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Numerical Simulation of Soda-Lime Silicate Glass Tempering

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Abstract: The paper deals with the computation of residual and transient stresses in a tempered glass plate. The modelling takes into account the viscoelastic behavior and the structural relaxation of glass. The evolution of stresses with time during the rapid cooling is computed. Simulation results are compared with experimental ones from the literature available. Levels of transient tensile stress in the surface are analysed.

1. INTRODUCTION

The determination of stresses during the tempering of plates made of soda-lime silicate glass is investigated. Several models exist to characterize the behavior of glass. The "Instant Freezing" theory developed by Bartenev in 1948 [1] [2] [3], is based on the hypothesis that the liquid glass is suddenly changed into an elastic glass solid at the transition temperature (T_g). In the "viscoelastic" theory, Indenbom (1962 [4] [2]) takes into account the relaxation of stresses (variations of the stresses in glass with time and temperature). Narayanaswamy (1969 [5] [6] [2]) developed a model that introduces both the state of the structure (i.e. the structural relaxation) and stress relaxation due to viscoelastic behavior.

The uniform tempering of an infinite plate of glass on its two sides is studied. The Narayanaswamy's model is used for the computation of transient and residual stress state through the thickness of the plate during the quenching. Then, computational results obtained using the Finite Element code MARC are compared with experimental results found in literature. The residual stress variations through the thickness of the plate is given. The well known interest of tempering is shown: the residual stress state is such that the core is under tension and the surface of the plate is under compression.
A tensile stress occurs on the plate surface at the beginning of the tempering. The dependence of this transient stress with the initial temperature and the cooling rate is presented.

This work is a preliminary study supported by the B.S.T.C. (Building Scientific and Technical Centre) for the application of tempered glass to building structure parts. The residual compressive stress in the surface provides a strengthening of the plate. So, the safety of glass elements in structures can be evaluated. The study of transient tensile stress in surface gives some information for the analysis of possible propagations of surface defects that may weaken the elements of glass.

2. MODELLING QUENCHING OF GLASS

2.1. Quenching

Quenching of glass consists in heating the whole plate to a temperature close to 650 °C and in cooling it down suddenly to room temperature within a few tens of seconds [7] [8]. Figure 1 presents the variations of temperature with time in surface and midplane according to the simulation results. Note that the cooling is more rapid at the surface than in the inner part of the glass. So, exists a gradient of temperature through the thickness of the glass plate that causes the residual stress state. In the first seconds of cooling, the difference of temperature between the surface and the midplane is greater than 150 °C.

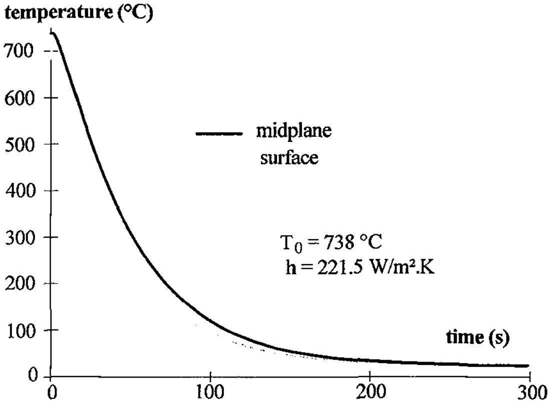

Figure 1: Surface and midplane temperatures versus time

2.2. Mechanical behavior of glass

2.2.1. Stress-strain relation

At room temperature (20°C), the glass can be modelled as an elastic solid and for high temperature (1100°C), the glass is a viscous fluid. For intermediate temperatures, the glass can be considered as a viscoelastic material.

The stress-strain relation can be written in two different ways for a viscoelastic material [9]: stress relaxation and creep. For a unidimensional load of strain, the relaxation of stress with time is:

$$\sigma(t)=\int_{-\infty}^{t} G(t-t')\frac{d\varepsilon(t')}{dt'}dt'$$

with σ = stress
ε = strain
G = modulus of relaxation

The previous relation can be generalized to a three-dimensional stress state by separating the behavior into a deviatoric part and a volume part with:

$$\begin{cases} s_{ij}(x,t)=\int_{-\infty}^{t} G_1(t-t')\frac{\partial}{\partial t'}e_{ij}(x,t')dt' \\ \bar{\sigma}(x,t)=\int_{-\infty}^{t} G_2(t-t')\frac{\partial}{\partial t'}\bar{\varepsilon}(x,t')dt' \end{cases}$$

with $\sigma_{ij} = s_{ij} + \bar{\sigma}\,\delta_{ij}$
$\varepsilon_{ij} = e_{ij} + \bar{\varepsilon}\,\delta_{ij}$
G_1 = modulus of relaxation in shear
G_2 = modulus of relaxation in hydrostatic pressure

We can also define the compliance of creep in shear and hydrostatic pressure (J_1 and J_2) in a similar way.

