. (33) that p_e *decreases* towards the direction of gravitational attraction, whereas the equilibrium of a material fluid, assumed perfect, under a gravitation field $\mathbf{g}$ and the kinetic pressure forces $-$ grad p, writes $\mathbf{g} = +(\mathrm{grad}\, p)/\rho$ and thus, of course, the contrary occurs for p. That p_e decreases as ρ increases, is consistent with the assumption that material particles are ether vortices, because the pressure of a perfect fluid is known to be lower in the vicinity of a vortex [19]; after all, this is common observation in meteorology. At first sight, Eq. (33) looks as an equilibrium equation for the ether at the macroscopic scale (this will be called hereafter "macroether"), but it is not true: the macro-ether as a whole is subjected only to the macropressure force $-$ grad p_e†; considering the macro-ether "minus the particles" does not improve the situation, e.g. because the gravity acceleration (33) can be the same in two domains, only one of which contains matter. The equilibrium of the macro-ether $\mathcal{E}$ itself is beyond the capacity of this mechanics, since $\mathcal{E}$ defines the inertial frame. Yet the microscopic motion of the ether relative to its *mean rest frame* $\mathcal{E}$ obeys Newton's second law. The motion of material particles is a particular kind of such microscopic motion.

Let us note finally that the foregoing microscopic considerations have an explanatory and explorative character but are logically unnecessary to the proposed gravitation theory. This, for the moment, reduces to Eq. (33), together with the transformation rule (24) of the mass force field entering the covariant form (2) of Newton's second law.

4.2 EQUATIONS FOR THE FIELD OF MACROSCOPIC ETHER PRESSURE

In the preceding section, the matter was passive under the gravitation (the field $\mathbf{g}$, Eq. (33)), because the spatial domain, though large as compared with the size of material particles, was assumed small enough to consider that grad p_e is uniform in it: this was the often discussed re-

† (per unit volume)

presentative macro-element of statistical physics. However, the presence of material particles (ether vortices) is related to a variation of the pressure p_e, as explained above. Without trying to find the law of this variation by a homogeneization process (which one day could become relevant), it remains to build a law by phenomenological considerations. First, *newtonian gravitation must be regained for an incompressible ether. This is immediate*, since Poisson's equation gives with (33):

$$\mathrm{div}(\mathrm{grad}\ p_e) = -\rho_e \,\mathrm{div}\,\mathbf{g} = 4\pi G\ \rho_e\ \rho = \Delta p_e, \qquad (35)$$

whenever ρ_e is uniform. Now it is readily seen that the last equation also has a sense for a compressible ether where ρ_e varies in space. The great accuracy of newtonian gravity means that the ether conpressibility plays little role in "usual" situations, in the sense that Eqs. (33) and (35), with uniform ρ_e, are a very good approximation of the equation we are looking for, at least for weak and slowly varying gravitational fields. Thus the spatial variation of ρ_e is expected to be scarce, so that the $p_e - \rho_e$ relationship may be linearized around some reference pressure and density $p_e^{\mathrm{ref}}(\rho_e^{\mathrm{ref}})$. Introducing the "sound" velocity c_e in the ether (depending on ρ_e for non-linear $p_e - \rho_e$ relationship), obtains

$$p_e - p_e^{\mathrm{ref}} = c_e\,(\rho_e^{\mathrm{ref}})^2\,(\rho_e - \rho_e^{\mathrm{ref}}),\ c_e(\rho_e)^2 = \mathrm{d}p_e/\mathrm{d}\rho_e. \qquad (36)$$

Eq. (36) allows to eliminate the ether density from Eq. (35):

$$\Delta p_e = (4\pi G/c_e^2)\,(p_e + K^{\mathrm{ref}})\rho,\ K^{\mathrm{ref}} = c_e^2\,\rho_e^{\mathrm{ref}} - p_e^{\mathrm{ref}},\ c_e = c_e(\rho_e^{\mathrm{ref}}). \quad (37)$$

