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Prediction of the load carrying capacity of bolted timber joints

L. Daudeville, L. Davenne, M. Yasumura

Abstract Failure of bolted timber joints is analyzed experimentally and numerically. In this study, the prediction of the load-carrying capacity of dowel-type joints with one dowel under static loading is based on the analysis of fracture in wood contrarily to most engineering methods that are based on the yield theory. Mechanical joints consist of glued laminated spruce members and steel dowels. In the different analyzed tests, the bolt loads the wood parallel or perpendicular to the grain. The wood member thickness is chosen sufficiently thin to avoid the fastener from presenting plastic hinges. The influences of different structural parameters such as the dowel diameter, the edge- and end-distances are investigated. The fracture propagation analysis is carried out with the Finite Element (FE) method in the framework of Linear Elastic Fracture Mechanics (LEFM). The only identified parameter is the critical energy release rate in mode I (G_{Ic}). The comparison between experimental and numerical results shows that the fracture must be considered for a correct prediction of the ultimate load and that LEFM can help to improve design codes.

Introduction

Most engineering methods for the design of bolted or nailed joints in timber are based on the Johansen's yielding theory (1949) (EC5, 1993) (Aune et al., 1986) which assumes plasticity in both the wood and the fastener. For the purpose of a reliable design of joints, fracture of wood should be considered in a large class of mechanical joints because it may lead to brittle failure especially in the case of a loading perpendicular to the grain. Some recommendations exist in design codes about spacing, end- and edge-distances ($a_{3,t}$ and $a_{4,t}$, Fig. 1) in order to avoid

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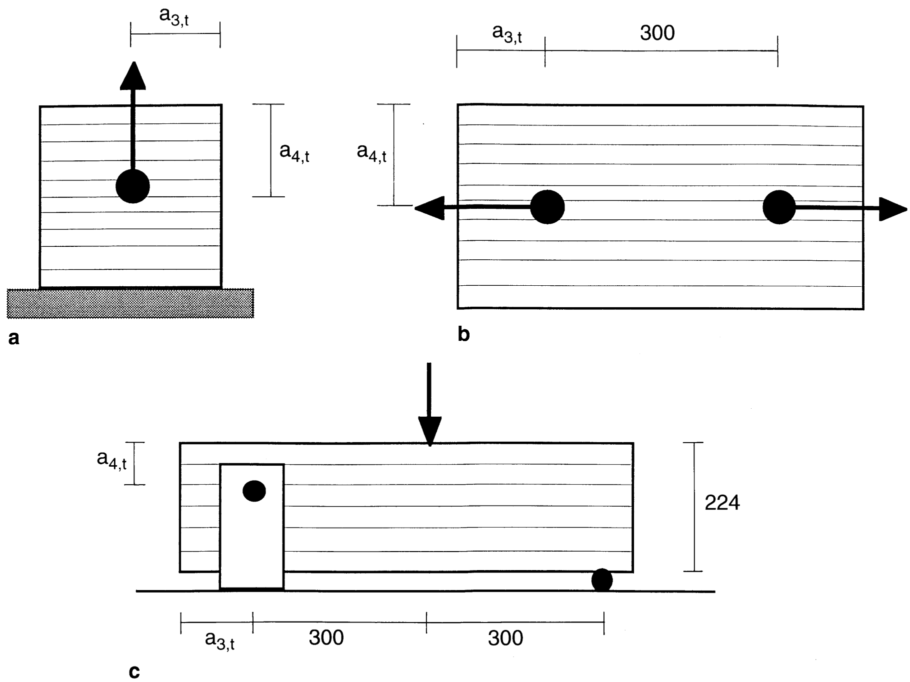

Fig. 1a–c. Specimens: **a** tension perpendicular to the grain, **b** tension parallel to the grain, **c** bending; $a_{3,t}$ = end-distance, $a_{4,t}$ = edge-distance

brittle failure in connections, but they are essentially based on empirical rules. The design recommendations for multiple fasteners joints are generally based on the response of the single bolt joint.

Application of fracture mechanics to structural elements concerned essentially beams with end splits, side cracks, knots and notches (Valentin et al., 1991). Fracture mechanics methods have been scarcely applied to the analysis of mechanical joints. Wilkinson et al. (1981), Rahman et al. (1991) and Bouchain et al. (1996) made FE analyzes of joints loaded parallel to grain. These studies are very interesting investigating the influences of bolt-spacing, of end- and edge-distances of a multiple fastener joint, but the ultimate loads were not predicted.

The aim of this paper is to show that LEFM provides an efficient tool for the prediction of the load-carrying capacity of timber mechanical joints. Coupled with an FE analysis, it allows completing engineering methods proposed in codes for the design of timber mechanical joints by taking into account the influence of a possible crack and of structural parameters.

Three test programs were carried out corresponding to three kinds of loading: tension perpendicular to the grain (Fig. 1a), tension parallel to the grain (Fig. 1b), and bending (Fig. 1c). In order to prevent the dowel from bending, the wood members are relatively thin compared to the dowel diameter (d). The influences of d, of the end- and edge-distances ($a_{3,t}$ and $a_{4,t}$) are investigated.

