Improvement of Probabilistic Models for Prediction of Missile Impact Effects on Reinforced Concrete Protective Panels using Experimental and Numerical Database

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ABSTRACT

A succession of military invasions and terrorist attacks requires the development of probabilistic models for predicting the effect of various local hard missile impacts effects on reinforced concrete (RC) protective panels. Because of the severity and irreversible repercussions of such events, the current work enhanced previously developed finite element calculations with the addition of experimental data from the literature. The improved model predicts the probabilistic models more accurately than the previous one and minimises statistical uncertainty owing to the incorporation of fresh data. The parameters investigated in the study are the penetration depth of the missile, perforation limit of the target, missile ballistic limit, and residual velocity of the missile. Among the components are residential slabs, bunkers, containments, aircraft shelters, and storage tanks. These models are built using probabilistic approaches and the Bayesian method. All aleatoric and epistemic uncertainties involved in missile impact contact with the target, geometrical configurations, material qualities, and measurement mistakes are all accounted for by the updated formulae. These models also take into consideration the strain rate effect, the multi-modal response of the structure, and numerous failure mode transitions, among other things. An assessment with experimental findings is carried out to establish the dependability and credibility of the updated

equations, and the acquired results demonstrate the trustworthiness of anticipated formulae. This study accommodates natural disasters and accidental events missile such as wind-borne missiles, and impacts due to pressure pipes debris, iron rods, etc.

Keywords: Missile Impact, Probabilistic Models, Bayesian Approach, Protective Structures, Uncertainties, HyperMesh, LS DYNA.

INTRODUCTION

The upcoming need for reinforced concrete (RC) structures designed against impulsive and extreme loads due to natural hazards (wind-borne missiles), industrial accidents (pipe break generated missiles such as turbine blades) or terrorist attacks (aircraft impact) remains an important issue. Military and nuclear groups have been working on impact loads due to tornadoes [Standohar et al., 2015; Cui et al., 2018], rock falls, steel pipes, and so on since the 17th century. However, most engineering constructions such as residential houses, bunkers, silos, and nuclear containments are being constructed with concrete because to its resilience, cost-effectiveness, and availability [Li et al., 2005]. Impacts cover a wide range of loadings; two limiting cases – hard and soft impacts – are generally considered depending on the impactor stiffness. If the impactor deforms relative to the target structure, the missile impact is soft or semi-hard [Giraldo and Pujol, 2019]. The impact of an aeroplane fuselage is a famous example [Sugano et al., 1993]. The hard missile hit is the inverse case. For example, a plane's engine, steel rod ejections owing to pressure leaking, large hardwood logs due to hurricanes, cruise missiles, and so on [Distler et al., 2021].

Impact loading produces a high strain rate ranging from 10^{-4} to 10^{2} per second [Bischoff and Perry, 1991]. A hard missile causes localised damage, but a heavy soft missile often provokes damage in the whole structure. Local effects of rigid missile impacts are calculated using procedures such as empirical equations, experimental studies, and numerical methods [Fang and Wu, 2017]. Figure 1 is captured as a part of the validation study discussed in validation section. The configuration of the slab is 1200mm X 1200mm and its depth is 120mm subjected to a missile mass of 2kg and velocity of 215m/s. The following are the damage consequences evaluated for design considerations in the event of a hard missile impact (Figure 1),

- Penetration Depth (x): maximum depth at which missile pierces into the target
- Perforation Limit (h_p): necessary depth of the target to terminate the residual velocity of the missile
- Residual Velocity (V_r): rear side missile velocity after perforating into the target
- Ballistic Limit (V_{bl}): maximal velocity of the missile to avoid perforation of the target