2.2.2. Generalised Maxwell's model

The generalised Maxwell's model is described by the rheological model presented in Figure 2. It is a particular case of the previous modelling of stress relaxation.

- for the deviatoric part:

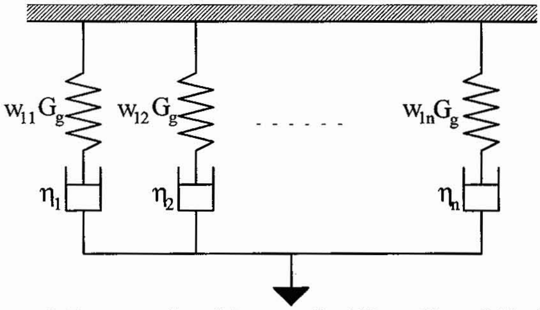

Figure 2: Representation of the generalised Maxwell's model in shear

$$G_1(t) = 2G_g\Psi_1(t)$$

$$\text{with } \Psi_1(t) = \sum_{i=1}^{n_1} w_{1i} \exp\left(-\frac{t}{\tau_{1i}}\right)$$

$$\tau_{1i} = \frac{\eta_i}{w_{1i}G_g}$$

G_g = instantaneous shear modulus

Note that the asymptotic shear modulus G_g is zero.

- and by analogy for the volume part:

$$G_2(t) = 3K_e - (3K_e - 3K_g)\Psi_2(t)$$

$$\text{with } \Psi_2(t) = \sum_{i=1}^{n_1} w_{2i} \exp\left(-\frac{t}{\tau_{2i}}\right)$$

K_e = asymptotic bulk modulus
K_g = instantaneous bulk modulus

2.3. Thermo mechanical behavior of glass

The glass is a viscoelastic material, and therefore its behavior depends on time. But it also varies with temperature.

2.3.1. Stabilised glass

When its structure is stabilised, the glass is a thermorheologically simple material [10] [11] [2]. This means that a variation of temperature can be replaced by a change of the scale of time (cf. Figure 3). So one of the two variables (time or temperature) can be eliminated. If the function of uniaxial relaxation at a reference temperature (T_{ref}) is known, this function can be calculated for another temperature with the reduced time.

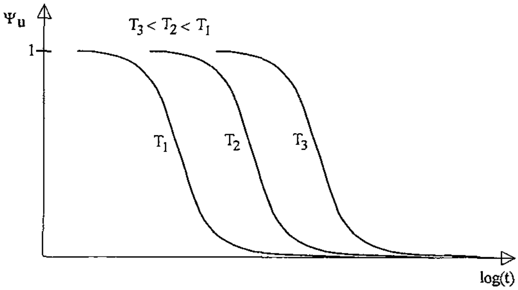

Figure 3: Variations of the uniaxial relaxation function with the logarithm of time at different temperatures

$$\Psi_{u_T}(t) = \Psi_{u_{T_{ref}}}(\xi)$$

$$\text{with} \quad \xi(t) = \int_0^t \frac{\tau_{ref}}{\tau(T,t')} dt' = \text{reduced time}$$

$$\tau_{ref} = \text{relaxation time at } T_{ref}$$

$$\tau(T,t') = \text{relaxation time at } T$$

2.3.2. Structural relaxation

During the tempering, the glass is cooled brutally so its structure isn't stabilised. Then, the structure state must be taken into account, that is the structural relaxation [2] [5] [6] [11] [12] [13].
When a glass is heated to a temperature T_2 (point A on Figure 4) below T_g (temperature of transformation), the structure of the glass continues to change to a state characteristic of the liquid glass (point B on Figure 4). This evolution is the direct structural relaxation represented by AB on Figure 4. This evolution is important if the glass temperature is very close to T_g. If the temperature is far below T_g, this phenomenon is too slow to be observed. If a glass is heated suddenly to a temperature T_1 above T_g, the state of the glass is below the straight line "liquid" (point C on Figure 4). Then, the configuration of the glass moves to the equilibrium state (point D on Figure 4). This evolution is named reverse structural relaxation (represented by CD on Figure 4).