The failure mode is a propagation of a crack parallel to the grain. Some embedding of the bolt into the wood was sometimes observed prior to failure.

A simplified approach is proposed for the prediction of the ultimate load of these tests. The crack propagation is analyzed by the use of LEFM and with the FE code CASTEM 2000 of CEA (Commissariat à l'Energie Atomique) on the assumption of elastic bodies. According to experimental observations, this assumption seems valid for a load perpendicular to the grain (Fig. 1a and 1c) but it is certainly coarse in the latter case (Fig. 1b). A two dimensional analysis is carried out in a plane perpendicular to the fastener because the bolt does not bend. The influence of friction was investigated (Talland, 1996). Essentially, it has an influence on the location of crack initiation. It seems possible to neglect friction at the steel and wood contacts in the study of propagation. The presented results are based on this assumption.

In the case of bending or tension parallel to the grain, the stress state is a combination of shear and tensile stress perpendicular to the grain. Thus, a possible mixed mode of fracture (mode I and II) is investigated, the two energy release rates G_I and G_{II} are computed by the local crack closure technique.

A Wu's criterion (1967) based on G_{Ic} is used for the propagation analysis. The critical energy release rate value is chosen in order to obtain the best comparison between the results of tests performed on joints and numerical results.

The ultimate load of each test is computed according to the present analytical theory used in (EC5, 1993). On one hand, it is based on the Johansen yield theory and on the other on a shear stress criterion. In order to compare experimental, numerical and analytical results, mean material parameters are used. The mean embedding strength of spruce is derived from experimental results (Chaplain, 1996). The design loads (EC5, 1993) are calculated with characteristic material parameters and compared with experimental results.

Experiment

All the tests were conducted on glued laminated spruce with no knots and no glue interface near the loaded hole. There was no allowance between the dowels and the wood. Two steel plates connected with steel dowels load the wood member. No plasticity in steel elements was observed.

Tension perpendicular to the grain

Tests were performed at LMT Cachan, France (Fig. 1a). The test results were already presented in Daudeville et al. (1996). The mean density and moisture content of the wood were 460 kg/m^3 and 10% respectively. Table 1 gives the structural parameters and the experimental maximum loads. The edge- and end-distances are greater than the EC5 minimum recommended distances.

Table 1. Specimen configurations and ultimate loads in tension perpendicular to the grain

Reference	d (mm)	t_2/d	$a_{3,t}/d$	$a_{4,t}/d$	P_{exp} (kN/m)	nb (COV %)	P_{EC5} (kN/m)
A1				4	140	5 (3)	112
A2	12	3	7	8	148	4 (10)	224
A3				12	143	3 (5)	247
B1	16	3	7	4	204	3 (12)	149
B2	20	3	7	4	230	4 (9)	187
C1				4	173	2 (1)	112
C2	12	3	25	8	223	2 (12)	224
C3				12	264	2 (10)	247

A stable crack propagation could be observed, especially with $a_{3,t} = 25d$. In general the crack propagation was not symmetrical. A crack could propagate on one side, stop its propagation, and then a crack could propagate on the other side of the loaded hole. The non-linearity of the load-slip curve was not very important.

Tension parallel to the grain

Tests were performed at BRI Tsukuba, Japan (Fig. 1b). The test results were already presented in Davenne et al. (1996). The mean density and moisture content of the wood were 406 kg/m^3 and 10% respectively. Table 2 gives the structural parameters and the experimental maximum loads. Some end-distance ($a_{3,t}$) values were chosen to be less than the minimum value recommended in EC5. For large end-distance values, an important non linearity of the load-slip curve due to the embedding of the two bolts into the wood could be observed (Fig. 2). For short end-distance values, the behaviour could be very brittle. 80% of the specimens developed a central crack.

Bending

Tests were performed at BRI (Fig. 1c). The test results were presented in (Yasumura et al., 1987). Table 3 gives the structural parameters and the experimental maximum loads. Other information are unknown.

Present theory of Eurocode 5

The present theory of EC5 is based on the Johansen's yield theory (1949). A shear criterion also has to be verified in order to avoid brittle failure with a load perpendicular to the grain. According to the classical yield theory classification (EC5-6.2.2.h), (Aune et al., 1986), the failure mode is a mode 1 double shear (different from the first mode of fracture).

Table 2. Specimen configurations and ultimate loads in tension parallel to the grain

Reference	d (mm)	t_2/d	$a_{4,t}/d$	$a_{3,t}/d$	P_{exp} (kN/m)	nb (COV %)	P_{EC5} (kN/m)
D1				2.5	266	4 (4)	
D2	8	2	3	4	324	5 (10)	233
D3				7	312	5 (15)	
D4				10	336	5 (13)	
E1				2.5	332	4 (41)	
E2	12	2	3	4	382	4 (30)	334
E3				7	421	5 (19)	
E4				10	422	5 (12)	
F1				2.5	300	3 (30)	
F2	16	2	2.5	4	531	3 (11)	425
F3				7	576	3 (1)	
G1				2.5	313	3 (20)	
G2	16	4	2.5	4	568	3 (11)	425
G3				7	621	3 (1)	
G4				10	715	3 (8)	
H1				2.5	459	5 (14)	
H2	20	2	3	4	541	5 (23)	506
H3				7	681	3 (28)	
H4				10	751	5 (9)	

