ON CONTINUUM DYNAMICS AND THE ELECTROMAGNETIC FIELD IN THE SCALAR ETHER THEORY OF GRAVITATION

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SPACETIME MANIFOLD AND METRIC

Two assumptions common with GR:

a) Our space and time measurements may be arranged so as to be described by a metric γ with $(+ - - -)$ signature on a 4-dimensional manifold V (the spacetime).

b) A continuous medium or a continuous field is defined by the expression of its energy-momentum tensor $\mathbf{T}$, which is a spacetime tensor depending on some state variables. The dynamical laws can be universally expressed in terms of $\mathbf{T}$, independently of the specific medium/field.

Several notions used in this theory can also be defined in GR:

The notion of a reference fluid

= a 3-D network $\mathcal{F}$ of *reference world lines*, each being time-like for a physically admissible reference fluid (Cattaneo 1958).

$\mathcal{F}$ may be defined by the associated unit tangent 4-vector field $\boldsymbol{U} = \boldsymbol{U}_{\mathcal{F}}$ (e.g. Rodrigues & Capelas de Oliveira 2007): the reference world lines are the integral curves of $\boldsymbol{U}_{\mathcal{F}}$.

Adapted charts: those for which the vector $\mathbf{x} \equiv (x^i) \in \mathbb{R}^3$ is constant on any reference world line (e.g. Møller 1952, Cattaneo 1969).

The notion of a reference fluid was thus first defined for GR (but is not commonly used there).

SPACE MANIFOLD ASSOCIATED WITH A REFERENCE FLUID

This notion too can be defined in GR (but has not been used yet):

Physical space associated with a reference fluid $\mathcal{F}$: the set $\mathrm{N}_{\boldsymbol{U}}$ of the reference world lines (MA, Arch. Mech. **48**, 551 (1996)). Depends on the reference fluid considered.

For a “normal” non-vanishing vector field on V, adapted charts do exist. They allow one to endow the set $\mathrm{N}_{\boldsymbol{U}}$ with a canonical structure of *3-D differentiable manifold* (MA, IJGMMP 2016).

So, given a reference fluid, we can define *spatial fields:* fields over the space manifold $\mathrm{N}_{\boldsymbol{U}}$ — whether scalar, vector or tensor fields. In particular, the spatial metric defined from γ by Landau & Lifshitz or Møller is a $(0\ 2)$ spatial tensor field $\boldsymbol{g}$. It makes $\mathrm{N}_{\boldsymbol{U}}$ a Riemannian manifold — but the spatial metric $\boldsymbol{g}$ depends on time.

GLOBALLY-SYNCHRONIZED REFERENCE FLUID

We consider a special reference fluid $\mathcal{E}$, with four-velocity vector field $\boldsymbol{U}_{\mathcal{E}}$, and we assume that it is globally synchronized:

There is a global space-time coordinate system (x^μ) which is adapted to $\mathcal{E}$, and in which the components of the spacetime metric γ verify

$$\gamma_{0i} = 0. \tag{1}$$

A chart in which (1) is true exists locally in a generic spacetime (e.g. Landau & Lifshitz). Condition (1) alone does not specify a unique reference fluid, even less a unique coordinate system.

The preferred character of the $\mathcal{E}$ assumed in the theory appears with the dynamics.

Newton's second law in a curved spacetime

$$\mathbf{F} + (E/c^2)\mathbf{g} = D\mathbf{P}/Dt_{\mathbf{x}}. \qquad (2)$$

$\mathbf{F}$: non-gravitational force; $\mathbf{g}$: gravity acceleration;
E: energy of the test particle: $E = m(v)c^2$ for a mass point, with

$$m(v) \equiv m_0\gamma_v, \qquad \gamma_v \equiv 1/\sqrt{1-(v^2/c^2)}. \qquad (3)$$

$\mathbf{v}$: 3-velocity (relative to the reference fluid $\mathcal{E}$): measured with local time $t_{\mathbf{x}}$ and modulus v defined with space metric $\boldsymbol{g}$:

$$v^i \equiv \frac{\mathrm{d}\,x^i}{\mathrm{d}\,t_{\mathbf{x}}} \equiv \frac{1}{\beta}\frac{\mathrm{d}\,x^i}{\mathrm{d}\,t}, \quad \beta \equiv \sqrt{\gamma_{00}}, \qquad v \equiv [\boldsymbol{g}(\mathbf{v},\mathbf{v})]^{1/2} = (g_{ij}v^i\,v^j)^{1/2}. \qquad (4)$$

$\mathbf{P} \equiv (E/c^2)\mathbf{v}$: momentum.
$D/Dt_{\mathbf{x}}$: relevant time-derivative in space manifold N_U endowed with time-dependent metric $\boldsymbol{g}$ (MA Arch. Mech. 1996).