As the compressibility $1/c_e^2$ tends towards 0, one may obviously expect that the solution of (37) with given boundary conditions for p_e(e.g. $p_e(r = \infty) = p_e^{(\infty)} = p_e^{\mathrm{ref}}$ for a problem with spherical symmetry), fixed ρ_e^{ref} and given material density ρ, tends towards the solution of Eq. (35) with imposed uniform ether density $\rho_e = \rho_e^{\mathrm{ref}}$ and the same ρ and boundary conditions. A precise mathematical study (which could use perhaps a penalty method) is not our purpose here, but it will be seen in the next section that this indeed happens in the crucial case of spherical symmetry. It seems unlikely that another second-order linear equation in p_e as (37) which is deduced from (35) by linearization of the $p_e - \rho_e$ dependence, could have the same property. *We thus postulate that the non-linear Eq. (35) is the field equation relating the pressure $p_e = p_e(\rho_e)$ in the barotropic ether to the material density ρ, in the static case* where the fields are time-independent. Note that in the linearized compressible case, the field $\mathbf{g}$ (Eq. (33)) derives from a potential which, contrary to the incompressible case, is not proportional to the ether pressure any more:

$$\mathbf{g} = -c_e^2\frac{\mathrm{grad}\ (p_e + K^{\mathrm{ref}})}{p_e + K^{\mathrm{ref}}} = -c_e^2\,\frac{\mathrm{grad}\,(\rho_e)}{\rho_e} = \mathrm{grad}\ U, \qquad (38a)$$

$$U = -c_e^2\,\mathrm{Log}\,(\rho_e),\ c_e = c_e(\rho_e^{\mathrm{ref}}). \qquad (38b)$$

Moreover, taking the divergence of the first equation in Eq. (38a), we get using Eq. (37) :

$$\operatorname{div} \mathbf{g} - \frac{\mathbf{g}^2}{c_e^2} = -4\pi G \rho. \tag{38c}$$

As a first application, let us examine the case of uniform material density. The newtonian theory is known to give meaningful results in that case, only after it has been modified in a rather artificial way [4]—[5]. It seems that a reasonable gravity field $\mathbf{g}$ in such case should be $\mathbf{g} = 0$. For a *uniform density* ρ, Eq. (35) has one solution giving uniform pressure, namely $p_e = 0$ (assuming of course that $p_e(\rho_e = 0) = 0$). Then Eqs. (33) and (37)—(38) do not apply, but clearly the macroscopic pressure forces cancel and hence $\mathbf{g} = 0$. We thus get the important result that the theory allows nil gravity $\mathbf{g}$ for uniform material density $\rho = \rho_0$. Of course, it makes more sense to assume a *macro-uniform* density : $\rho = \rho_0 + \rho_1$ with ρ_1 fluctuating around 0; apparently this case has not been investigated in newtonian theory. In that case it is expected that $\mathbf{g}$ will fluctuate around 0. This is not immediate to check, since Eqs. (35) or (37) imply a non-linear dependence of p_e on ρ and Eq. (38c) shows directly the non-linear dependence of $\mathbf{g}$ on ρ.

Let us turn to the unsteady situation where the fields p_e and ρ depend on the time t. Then the continuity equation for the velocity field $\mathbf{v}$ of the ether, not yet stated, plays an important role. *It is assumed that the macro-ether is conserved.* The usual continuity equation holds thus in any solid reference frame :

$$\frac{\partial \rho_e}{\partial t} + \operatorname{div}(\rho_e \mathbf{v}) = 0. \tag{39}$$

As in usual acoustics, consider the perturbation of a large-scale flow of the macro-ether by a vibration of a small amplitude and low velocity (e.g. [20, § 63)]. We can use Eq. (39) with $\mathbf{v}$ being the velocity of the disturbed motion with respect to the undisturbed one, the *latter* being assumed to define the fluid inertial frame $\mathscr{E}$. Referring the spatial position $\mathbf{x}$ and the velocity $\mathbf{v}$ to the solid reference frame $\mathscr{R}_\mathscr{E}$ which is tangent to the undisturbed motion at a given point $\mathbf{x}_0$ (this latter being bound to the undisturbed motion i.e. to the frame $\mathscr{E}$), we make a local decomposition :

$$\rho_e = \rho_{e0} + \rho_{e1}, \; p_e = p_{e0} + p_{e1}, \; \rho_{e1} \ll \rho_{e0}; \; p_{e1} \ll p_{e0}, \; |\mathbf{v}| \ll c_e(\rho_{e0}). \tag{40}$$

Since $\mathscr{R}_\mathscr{E}$ follows the local undisturbed fluid motion which is assumed to be very slow (as the today accepted expansion), the time derivatives of p_{e0} and ρ_{e0} are negligible with respect to those of p_{e1} and ρ_{e1}, as far as $\mathbf{x}$ remains in a small neighborhood of $\mathbf{x}_0$. Thus Eq. (39) rewrites up to the first-order terms :

$$\frac{\partial \rho_{e1}}{\partial t} + \rho_{e0} \operatorname{div} \mathbf{v} = 0. \tag{41}$$