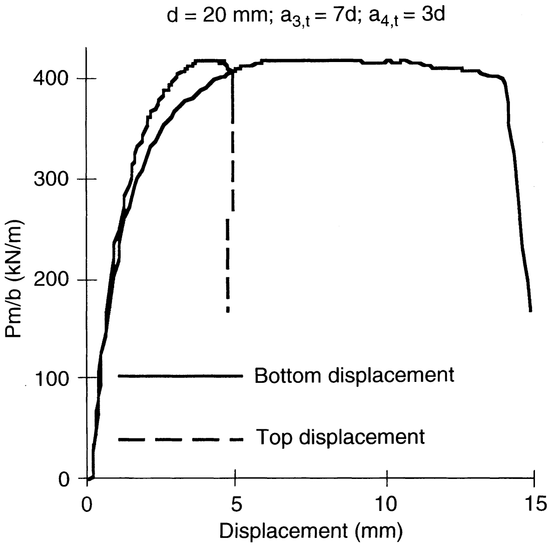

Fig. 2. Load-slip curves for a tension parallel to the grain

EC5 recommendations are based on characteristic values (five percentile values, subscript k). The design load on the dowel per unit wood member thickness for the three problems is examined here (Fig. 1):

$$P_{EC5} = \min\begin{cases} (k_{mod}/\gamma_M) f_{h,\alpha,k} d & (1) \\ 2/3 f_{v,k} a_{4,t} & (2) \end{cases}$$

$f_{h,\alpha,k}$ is the characteristic embedding strength for a load at an angle α with the grain, d is the bolt diameter. k_{mod} takes into account the variation of the loading with time (EC5-3.1.7), it is chosen: $k_{mod} = 1$ and $\gamma_M = 1$. For glulam, the characteristic density (ρ_k) and shear strength ($f_{v,k}$) can be obtained with the mean density (ρ) from the norm prEN1194.

Note that criterion (2) does not take into account the real degradation mode that is a cracking parallel to the grain.

The characteristic embedding strength ($f_{h,\alpha,k}$) (N/mm^2) is (EC5-6.5.1.4):

Table 3. Specimen configurations and ultimate loads on the dowel in bending

Reference	d (mm)	t_2/d	$a_{3,t}/d$	$a_{4,t}/d$	P_{exp} (kN/m)	nb (COV %)	P_{EC5} (kN/m)
I1	16	4	4	4	184	3 (15)	119
I2				7	260	3 (4)	209
J1	16	8	4	4	187	3 (10)	119
J2				7	253	3 (7)	209
K1				4	244	3 (17)	119
K2	16	4	7	7	287	3 (7)	209
K3				10	465	3 (16)	267
L1				4	193	3 (35)	119
L2	16	8	7	7	356	3 (6)	209
L3				10	423	3 (5)	267

$$\begin{cases} f_{h,0,k} = 0.082(1 - 0.01d)\rho_k \\ f_{h,90,k} = f_{h,0,k}/k_{90} = f_{h,0,k}/(1.35 + 0.015d) \end{cases} \tag{3}$$

d is in mm, ρ_k is in (kg/m^3). (3) is derived from experimental determinations of the embedding strength according to an ASTM test (Larsen, 1973) (Fig. 3).

EC5 recommends an end-distance greater than 7d and an edge distance greater than 4d (load perpendicular to the grain) and 2d (load parallel to the grain). Note that the yield theory (1) does not take into account the influence of these distances. The edge-distance is taken into account in (2).

The comparison between design loads (P_{EC5}) and experimental ones (P_{exp}) is important, from a safety point of view. But it in order to estimate the validity of a model by comparisons with mean experimental results, it is necessary to use mean material values in (1)–(3) (no subscript k).

An experimental program for the determination of the mean embedding strength ($f_{h,\alpha}$) of spruce was carried out (Chaplain, 1996). The ASTM tests used in (Larsen, 1973) were performed on 179 specimens with a unique dowel diameter (14 mm). The mean density and moisture content were 400 kg/m^3 and 8.7% respectively. Fig. 3 shows the specimens and the observed degradation modes. Note that a central crack is generally observed for a load parallel to the grain as observed in joint tests.

It is assumed that the dependence of $f_{h,\alpha}$ with d proposed by Larsen in (3) is valid. Then the mean embedding strength of spruce parallel to the grain ($f_{h,0}$) for every density and bolt diameter can be obtained with the experimental strength for the considered diameter and mean density (36.2 MPa) (Chaplain, 1996):

$$\begin{cases} f_{h,0} = 0.106(1 - 0.01d)\rho \\ f_{h,90} = f_{h,0}/k_{90} = f_{h,0}/(1.35 + 0.015d) \end{cases} \tag{4}$$

The calculation of $f_{h,90}$ with k_{90} given in (3) and (4) gives an excellent concordance with the experimental value obtained in tests (24.3 MPa).