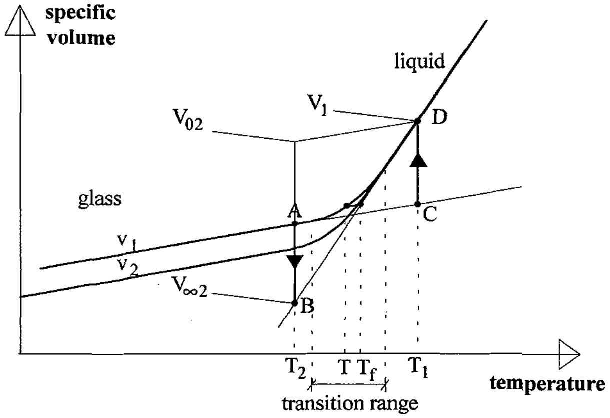

Figure 4: Variations of the specific volume of glass with temperature for two different rates of cooling

In 1946, Tool ([14]) has defined the fictitious temperature (T_f) to characterize the state of the structure of glass. He uses the variations of the volume of glass to describe the variations of T_f (cf. Figure 4):

- $T_f = T$ if T is above the transition range

- T_f = the intersection between the "liquid" and "glass" straight line if T is within the transition range
- T_f = the intersection between the "liquid" straight line and the parallel to the "glass" straight line if T is below the transition range

According to the previous definition of T_f, the response function $M_v(t)$ gives a relationship between the specific volume and the fictitious temperature:

$$M_v(t)=\frac{V(t)-V_{\infty,2}}{V_{0,2}-V_{\infty,2}}=\frac{T_f-T_2}{T_1-T_2}$$

with T_1-T_2 = step of temperature
T_f = instantaneous fictitious temperature at T_2
V = instantaneous specific volume
$V_{0,2}$ = volume just after the temperature change
$V_{\infty,2}$ = equilibrium volume at temperature T_2

The dependency of the response function with temperature is obtained by using the reduced time ξ. $M_v(t)$ can be considered as the volume relaxation function that allows to define the fictitious temperature:

$$T_f(t)=T(t)-\int_0^t M_V[\xi(t)-\xi(t')]\frac{dT(t')}{dt'}dt'$$

The response function can be replaced by the discrete form: $M_v(\xi)=\sum_{i=1}^{n} C_i \exp\left(-\frac{\xi}{\lambda_i}\right)$

A classical Arrhenius model is assumed to be valid to represent the influence of temperature in the viscoelastic behavior i.e. in the reduced time:

$$\xi(t,T,T_f)=\int_0^t \frac{\tau_{ref}}{\tau(T,t')}dt'=\int_0^t a(T(t),T_f(t))dt'$$

with $\ln(a(T,T_f))=-\frac{H}{R}\left(\frac{1}{T_{ref}}-\frac{x}{T}-\frac{1-x}{T_f}\right)$
x = constant ($0<x<1$)
H = energy of activation
R = constant of perfect gas

The fictitious temperature is calculated with the algorithm [15]:

$$T_{f_i}(t)=\frac{T_{f_i}(t-\Delta t)+T(t)\frac{\Delta t}{\lambda_i}}{1+\frac{\Delta t}{\lambda_i}}$$

$$\lambda_i=\lambda_{iref}\exp\left\{-\frac{H}{R}\left[\frac{1}{T_{ref}}-\frac{x}{T(t)}-\frac{1-x}{T_f(t-\Delta t)}\right]\right\}$$

$$T_f(t)=\sum_{i=1}^{n} C_i T_{f_i}(t)$$

$$T_{f_i}(0)=T_0$$

The fictitious temperature is also used to express the thermal strains:

$$\begin{aligned}\varepsilon_{th} &= \beta_g(T(t)-T_f(t))+\beta_l(T_f(t)-T_0)\\ &= \beta(t)(T(t)-T_0)\end{aligned}$$

with β_g = linear expansion coefficient of solid glass
β_l = linear expansion coefficient of liquid glass
T_0 = initial temperature
β = equivalent linear expansion coefficient

Figure 5 gives the evolution of the equivalent linear expansion coefficient that includes the fictitious temperature effect with respect to the actual temperature.