The disturbed motion is assumed to obey Newton's law, as the microscopic motion; to write this, we consider that the pressure force (here the mere force in Newton's law) is due only to the deviation of the pressure from the pressure in the undisturbed fluid:

$$-\frac{\operatorname{grad} p_{e1}}{\rho_{e0}} \approx -\frac{\operatorname{grad} p_{e1}}{\rho_e} = \frac{d\mathbf{v}}{dt} \approx \frac{\partial \mathbf{v}}{\partial t}, \tag{42}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\operatorname{grad} p_{e1}}{\rho_{e0}} = 0. \tag{43}$$

With (41) and since **v** is curl-free (this has been proved for the motion of the inertial frame with respect to a solid; for the disturbed motion it follows from the argument of "isentropy": see after Eq. (22)), this gives in the classical way d'Alembert's equation:

$$\Delta p_{e1} - \frac{1}{c_e^2} \frac{\partial^2 p_{e1}}{\partial t^2} = 0, \; c_e^2 = \frac{dp_e}{d\rho_e}(\rho_{e0}). \tag{44}$$

The undisturbed fields p_{e0} and ρ_{e0} of the local decomposition (40) are assumed to obey Eq. (35). Summing this and (44) gives to the first order:

$$\Delta p_e - \frac{1}{c_e^2} \frac{\partial^2 p_e}{\partial t^2} = 4\pi G \rho \, \rho_e, \; p_e = p_e(\rho_e), \; c_e = c_e(\rho_e). \tag{45}$$

This is the general equation which is stated for relating the fields of material density ρ and macro-ether pressure p_e. It clearly admits gravitational (pressure) waves propagating at the sound velocity $c_e(\rho_e)$ in the compressible ether, which *varies* with ρ_e and thus in space, unless if the barotropic relationship $p_e = p_e(\rho_e)$ is linear. In deriving these equations, no restriction was imposed on the large-scale motion of the macro-ether and *this motion is not determined by any equation of the theory*. As in newtonian theory, the motion of an inertial frame is phenomenologically determined from the absence of inertial forces in it — but here this motion is an arbitrary (curl-free) fluid motion. In the present state of knowledge, a uniformly expanding ether is expected to give a good description at a large (inter-galactical) scale, and this amounts to nearly rigid motion at smaller scales [15] because the acceleration due to expansion (Eq. (26)) is negligible as compared with the relevant accelerations (of the stars, planets, vehicles...).

4.3 CENTRAL STATIC SOLUTION FOR LINEARIZED ETHER COMPRESSIBILITY

Since the Lorentz form (5) of the mass force field (necessarily with uniform magnetic part, sect. (3.3)) is known to be a very good approximation in our solar system, we already can state that the macro-ether (or privileged inertial frame) has very nearly a rigid motion in this region.

This is connected with the other empirical fact that the solar system is nearly an isolated system, with the Sun alone having the main gravitational effects. If all matter were concentrated in one and alone body at equilibrium, there would be no reason for the ether to move relative to this body. Let us thus consider the case of one spherical body with mass M and radius R, at rest in the rigid ether, and refer the space to spherical coordinates r, θ, Φ from the centre. Of course the equations of sect. (4.2) are written in terms of the euclidean metric and the newtonian equation ($\mathbf{a}_e = \mathbf{g}$) is used. In the forthcoming paper another interpretation is studied. For $r > R$, Eq. (35) is Laplace's equation, with spherically symmetrical solution :

$$p_e = A + B/r = p_e^{(\infty)}(1 - r_0/r). \tag{46}$$

The linearization (36) of the $p_e - \rho_e$ relationship, not used for obtaining (46), may hence be made with $p_e^{ref} = p_e^{(\infty)}$, and the field $\mathbf{g}$ (Eq. (38 a)) writes :

$$\mathbf{g} = \frac{\mathbf{g}\cdot\mathbf{r}}{r}\frac{\mathbf{r}}{r} = -g\frac{\mathbf{r}}{r},\ g = \frac{r_0\, p_e^{(\infty)}}{r^2(\rho_e^{(\infty)} - r_0\, p_e^{(\infty)}/(c_e^2 r))}. \tag{47}$$