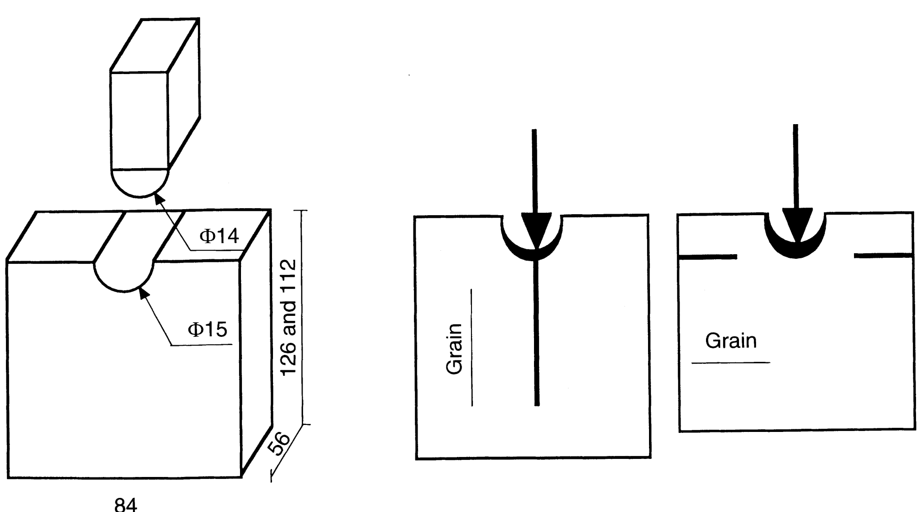

Fig. 3. ASTM specimens and degradation modes (Chaplain, 1996)

According to the present theory, the mean ultimate load per unit thickness is used in EC5:

$$P = \min \begin{cases} f_{h,\alpha} d \\ 2/3 f_v a_{4,t} \end{cases} \tag{5}$$

Table 4 gives the characteristic and mean parameter values used in (1)–(5) relative to the three configurations (a), (b) and (c) (Fig. 1) with d = 16 mm.

The EC5 design loads are given in Tables 1, 2 and 3. The mean ultimate loads according to (1)–(5) are given in Fig. 8–12.

Modeling

The modeling assumptions are:

H1: Plane stress state – H2: Wood is linear-elastic to failure – H3: The bolt is a rigid body – H4: The transverse T and radial R directions are not distinguished – H5: Friction, on the wood and bolt contact is neglected.

The assumption H5 is valid for propagation analyzes (Talland, 1996) but friction is important for the determination of the crack initiation location.

In a plane stress problem, the radial R and transverse T directions cannot be distinguished, so a mean behavior is considered . The elastic moduli were chosen by extrapolation of the results (Guitard, 1987) with respect to the mean density. x direction corresponds to the L direction.

$$\begin{cases} E_x = E_L = 15000\,\text{MPa}; \quad E_y = \frac{E_T + E_R}{2} = 600\,\text{MPa} \\ G_{xy} = \frac{G_{TL} + G_{RL}}{2} = 700\,\text{MPa}; \quad v_{xy} = 0.5 \end{cases} \tag{6}$$

Crack propagation analysis

LEFM assumes that all non linear phenomena are concentrated at the crack tip. Non linear fracture mechanics or damage mechanics consider a process zone where non linear damage phenomena occur. For the studied problem, the size of the process zone can be neglected compared with the dimensions of the crack and of the structure. In such a case, the LEFM approach can be applied.

The energy release rate is:

$$G(P,a) = G_I + G_{II} = -\frac{1}{t_2}\frac{\partial W}{\partial a} \tag{7}$$

Table 4. Density and strength: mean and characteristic values

Tests (Fig. 1)	(a)	(b) and (c)
ρ (kg/m^3)	460	406
ρ_k = 0.95 ρ (kg/m^3) (prEN1194)	437	386
f_v (N/mm^2) (CTBA, 1995)	8	8
$f_{v,k}$ (N/mm^2) (prEN1194)	3.5	2.8
$f_{h,0}$ (N/mm^2) (4)	41	36.1
$f_{h,0,k}$ (N/mm^2) (3)	30.1	26.6
$f_{h,90}$ (N/mm^2) (4)	25.8	22.7
$f_{h,90,k}$ (N/mm^2) (3)	18.9	16.7

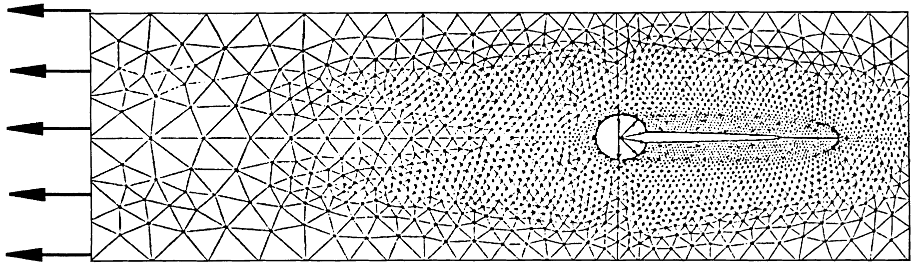

Fig. 4. Finite element mesh for a tension parallel to the grain and a central crack

P is the load applied to the structure, W is the potential energy, a is the crack length, t_2 is the wood member thickness.