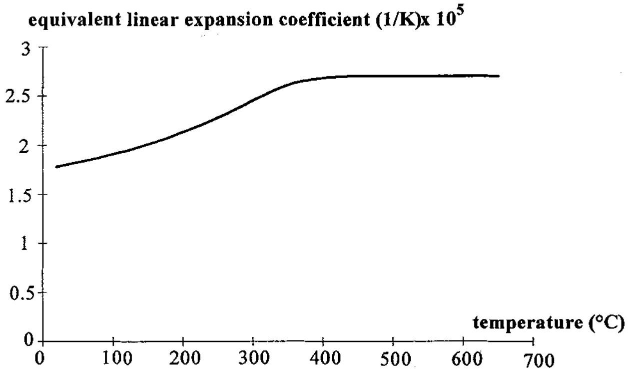

Figure 5: Equivalent linear expansion coefficient versus temperature

3. UNIFORM QUENCHING OF AN INFINITE PLATE

3.1. Modelling

The uniform tempering of an infinite glass plate is modelled (cf. Figure 6). The cooling of glass by air cast going out of the nozzles is modelled by forced convection which is characterized by a thermal transfer coefficient (h) and the air temperature far away from the plate (T_{ext} = 20° C).

The two major assumption of the modelling are:
- the initial temperature is uniform
- the heat transfer due to radiation is neglected

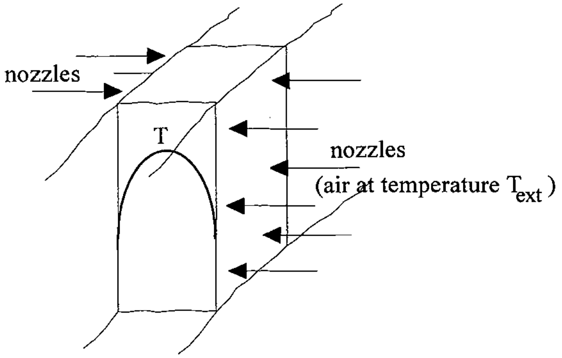

Figure 6: Uniform and symmetrical quenching of an infinite plate

The problem requires a thermo-mechanical calculation. First, the resolution of the heat equation allows to find the temperature at each time and for each point of the structure. Then, a time dependent mechanical problem is solved. The Narayanaswamy's model allows to know the temperature dependency of the glass behavior. Thermal strains constitute the mechanical problem loading.

3.2. Mesh and boundary conditions

The calculations concern the modelling of the tempering of an infinite plate with the finite element method (code MARC). The mesh represents a eighth of a piece of the infinite plate. The boundary conditions are: forced convection on the upper surface, symmetry conditions for 3 sides of the elementary volume and conditions to prevent rotation of the two other sides.

The problem to solve is independent of the in-plane coordinates y and z (cf. Figure 7). So, the mesh is refined only in the direction x. 3-D elements are used.

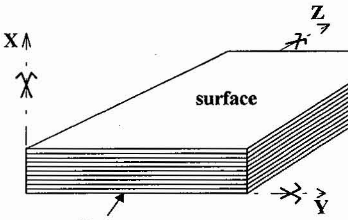

Figure 7: View of the 3-D mesh

3.3. Data

The mechanical characteristics of glass are [16]:

- Young's modulus: $E = 7.2.10^{10}$ Pa
- Poisson ratio: $\nu = 0.2$
- linear expansion coefficient of solid glass: $\alpha_g = 9.10^{-6}\ K^{-1}$
- linear expansion coefficient of liquid glass: $\alpha_l \approx 3\alpha_g = 27.10^{-6}\ K^{-1}$
- density: $\rho = 2500\ kg/m^3$

The viscoelastic characteristics of glass in relaxation are (T_{ref} = 596°C):

deviatory part [12]

w_{1i}	τ_{1i}
0.09256167	0.0002219596
0.1021670	0.001672941
0.1387652	0.02163904
0.1990315	0.2103019
0.2605422	0.9835313
0.2069324	3.376455

volume part [12]

w_{2i}	τ_{2i}
0.02223075	0.000030801
0.02240328	0.00061147
0.02871376	0.0012435
0.2136628	0.011836
0.3940633	0.073738
0.3189240	0.125040

parameters for T_f [15]

C_i	λ_i
0.0561	51.67
0.5074	231.55
0.2163	629.36
0.1320	1710.9
0.0408	4650.0
0.0421	20845

The characteristics for the relation time-temperature are [15]:

- $x = 0,5$
- $\frac{H}{R} = 76200\ K$

The dependency of thermal characteristics with fictitious temperature is neglected. According to [17], the thermal conductivity and the specific heat vary with temperature as:

$$\lambda = 0.975 + 8.581\ 10^{-4}T \quad \text{(W/m.K)} \quad \text{(T in °C)}$$

$$\left.\begin{array}{ll} C_{p,l} = 1433 + 6.5\,10^{-3}T & \text{for liquid glass } (T > T_g \approx 850\,K) \\ C_{p,g} = 893 + 0.4T - 1.8\,10^{-7}/T^2 & \text{for solid glass } (T < T_g) \end{array}\right. \quad \text{(J/kg.K)} \quad \text{(T in K)}$$

These values are in good agreement with various data from literature [15] [18].