Many empirical equations are based on actual data, with a few drawn largely from theoretical and analytical techniques such as the theory of energy absorption [Laczak and Karolyi, 2017]. In establishing the formulae, a proper curve-fitting analysis is well used. In [Corbett et al., 1996], the authors describe a few well-explained formulations of hard missile local impacts. Various organizations established significant formulas in the early nineteenth century. Because of age-old estimation, the range of variables for empirical formulations is limited to lower instances of impact loadings, such as a smaller mass of a missile with a smaller diameter. Extrapolation of such statistical data may have undesirable effects. The well-known Modified Petry formula by the United States (US) is solved analytically for penetration depth using equations of motion. However, a standard penetrability factor is advised for all concrete classes, which is a disadvantage. The Army Corps of Engineers (ACE) formula was statistically fit based on testing data below 48MPa and a missile diameter of 155mm. The present missile designs and concrete compressive strength are far advanced in their ranges, which is the limitation of these formulae. The modified National Defense Research Committee (NDRC) formula is based on penetration theory, and a good approximation is determined by experimental data. Because this formulation is based on the principle of penetration, extrapolation may provide realistic findings. However, [Ranjan et al., 2014] demonstrate a significant mistake in forecasting penetration depth. The Ammann and Whitney formula proposed in 1976 is comparable to the ACE and NDRC equations for forecasting high-velocity fragments (greater than 300m/s), which are generally focused on nuclear facility defence. The Whiffen formula was devised in 1943 by the British Road Research Laboratory based on the creation of fragments utilizing wartime bomb data available in the United Kingdom on RC structures. Similarly, various formulas have been presented in the past and the present century for determining missile penetration depth, each with its own set of pros and disadvantages. The theory of empirical formulation is well explained in [Kennedy, 1976]. The inclusion of fibres in the panels reduces the penetration depth of the target and residual velocity of the missile due to higher resistance [Tran et al., 2022; Daneshvar et al., 2020]. Many perforation limit equations are based on the penetration depth principle [Li et al., 2005]. Popular perforation limit formulations recommend doubling the missile's penetration depth, which may result in uneconomical provisioning. However, the missile's ballistic limit and residual velocity are few [Boyce et al., 2001; Grisaro and Dancygier, 2014] and do not offer adequate experimental data. It was discovered that if the missile did not collide with the rebar mesh, the comparatively low reinforcement ratio (less than 4%) had almost no effect on the projectile's ballistic limit [Fang and Wu, 2017]. Theoretically, [Ben-Dor et al., 2009] discovered that (i) the ballistic limit of a multilayered concrete shield did not depend on the order of the slabs in the shield; (ii) the monolithic shield was superior to any layered shield of the same thickness; and (iii) the largest decrease in ballistic limit velocity occurred when a shield was divided into several slabs of the same thickness. In 1946 (NDRC, 1946), a formulation of the ballistic limit of various nose missile forms was developed; nevertheless, the chosen range of values is lower. It was discovered that the rear strengthened CFRP fabric may enhance the ballistic limit by approximately 18% [Fang and Wu, 2017]. [Barr, 1990] proposed a method to predict the ballistic limit of the projectile perforating the RC/steel composite target, assuming the back steel liner is extra reinforcement to the target. (Grisaro and Dancygier, 2014) advocated converting the back steel liner to an extra certain depth of concrete slabs with the same ballistic limit to test the perforation resistance of the RC/steel composite target affected by non-deforming projectiles. The formulations are difficult to understand and are interconnected with several extra equations, which is a drawback of these investigations. Finally, the results of now well-accepted empirical formulations include (i) not matching with existing ranges, (ii) surrendering to mistakes in comparison with testing, (iii) uneconomical design guides, and (iv) difficulties in application.

Experimentation is an alternative method for investigating local missile impacts, although it is time consuming and expensive [Said and Mouwainea, 2022]. Although the experimental test technique yields valid data, numerical solutions are favored owing to their low cost and shorter time length [Thai and Kim, 2017]. By doing a numerical parametric assessment using empirical equations and experimental results [Ranjan et al., 2014; Kojima, 1991], FE outcomes are better suited to studies than empirical outcomes [Thai et al., 2021; Wang et al., 2022]. [Terranova et al., 2018] calculated the impactor's mass and the tensile strength of concrete. The velocity of the missile has a significant impact on the perforation resistance of the concrete target.