The computation of the global energy release rate can be carried out by means of two FE calculations 1 and 2. The crack propagation is modeled by separating two connected lines (Fig. 4–5):

$$G = \frac{P^2}{2(a_2 - a_1)}\left(\frac{1}{k_2} - \frac{1}{k_1}\right)t_2 \tag{8}$$

With $\Delta a = a_2 - a_1 \ll a$; k stiffness; P(N/m).

Eq. (8) is obtained on the assumption that the applied load P does not vary during the elementary crack increment. A similar relation can be obtained with prescribed displacement. Both methods lead to very similar results (Laflotte, 1997).

Eq. (8) can be used only in the case of a pure mode of fracture in association with a Griffith's criterion (1920). Because of the orthotropic feature of wood, a partition of modes is necessary in order to use the following criterion issued from the Wu's one (1967) that was originally based on stress intensity factors:

$$\sqrt{\left(\frac{G_I}{G_{Ic}}\right)} + \left(\frac{G_{II}}{G_{IIc}}\right) = 1 \tag{9}$$

The previous criterion is the Griffith's one in the case of a pure mode I or II. G_I and G_{II} are obtained separately by a local method, the crack closure technique that is based on the necessary work to close the crack during a propagation Δa:

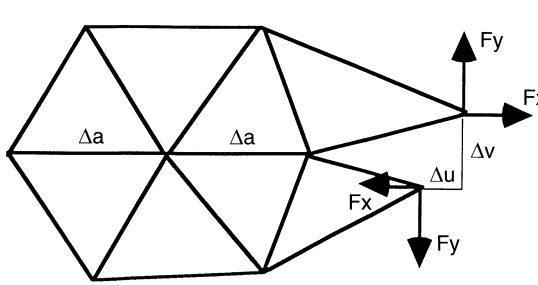

Fig. 5. Crack closure technique

$$G_I = \frac{1}{2t_2}\frac{F_y \cdot \Delta v}{\Delta a}; G_{II} = \frac{1}{2t_2}\frac{F_x \cdot \Delta u}{\Delta a} \tag{10}$$

where F_x and F_y are the nodal forces in the grain (x) and perpendicular to the grain (y) directions (obtained in the first FE computation). Δu and Δv are the relative displacements of the released node in the x and y directions (second FE computation) (Fig. 5).

In both the local and the global method, the mesh refinement at the crack tip is constant during the crack propagation.

According to Valentin et al. (1991), Mansfield-Williams (1995) and Petersson (1995) a good approximation is:

$$G_{IIc} = 3.5 G_{Ic} \tag{11}$$

The load P that leads to a crack propagation of a joint is computed for different crack lengths with (8)–(11) and for a given G_{Ic}. The maximum load gives the calculated load carrying capacity P_{calc} of the joint and the critical crack length. Fig. 6 shows the computed load with respect to the crack length of a joint loaded perpendicular to the grain.

Fracture initiation

Experimentally, the initiation of a crack was always observed at $\theta = 0°$ (Fig. 7) for a perpendicular tension. Under bending, the experimental observations show that the initiation angle is $\theta = 0°$ but the way of propagation (from the dowel to the edge or from the dowel to the central load) is a priori unknown. In the case of a tension parallel to the grain, θ was in general close to zero but sometimes an initiation at an angle θ between 0° and 45° was observed.

Results

Identification

The fracture process is a priori unknown (initiation location, one non-symmetrical crack or two symmetrical cracks). Thus different fracture processes were

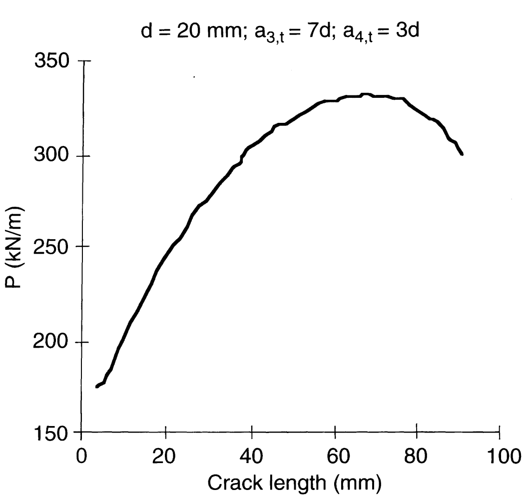

Fig. 6. Load versus crack length in tension perpendicular to the grain

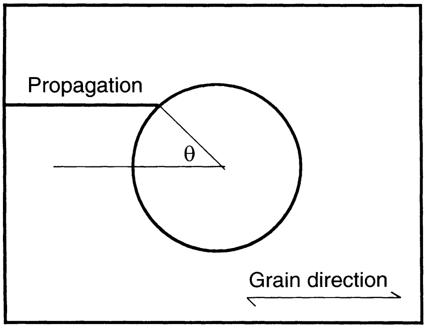

Fig. 7. Initiation of cracking

analyzed for each test. The best concordance between the calculated and experimental maximum loads gives the fracture process.