This shows that $\mathbf{g}$ is like $1/r^2$ at large r, and equivalent to the newtonian expression if

$$r_0 = GM\frac{\rho_e^{(\infty)}}{p_e^{(\infty)}}, \tag{48}$$

but this way of obtaining the value of r_0 is not entirely satisfying, although it is often used also in general relativity [1]—[4] : once the equations are stated, the form of which is "close" to the newtonian form, it would be desirable that the adjustment of the constants be internal to the theory. This is possible here, at least when the material density in the central body is assumed to be uniform. Eq. (38c) writes for spherical symmetry :

$$\frac{dg}{dr} + 2\frac{g}{r} + \frac{g^2}{c_e^2} = 4\pi G\rho(r) = k(r)\, c_e^2,\ \mathbf{g} = -g\cdot\frac{\mathbf{r}}{r}, \tag{49}$$

a Riccati equation which by the change $g = c_e^2(y - 1/r)$ takes the form

$$y' + y^2 = k. \tag{50}$$

If $\rho(r) \equiv \rho_0$ or $k = \text{Const}$ (for $0 \leqslant r \leqslant R$), Eq. (50) admits $y = \sqrt{k}$ th $(r\sqrt{k})$ as a particular solution. The general solution of Eq. (49) is then found to be

$$\frac{g(r)}{c_e^2} = \frac{\sqrt{k}}{\mathrm{sh}(r\sqrt{k})\,\mathrm{ch}(r\sqrt{k}) + A\,\mathrm{ch}^2(r\sqrt{k})} + \sqrt{k}\,\mathrm{th}(r\sqrt{k}) - \frac{1}{r}. \tag{51}$$

As $r \to 0$, this is equivalent to $-1/r$ and gives thus unbounded *repulsion*, *unless if* $A = 0$ in which case it tends towards 0. Similarly, in the newtonian (incompressible) theory the solution is uniquely determined by the requirement that g remains bounded as $r \to 0$. Here the exterior solution (47) has a singularity at

$$r'_0 = r_0\, p_e^{(\infty)}/(\rho_e^{(\infty)} c_e^2). \tag{52}$$

It is obvious that r_0 cannot be in the range of the exterior solution (46) for the ether pressure, since p_e and hence ρ_e would then *change sign* at $r = r_0$. Contrary to general relativity, there is no question about the reality of the singularity at r_0, because the present theory *would not apply* to a spherical massive body whose the radius R at equilibrium would be less than r_0. In the same way, an unbounded repulsion would occur for $r < r'_0$ and Eqs. (47) and (38a-b) imply that the ether density would cancel at r'_0; hence the field of the central body at equilibrium has no singularity (if the theory applies), it is:

$$\frac{g(r)}{c_e^2} = \begin{cases} r'_0/(r^2(1 - r'_0/r)) & \text{if } r > R \\ \dfrac{\sqrt{k}}{\mathrm{sh}(r\sqrt{k})\,\mathrm{ch}(r\sqrt{k})} + \sqrt{k}\,\mathrm{th}(r\sqrt{k}) - \dfrac{1}{r},\ k = \dfrac{4\pi G \rho_0}{c_e^2} & \text{if } 0 \leqslant r < R. \end{cases} \tag{53}$$

Assuming that $\xi = r\sqrt{k} = (3GM(r)/(c_e^2 r))^{1/2} \ll 1$ (with $M(r) = (4/3)\pi\rho_0 r^3$), the interior solution is expanded as

$$g_{int}(r) = c_e^2\sqrt{k}\left(\frac{\xi}{3} + \frac{43}{90}\xi^3 + 0(\xi^5)\right) = \frac{GM(r)}{r^2}\left(1 + 4.3\,\frac{GM(r)}{c_e^2 r}\right) + c_e^2\sqrt{k}\,O(\xi^5). \tag{54}$$

The continuity of g at $r = R$ gives then:

$$r'_0 = R(\varepsilon + 3.3\,\varepsilon^2 + O(\varepsilon^3)),\ \varepsilon = \frac{GM}{c_e^2 R}, \tag{55}$$

$$g_{ext}(r) = \frac{GM_e}{r^2\left(1 - \dfrac{r'_0}{r}\right)}\quad M_e = M(1 + 3.3\,\varepsilon + O(\varepsilon^2)). \tag{56}$$

Hence *the newtonian solution (exterior and interior) is regained as* $\varepsilon \to 0$. Of course, this happens if one makes c_e tend towards $+\infty$, but physically c_e is determined (see the next paper). Thus Eqs. (54) and (56) mean that the newtonian solution is a better approximation of the presented theory, when $r/R \gg \varepsilon$ and also when $r^2 \ll 1/k$, or at given r when the ratio M/R is low — in general: *when the field* $\mathbf{g}$ *is weak.* This is a well-known result in general relativity, but *here the solution has been adjusted internally to the proposed theory* (this is also done for general relativity by W. Thirring [8, §4.4]). The question of internal consistency is not merely of a formal interest, but has consequences on the obtanied predictions. Indeed, Eq.