4. RESULTS AND DISCUSSION

4.1. State of residual stresses

The surface cools down more rapidly than the inside of the glass. Then the surface contracts more rapidly than the core of the plate. The midplane is in compression and by equilibrium, the surface is in tension. By 580 °C ($\approx T_g$), the surface of the glass plate becomes suddenly solid, while the inner part still remains liquid. The inside continues to cool down, then to contract and the surface comes in compression while the inside remains in tension.

So, during quenching, the sign of transient longitudinal stresses changes as we can see in Figure 8. The residual stresses are: compression in surface and tension in the midplane of the plate. Therefore, at the end of quenching, the glass is prestressed. This thermal treatment strengthens the glass.

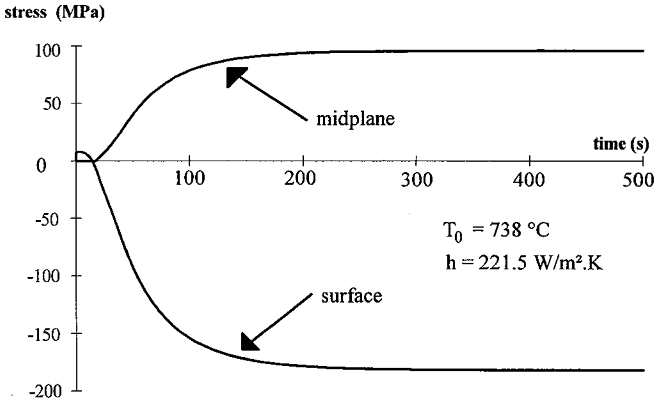

Figure 8: Variations with time of the longitudinal stresses at the surface and in the midplane of the plate

4.2. Comparison of our calculations with experimental results

4.2.1. Variation of residual stress through the thickness of the plate

Figure 9 represents the variations of residual stresses through the thickness of the plate. The results of our calculations are compared with experimental results found in [2] (optical experiments). Tempered glass is transparent and birefringent. Optical experiments constitute an appropriate method to measure stresses. A pencil of rays goes through the glass parallel to the plane of the plate. The birefringence is proportional to the integral of the difference of principal stresses perpendicular to the pencil of rays. The birefringence is measured by the optical path difference. A measure of birefringence is done for each position in the thickness in order to obtain the experimental curve (cf. Figure 9).

The shape of stresses (σ_{yy} or σ_{zz}) is parabolic. The compression in surface is twice greater than the midplane tension. A good agreement between calculations and experimental results is obtained.

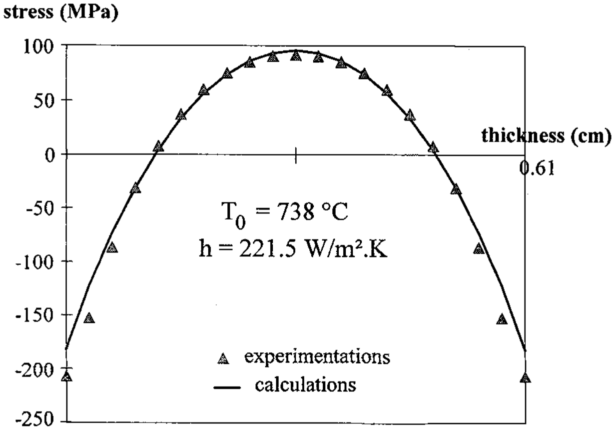

Figure 9: Variations of the residual stresses in the thickness of the plate

4.2.2. Variations of midplane residual stress with h and T_0

In order to test the quality of calculations, the state of residual stresses is predicted for a large range of quenching. Figure 10 shows the midplane tensile residual stress versus initial temperature for different cooling rates. A good agreement can be observed calculations and experimentations [2].