The only unknown material parameter G_{Ic} is identified in order to obtain the best comparison between experimental and numerical results. Note that the knowledge of the fracture process is very important. For instance for a tension perpendicular to the grain, the propagation of two symmetrical cracks needs

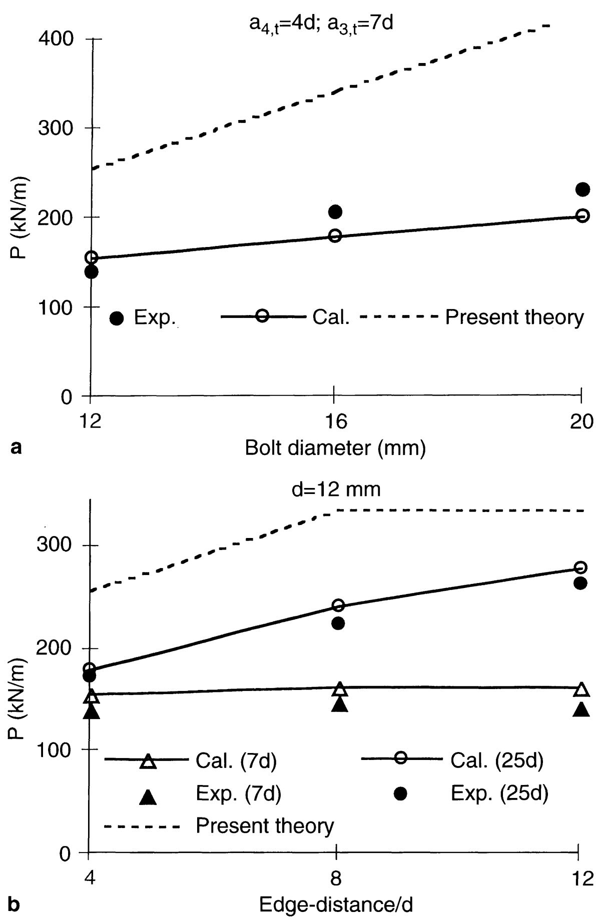

Fig. 8a, b. Tension perpendicular to the grain about twice the energy necessary to propagate one single crack. Fig. 8–12 give the calculated load-carrying capacities of joints with:

$$G_{Ic} = 100\,\mathrm{Nm/m^2} \tag{12}$$

This value is about half of the fracture energy G_f obtained with classical fracture tests (Daudeville et al., 1996) and can be considered as low. Note that G_f and G_{Ic} are equal for a perfectly brittle material only and that the ultimate load depends on the square root of G_{Ic}. Thus, an error of 50% on the estimation of G_{Ic} leads to an error of 22.4% only on the estimation of the ultimate load.

Discussions

Tension perpendicular to the grain (Fig. 1a, Fig. 8–9).
According to the simulations, the more probable fracture process from the initiation to the maximum load is the propagation of only one crack on one side of the loaded hole. A second crack can be obtained after this maximum load.

Fig. 8a gives the influence of the dowel diameter, Fig. 8b gives the influence of the edge-distance for two end-distances. LEFM allows correct predictions and gives the good trends. The present theory's EC5 predictions are much greater than experimental results. It is alarming to notice that the design load is on the unsafe side for joints A2, A3 and C2.

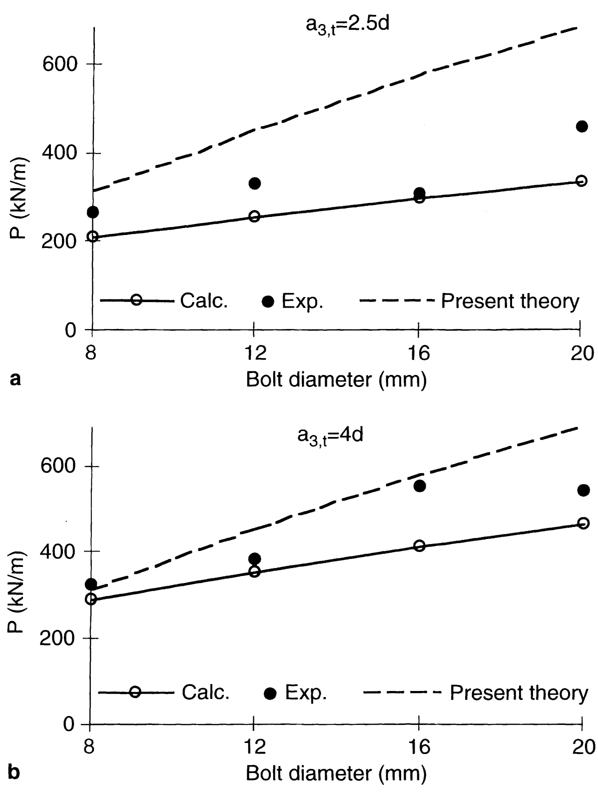

Fig. 9a, b. Tension parallel to the grain – Influence of bolt diameter – Short end-distances