(56) shows that the coefficient of G/r^2 in the expression of $\mathbf{g}$ at large r (the newtonian active mass), is not exactly the inertial mass M, and its difference from M depends on M/R, hence depends on the massive body. It is not obvious that this first difference alone has completely negligible consequences in celestial mechanics, even in the solar system. For the Sun, the Earth, the Moon and Jupiter respectively, ε is approximately: $2. \times 10^{-6}$, 6.9×10^{-10}, 3×10^{-11}, 2×10^{-8}, if c_e is taken to be equal to the light velocity $c \approx 3 \times 10^5$ km/s. However, we stop our comments here, because the theory is reinterpreted in the next paper and gives differente results.

The motion of a test particle subjected only to the central gravitation field (47) of the spherical body obeys Kepler's second law, i.e. it is a plane motion (which may thus be analysed as a motion with the colatitude $\theta \equiv \pi/2$) such that $r^2\, \mathrm{d}\Phi/\mathrm{d}t = h = \text{Const}$. With $u = 1/r$ and Eq. (56), the radial equation of motion writes [21], to the first order in r'_0/r:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\Phi^2} + u = \frac{g(r)}{h^2u^2} \approx \frac{GM_\varepsilon}{h^2}\left(1 + \frac{GM_\varepsilon}{c_e^2}\,u\right) = A + \alpha u,\; A = \frac{GM_\varepsilon}{h^2}. \qquad (57)$$

Setting $\psi = \Phi\sqrt{(1-\alpha)}$, Eq. (57) becomes

$$\frac{\mathrm{d}^2u}{\mathrm{d}\psi^2} + u = \frac{A}{\sqrt{(1-\alpha)}} = A_1. \qquad (58)$$

with general solution:

$$u = C\sin(\psi - \psi_0) + A_1 = C\sin\left[(\Phi - \Phi_0)\sqrt{(1-\alpha)}\right] + A_1. \qquad (59)$$

The newtonian theory is regained if $\alpha = 0$: for planetary orbits, the constants C and A are such that Eq. (59), with $\alpha = 0$, represents an ellipse with large half-axe a and eccentricity e satisfying $A = 1/(a(1-e^2))$ [4]. Thus for $\alpha \ll 1$, Eq. (59) represents a bounded curve which repeats, not after an exact revolution ($\Delta\Phi = 2\pi$), but after a little more than that: the *advance in the perihelion*

$$\delta\Phi = 2\pi\left(\frac{1}{\sqrt{1-\alpha}} - 1\right) \approx \pi\alpha = \pi\frac{GM_\varepsilon}{h^2}\frac{GM_\varepsilon}{c_e^2} = \pi\frac{GM_\varepsilon}{c_e^2\, a(1-e^2)} \qquad (60)$$

is predicted. With $c_e = c$, Eq. (60) gives only one sixth of the advance predicted by general relativity, which agrees well with observations [4].

5. CONCLUSION

An old, classical idea subtends this paper: the absolute space of Newton's theory has to be a physical substratum, otherwise it is impossible to understand why Nature seems in fact privilege some reference bodies, and also how interactions can propagate through interstellar empty space. The originality may lie in the deliberate choice to explore this possibility in a strict, naive sense though with an exigence of logical consistency. Only a perfect fluid could fill the space left by material bodies without braking any motion, and only the pressure force can be exerted by such a fluid. Another point is essential in this work: the fluid must be inertial, i.e. Newton's theory in the usual sense implies that the fluid has in fact a rigid motion, that is, rigid with respect to the euclidean metric. In order to justify the fluid nature of the substratum, it appears then of an immediate necessity to generalize newtonian mechanics to the case where the inertial fluid is indeed deformed in terms of the euclidean metric, and this has an interesting application; of the three simplest possibilities: rigid motion, contraction and expansion, only the third one leads to a stable universe — in a sense that is precised. In connection with this, the generalization has the other advantage that no artificial modification of newtonian attraction (such as a repulsive force increasing with distance) needs to be postulated.