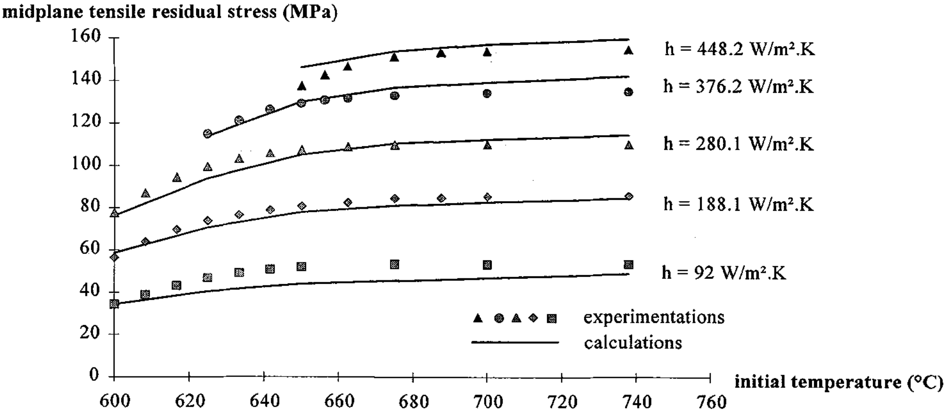

Figure 10: Variations of the midplane residual stress with the initial temperature for different cooling rates

Note that such results are obtained with absolutely no identification of parameters. Some analogous comparisons between experimental and computational results have already been performed [2] [15]. In these studies, results are not good as the presented ones. We think that is due in one hand to the dependence of C_p an λ with temperature that is taken into account in our modelling, and in another hand to a correct description of the viscoelastic behavior in the transition range obtained recently in [12].

4.3. Influence of process parameters h and T_0

Variations of tensile transient stresses with the heat transfer coefficient (h) and the initial temperature (T_0) are shown.. For each value of h, the transient stress increases as the initial temperature decreases. The greater h, the higher this variations with the initial temperature.

The value of T_0 is an important parameter for industrial process. If T_0 is very low (650°C), the plates of glass break at the beginning of the cooling. If T_0 value is very high (700°C), the glass is not stiff enough at the beginning of the cooling and rollers can mark the plate.
Transient stresses depend on the heat transfer coefficient because the higher h, the greater the thermal gradient at the beginning of the cooling.

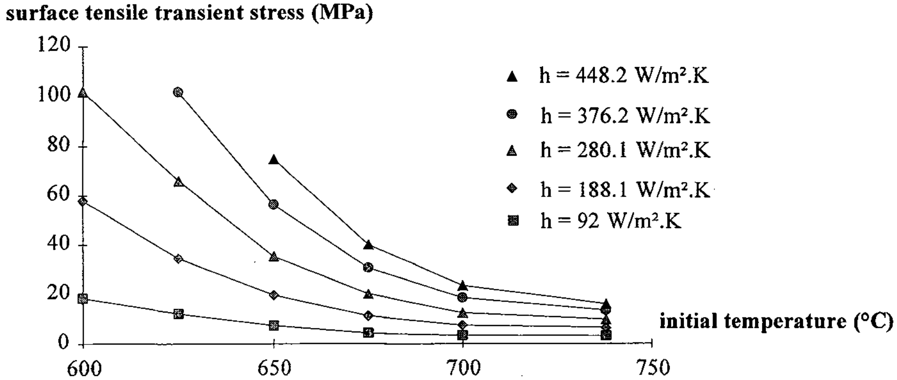

Figure 11: Variations of the maximum transient stresses with the heat transfer coefficient and the initial temperature

5. CONCLUSIONS

Narayanaswamy's model accurately describes the behavior of soda-lime silicate glass. Our computation is consistent with the experimental results for residual stresses in the thickness of the glass plate and for residual stresses on the surface for different temperings.

Transient and residual stresses of tempered glass are predicted. Nevertheless, some observations can be made. The temperature is assumed to be uniform at the beginning of the cooling. But, when the heating is rapid, the temperature is smaller at the surface than in the inner part of the glass. Besides, during the transfer of the plate from the oven to the nozzles, the glass is cooling down. Another hypothesis is made on the nature of the thermal loading: a forced convection, characterized by a heat transfer coefficient that includes the neglected radiation effect.

The good concordance between experimental and computational results seems to valid a posteriori these asumptions.

Simulation results show that the surface is in compression at the end of the temper. This explains the strengthening effect of tempering. The knowledge of the transient and residual stress state allows to study its influence on defects (such as cracks) and to evaluate the lifetime of glass parts of structures.

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