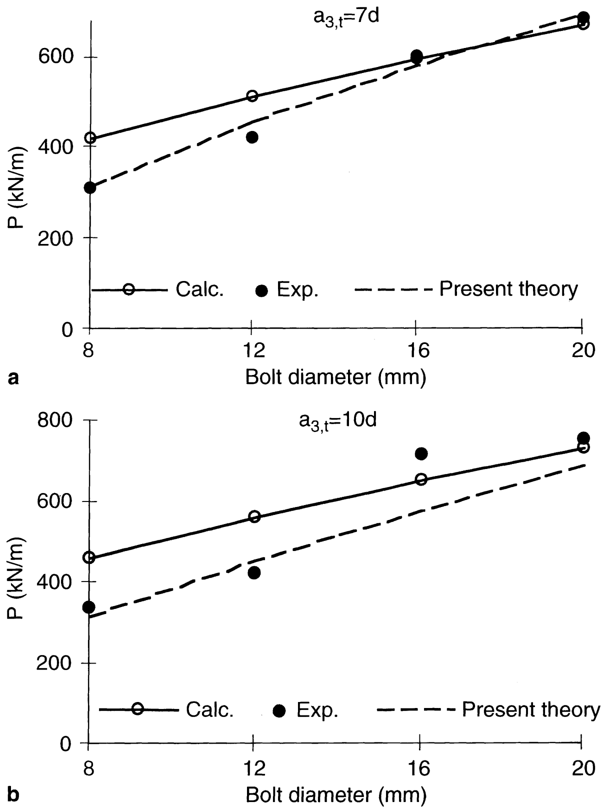

Fig. 10a, b. Tension parallel to the grain – Influence of bolt diameter – Conforming to the EC5 end-distances

In that case, the present theory of EC5 cannot predict correctly the load-carrying capacity of the joint because the main degradation that leads to failure is cracking.

Also note that the yield theory does not take into account the influences of the end- and edge-distances that are correctly described by the FE analysis.

Tension parallel to the grain (Fig. 1b, Fig. 9–11)

The experimental results of Fig. 9–11 with d = 16 mm are an average of tests F and G. According to the simulations, the more probable fracture process is a cracking in the mid-plane (Fig. 4). This is confirmed by the experimental observations.

Fig. 9–10 give the influence of the bolt diameter for different end-distances. The FE analysis gives correct results and trends.

Fig. 11 gives the influence of the end-distance for two bolt diameters. The yield approach gives good results for an end-distance greater than 7d that is the minimum requirement according to EC5. This is normal because the embedding strength was identified for an end-distance close to that value. Also note that the yield theory does not take into account the influence of the end-distance. LEFM could give information on this influence if it was decided to reduce the present minimum requirement.

In that case, the present design recommendations of EC5 are correct because yielding occurs (Table 2).

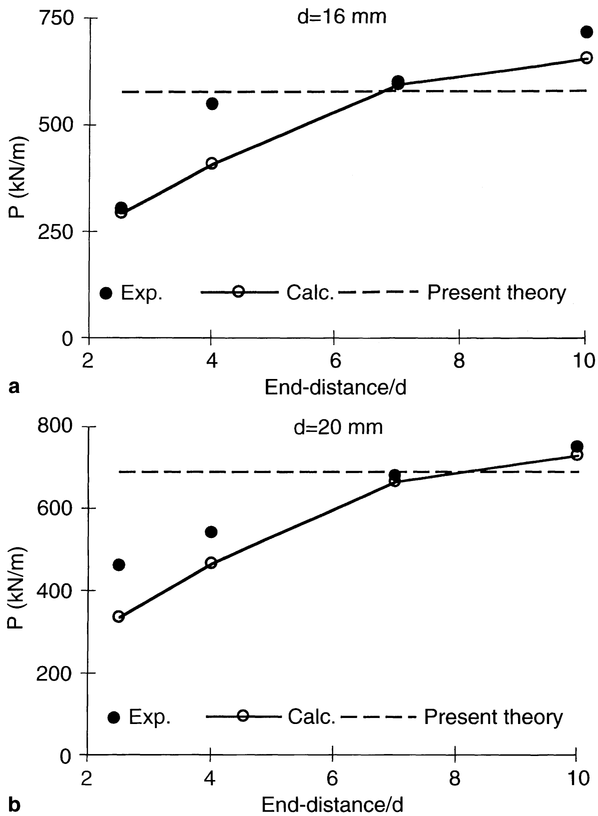

Fig. 11a, b. Tension parallel to the grain – Influence of the end-distance

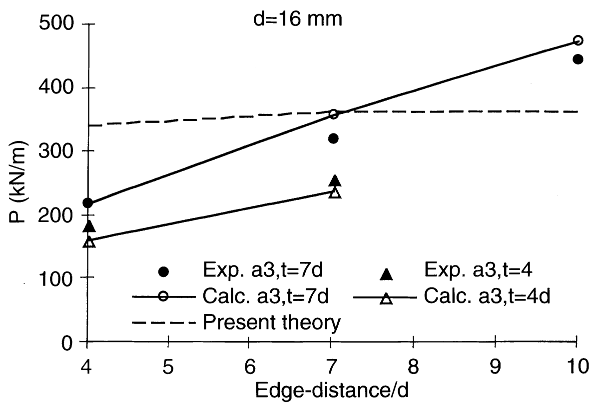

Fig. 12. Bending. Influence of the end- and edge-distances

Bending (Fig. 1c, Fig. 12)

The experimental results of Fig. 12 are an average of, on one hand, tests I and J and, on the other hand of tests K and L.