THE GRAVITY ACCELERATION

Newton's 2nd law (2) is compatible with geodesic motion of GR, provided a peculiar (velocity-dependent) form of the gravity acceleration $\mathbf{g}$ is assumed (MA Arch. Mech. 1996).
But we assume a different, simpler form:

$$\mathbf{g} = -c^2 \frac{\mathrm{grad}_g \beta}{\beta}, \quad (\mathrm{grad}_g \beta)^i \equiv g^{ij}\beta_{,j} \, , (g^{ij}) \equiv (g_{ij})^{-1}. \qquad (5)$$

($\beta \equiv \sqrt{\gamma_{00}}$). Came out from a mechanism/interpretation of gravity as a pressure force (MA 1993).

Can also be *derived* by demanding that (i) the metric field γ should be a spatial potential for a space vector $\mathbf{g}$, and (ii) the law of motion (2) should imply geodesic motion for "free" test particles in a *static* metric (MA Arch. Mech. 1996).

FOUR-ACCELERATION OF A MASS TEST PARTICLE

Using Newton's 2nd law (2) with the gravity acceleration (5), we can compute

$$A^0 = \frac{1}{2\beta^2}\, g_{jk,0}\, U^j\, U^k + \frac{\gamma_v}{\beta}\,\frac{\mathbf{F.v}}{m_0\, c^3},$$

$$A^i = \frac{1}{2}\, g^{ij} g_{jk,0}\, U^0\, U^k + \gamma_v\, \frac{F^i}{m_0\, c^2}. \qquad (6)$$

Here $\mathbf{U}$, with components U^μ, is the 4-velocity of the mass point. ($\mathbf{U} \neq \boldsymbol{U}$, the 4-velocity field of the preferred reference fluid.)

Newton's second law for a dust continuum

Dust: a continuum made of coherently moving, non-interacting particles, each of which conserves its rest mass.
Energy-momentum tensor:

$$T^{\mu\nu} = \rho^* c^2 U^\mu U^\nu. \tag{7}$$

Mass conservation: $(\rho^* U^\nu)_{;\nu} = 0$. With (7), we get thus:

$$T^{\mu\nu}_{;\nu} = \rho^* c^2 U^\mu_{;\nu} U^\nu = \rho^* c^2 A^\mu. \tag{8}$$

Inserting the 4-acceleration (6) gives:

$$T^{0\nu}_{;\nu} = b^0(\mathbf{T}) + \frac{\mathbf{f}.\mathbf{v}}{c\beta}, \qquad T^{i\nu}_{;\nu} = b^i(\mathbf{T}) + f^i, \tag{9}$$

where

$$b^0(\mathbf{T}) \equiv \frac{1}{2\beta^2} g_{ij,0} T^{ij}, \quad b^i(\mathbf{T}) \equiv \frac{1}{2} g^{ij} g_{jk,0} T^{0k}. \tag{10}$$

Dynamical equation for a general continuum

The same dynamical equation for the **T** tensor must apply to any kind of continuum:

Expresses the mass-energy equivalence and the universality of the gravitation force in the framework of assumption (b).

$\Rightarrow$ We assume that the dynamical equation derived for a dust in the presence of a field of external force density **f**, Eq. (9) with (10), is true for a general continuum.

Field tensor and first group of Maxwell eqs

As usual, we assume that the field tensor ***F*** derives from a 4-potential **A**:

$$F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu} = A_{\nu;\mu} - A_{\mu;\nu}. \tag{11}$$

Equivalent to assuming

- that ***F*** is antisymmetric ($F_{\mu\nu} = -F_{\nu\mu}$);
- and that the first group of the Maxwell equations is satisfied:

$$F_{\lambda\mu\,,\nu} + F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} = F_{\lambda\mu\,;\nu} + F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} = 0. \tag{12}$$

LORENTZ FORCE IN A GRAVITATIONAL FIELD

The expression of the Lorentz force should

- be a space vector, invariant by the transformations $x'^0 = \varphi(x^0), \quad x'^i = \psi^i(x^1, x^2, x^3)$;
- reduce to that valid in SR, when the gravitational field vanishes.