The pressure action of the inertial fluid or ether is equivalent to a mass force acting indifferently on all material objects, if the elementary particles of matter have the same "mass" density than the ether. This is no mystery, if these particles are themselves made of ether and this has an extremely small compressibility so that the "sound" velocity in ether is that of light. However the pressure which may be responsible for gravitation must be the macroscopic ether pressure. Thus the microscopic field of ether pressure might contain also the other, shorter range physical interactions.

Newtonian gravitation is immediately regained as a limiting case of the postulated field equation which relates the ether pressure to the material density, corresponding to a nil ether compressibility. Moreover the solution of the spherical static problem tends towards the newtonian one when the compressibility tends towards zero. In unsteady situations, the compressibility leads naturally to "acoustic" (pressure) waves in ether, i.e. to gravitational waves. However, the theory, at this stage, cannot describe gravitation completely since it does not allow for relativistic effects — and this traduces in the poor prediction of the advance in the perihelion of planetary orbits. In the forthcoming paper, the "relativistic" effects are accounted for naturally and efficiently by the ether theory.

Acknowledgements. I would like to kindly thank Mr. J. F. Heitz for helpful discussions. The interesting comments on the manuscript by Profs. P. Guélin and J. D. Weber are also gratefully thanked.

Received September 21, 1992

REFERENCES

1. J. L. SYNGE, *Relativity, the general theory*, North-Holland, Amsterdam, 1964.
2. V. FOCK, *The theory of space, time and gravitation*, Pergamon, Oxford, 1964.
3. L. LANDAU, E. LIFSHITZ, *Théorie des champs*, Mir, Moscow, 1970.
4. S. MAVRIDES, *L'univers relativiste*, Masson, Paris, 1973.
5. O. HECKMANN, E. SCHÜCKING, *Newtonsche und Einsteinsche Kosmologie, Handbuch der Physik*, Bd. LIII, Springer, Wien-New York, 1959, p. 489–519.
6. A. TRAUTMAN, *Sur la théorie Newtonienne de la gravitation*, C.R. Acad. Sc. Paris, série I, **257**, 617–620 (1963).

7. P. MAZILU, *Actio-reactio equations with finite wave speed derived from the principle of inertia*, Acta Mech., **79** (1989), 233.
8. W. THIRRING, *Classical field theory*, Springer, New York—Wien, 1986.
9. L. ROMANI, *Théorie générale de l'univers physique (Réduction à la Cinématique)*, (2 vols), Blanchard, Paris, 1975 and 1976.
10. M. A. TONNELAT, *Histoire du principe de relativité*, Flammarion, Paris, 1971.
11. A. TRAUTMAN, *Sur les lois de conservation dans les espaces de Riemann*, Les théories relativistes de la gravitation, 1(13) — 116, Ed. du CNRS, Paris, 1962.
12. J. J. MOREAU, *Cinématique de variétés C^1 et transport de mesures scalaires*, Séminaire d'Analyse Convexe, 4—1 to 4—44, Université de Montpellier, 1982.
13. A. P. FRENCH, *Newtonian mechanics*, Norton, New York—London, 1971.
14. J. J. MOREAU, *Sur la notion de système de référence fluide et ses applications en aérodynamique*, Congrès Nat. Aviation Française, rapport n° 368 (mémoire M87), Ministère de l'Air, Paris, 1945.
15. S. J. PROKHOVNIK, *Logique de la relativité restreinte*, Gauthiers-Villars, Paris, 1969. English orig. edn.: The Logic of special Relativity, Univ. Press, Cambridge, 1967.
16. J. DIEUDONNE, *Calcul infinitésimal*, Hermann, Paris, 1968.
17. M. ARMINJON, *Limit distributions of the states and homogenization in random media*, Acta Mech., **88**, (1991), 27.
18. M. ARMINJON, *Macro-homogeneous strain fields with arbitrary local inhomogeneity*, Arch. Mech., **43**, 2 (1991) ~~(to appear)~~, 191.
19. H. POINCARE, *Théorie des tourbillons*, Georges Carré, Paris, 1893 (new impression: Sceaux, Jacques Gabay, 1990).
20. L. LANDAU, E. LIFSHITZ, *Mécanique des fluides*, Mir, Moscow, 1971.
21. P. BROUSSE, *Mécanique*, Armond Colin, Paris, 1968.