The solicitation on the bolt depends a lot on the edge-distance ($a_{4,t}$). The higher $a_{4,t}$, the more sheared is the wood around the bolt. In this case, a mixed mode of fracture occurs (tearing + shear).

The way of propagation depends on $a_{4,t}$. According to the simulations, the more probable fracture process from the initiation to the maximum load, is always a crack propagation from the bolt to the edge except for $a_{4,t} = 10d$. In the latter case the crack propagates from the bolt to the central load.

Once again, LEFM gives excellent results and is able to predict the influence of the structural parameters. The present theory of EC5 does not give the correct trends and predictions of the load-carrying capacity.

Conclusion

LEFM is a simplified approach consisting in the comparison of the energy release rate with a critical value. This method has been applied for the determination of the load carrying-capacity of mechanical joints with a single bolt. Correct predictions confirm that failure of joints is fracture controlled. This approach can be considered as a possible tool to complement the present codes.

Present theory of Eurocode 5 is based on the yield theory and on a shear criterion. The influences of structural parameters that are not taken into account in the present theory may be considered with LEFM.

The yield theory is valid when the load is parallel to the grain. This approach is very simple, it is analytical. It is particularly convenient when the main degradation phenomenon is plasticity rather than splitting. In complex joints, perpendicular to the grain stresses generally exist.

The shear criterion does not take into account the actual degradation process for a load perpendicular to the grain and does not allow correct predictions of the load carrying capacity of single bolt joints.

References

Aune P, Patton-Mallory M (1986) Lateral load-bearing capacity of nailed joints based on the yield theory: Theoretical developments. Res. Pap. FPL 469, Madison, WI. U.S. Dept. of Agriculture, Forest Service, Forest Products Laboratory

Bouchain A, Racher P, Vergne A (1996) Effet des conditions aux limites sur le comportement des assemblages bois boulonnés, Materials and Structures 29: 372–382

Chaplain M (1996) Comportement sous sollicitations sévères des assemblages de structures en bois, Thesis, Université Blaise Pascal, Clermont-Ferrand

C.T.B.A. 1995: Étude des caractéristiques mécaniques du sapin et de l'épicéa – France entière, Rapport interne, C.T.B.A. Paris, France

Daudeville L, Davenne LM, Yasumura M (1996) Experimental and numerical analysis of failure in bolted joints, International Wood Engineering Conference '96, New Orleans USA, V. Gopu Ed. Louisiana State Univ. 4: 129–135

Daudeville L, Yasumura M (1996) Failure analysis of timber bolted joints by fracture mechanics, Materials and Structures 29: 418–425

Davenne L, Daudeville L, Yasumura M (1996) Failure of bolted joints loaded parallel to the grain: Experiment and Simulation, Proceedings of the 29th CIB-W18 Meeting, Bordeaux, France, paper 29-7-8

EC5 (Eurocode 5) (1993) Design of timber structures – Part 1-1: General rules and rules for buildings, ENV 1995-1-1, CEN, Brussels, Belgium

Griffith A (1920) The phenomena of rupture and flow in solids, Philosophical Transactions of the Royal Society of London, Series A, 221: 163–198

Guitard D (1987) Mécanique du matériau bois et composites. Cépadues-Editions, Toulouse, France

Johansen KW (1949) Theory of timber connections, Inter. Assoc. of Bridge and Structural Engineering, Bern, Switzerland, 9: 249–262

Laflotte J (1997) Méthodes simplifiées de calcul des structures en bois, Master thesis, CUST Clermont-Ferrand

Larsen HJ (1973) The yield load of bolted and nailed joints. Proceedings IUFRO 5-02, South-Africa, 646–654

Mansfield-Williams HD (1995) A new method for determining fracture energy forward shear along the grain, Proceedings of the 28th CIB-W18 Meeting, Copenhagen, Denmark, paper 28-19-2
Petersson H (1995) Fracture design analysis of wooden beams with holes and notches, Proceedings of the 28th CIB-W18 Meeting, Copenhagen, Denmark, paper 28-19-3
Rahman MU, Chiang YJ, Rowlands RE (1991) Stress and failure analysis of double-bolted Joints in Douglas-fir and Sitka spruce, Wood and Fiber Science, 23(4): 567–589
Talland O (1996) Simulation de la rupture d'un assemblage en bois boulonné, Master thesis, ENS Cachan
Valentin G, Boström L, Gustafsson PJ, Ranta-Maunus A, Gowda S (1991) Application of fracture mechanics to timber structures. RILEM state-of-art Report, Technical Research Centre of Finland, Espoo, Finland, Research Notes 1262
Wilkinson TL, Rowlands RE (1981) Influence of elastic properties on the stresses in bolted joints in wood. Wood Sci. 14(1): 15–22
Wu EM (1967) Application of fracture mechanics to anisotropic plates, Journal of Applied Mechanics, Series E, 34(4): 967–974
Yasumura M, Murota T, Sakai H (1987) Ultimate properties of bolted joints in glued laminated timber, Proceedings of the 20th CIB-W18 Meeting, paper 20-7-3