Leads to

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \wedge \frac{\mathbf{B}}{c}\right), \qquad (\mathbf{a} \wedge \mathbf{b})^i \equiv e^i{}_{jk}\, a^j\, b^k. \tag{13}$$

Electric and magnetic spatial vector fields have components

$$E^i \equiv \frac{F^i{}_0}{\beta}, \quad B^k \equiv -\frac{1}{2} e^{ijk} F_{ij}. \tag{14}$$

EQN FOR ENERGY-MOMENTUM TENSOR OF EM FIELD

Let $\mathbf{T}_{\text{field}}$ be the energy-momentum tensor of the e.m. field:

$$T_{\text{field}}^{\mu\nu} \equiv \left(-F^{\mu}{}_{\lambda} F^{\nu\lambda} + \frac{1}{4} \gamma^{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} \right) / 4\pi. \qquad (15)$$

Total energy-momentum is $\mathbf{T} = \mathbf{T}_{\text{charged medium}} + \mathbf{T}_{\text{field}}$. Obeys eqn (9) for continuum dynamics, without non-gravitational external force. (Since we assume eqn (9) is universal.)

Charged medium obeys eqn (9), with Lorentz external force. Combining the two using their linearity we get:

$$T_{\text{field}\ ;\nu}^{0\nu} = b^0(\mathbf{T}_{\text{field}}) - \frac{\mathbf{f.v}}{c\beta}, \qquad T_{\text{field}\ ;\nu}^{i\nu} = b^i(\mathbf{T}_{\text{field}}) - f^i. \quad (16)$$

The e.m. field behaves as a "material" continuum subjected to the gravitation and to the opposite of the Lorentz force.

Gravitationally-modified second group

Eq. (16) is equivalent to this *gravitationally-modified 2nd group:*

$$F^{\mu}{}_{\lambda}\, F^{\lambda\nu}{}_{;\nu} = 4\pi b^{\mu}\left(\mathbf{T}_{\text{field}}\right) - 4\pi F^{\mu}{}_{\lambda}\, \frac{J^{\lambda}}{c}. \tag{17}$$

- For a constant gravit. field, reduces to eqn in metric theories.
- In a variable gravitational field ($\Rightarrow b^{\nu} \neq 0$), leads to **macroscopic charge production/destruction**:

$$\hat{\rho} \equiv \left(J^{\mu}\right)_{;\mu} = c\left(\left(\boldsymbol{F}^{-1}\right)^{\mu}{}_{\nu}\, b^{\nu}(\mathbf{T}_{\text{field}})\right)_{;\mu}. \tag{18}$$

- $\boldsymbol{F}^{-1}\ \exists?$ $\quad \det \boldsymbol{F} \neq 0$ is equivalent to $\mathbf{E}.\mathbf{B} \equiv \boldsymbol{g}(\mathbf{E}, \mathbf{B}) \neq 0$.

Consistency with trajectories of photons

Photon trajectories: defined by Newton's second law (2), with $E = h\nu \quad (\nu \equiv \mathrm{d}\, n / \mathrm{d}\, t_{\mathbf{x}})$, without external force **F**.

$\Rightarrow$ Link with eqs. for the e.m. field needs to consider a "dust of photons". I.e.,

$$(T_{\text{field}})^{\mu\nu} = V^{\mu} V^{\nu}. \tag{19}$$

Happens to mean exactly that $\boldsymbol{F}$ is a "null" field.

For such a field, one shows, Maxwell's 2nd group (17) says exactly:

– that the trajectories of the e.m. energy flux, with velocity **u** defined from $\mathbf{T}_{\text{field}}$ by $T^{00}\, u^i = c T^{i0}$, are photon trajectories;

– that one has the continuous form for dust of the energy equation.

CONCLUSION

Particle dynamics: Extension to curved spacetime of the relativistic form of Newton's 2nd law, with $\mathbf{g}$ derived from a spatial potential.

Continuum dynamics: induced from that, when applied to a dust.

Maxwell eqs deduced from continuum dynamics,
using the energy-momentum tensor of the e.m. field.

Consistent with photon dynamics as defined by Newton's 2nd law.

Macroscopic charge production/destruction in a variable gravitational field. Dangerous, but: possible link with magnetic fields of astronomical objects, e.g. Earth?

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