According to [Pham and Hao, 2016] the energy absorption and pattern of damage for a missile of low velocity with a larger mass and smaller mass with higher velocity have distinct outcomes while having the same kinetic energy. This is a distinct disadvantage of empirical and experimental investigations. However, because of sophisticated FE tools, the worst-case scenario, i.e., more mass at a higher velocity, may be done using virtual FE codes. The current study aims to evaluate the effect of local damage characteristics such as penetration depth, perforation limit, missile ballistic limit, and residual velocity on the performance of RC panels. These unique probabilistic models have been constructed with better accuracy based on experimental [Berriaud et al., 1978] and numerical results [Gangolu et al., 2022]. Significant data is required for the development of probabilistic equations. Even though innovative formulations for these quantities of interest have been proposed [Gangolu et al., 2022], the current study is intended to combine experimental results with finite element (FE) simulations to have substantial data in reducing the error involved in the predictions. This research improves the design standards for hard missile impact on RC slabs for house residential buildings, storage tanks, aircraft shelters, and defensive structures. These formulations were created using dimensionless functions that alter the missile impact scenario. The generated probabilistic models were calculated using the Bayesian method as detailed in [Kapteyn et al., 2021]. The Bayesian statistical framework is a general, rational, and powerful model update tool that can handle a variety of challenges, including measurement mistakes, incomplete experimental data, nonunique solutions, and modelling errors caused by any model approximating the real system [Goller et al., 2012]. The suggested equations follow the conventional technique by [Gardoni et al., 2022]. These unique formulations take into consideration the strain-rate influence, as well as any aleatoric and epistemic errors in material and geometrical features. Furthermore, the present models consider failure mode transitions and interaction, as well as the structure's multi-modal response. A rigorous numerical validation of previously developed probabilistic models is required to determine the dependability of presented equations, as stated in the next section.

FINITE ELEMENT VALIDATION

Experiment results on the influence of critical parameters such as penetration depth, displacement, ballistic limit, residual velocity, impact force, and so on necessitate the use of advanced test setup and equipment. However, FE simulations result in the flexible capture of predicted parameters, resulting in cost-effectiveness and reduced time. In this study, HyperMesh is used for modelling reinforced concrete slabs, and missiles [Altair, 2003]. However, The HyperMesh file is exported to the commercial FE application LS-DYNA, which incorporates material properties, boundary constraints, contact cards, element forms, termination time, and other post-processing information [LSTC, 2006]. The number of interests chosen for validation of the current study is hard missile penetration depth, missile impact force, and panel damage pattern. Numerous probabilistic experiments involving missile impact produced satisfactory results based on this FE approach. The numerical analysis was studied by [Gangolu et al., 2022; Gangolu et al., 2022 a] and explained as follows.

Material Models and Structural Configuration

The primary challenge is determining an adequate concrete material model. For determining the exact stochastic fracture-mechanical properties of concrete, relevant concrete models and probability distribution functions are employed. These chosen models and functions aid in exact performance, resulting in the construction of accurate probabilistic models for RC panels. The explicit integration strategy is based on the second-order central difference integration scheme, which is being applied in the current study. Because dynamic analysis is prevalent in impact loading, all the material models considered are strain rate sensitive. The Winfrith concrete model is more suited to missile impact loads than the Continuous Surface Cap Model (CSCM). With 10% erosion, maximum and least primary strain are guaranteed (Add Erosion). Although the number of inputs is lower than in other concrete models, this model matches experimental results well. Using the Winfrith concrete model [Thai et al., 2021] studied the residual velocity of the missile and scabbing area of the RC slab subjected to hard missile [Vepsä et al., 2011]. However, the obtained difference between these chosen quantities of interest is 4.0% and 3.6%. Similarly [Chung et al., 2015] verified three concrete models, the CSCM model (MAT_159), Concrete Damage Rel. 3 (MAT_072R3) and Winfrith Concrete model (MAT_084-085) subjected to hard missile impact upon the slab. With an accuracy close to 95%, the Winfrith concrete model's displacement response pattern is following the measured result in the punching case (hard missile). Plasticity with Mohr-coulomb response via third stress invariant to address triaxial extension in compression and tension and strain-softening behaviour in tension to make material regular via fracture energy, crack width, and aggregate size is provided by this model. Compressive and tensile strength inputs can be supplied as two independent variables. The present model's auto-generation capabilities are limited. The referred code supplies the corresponding inputs for the Winfrith concrete model. This paradigm is explored in further depth in [Wu et al., 2012]. Nonetheless, Winfrith model can include smeared reinforcement, providing another material model for reinforcement bars. The Plastic Kinematic model uses elastoplastic reinforcement bars with a failure strain of 20%. The hardness of the missile is ensured with a Rigid material model. To replicate the field set-up, six degrees of freedom are released for the missile. This Rigid material certifies with high failure strain eventually leading to no erosion. A Rigid material model ensures the missile's toughness. Six degrees of freedom are granted to the rocket to imitate the field setup. This Rigid material is certified with a high failure strain, resulting in no erosion. Under missile impact, a solid model with three-dimensional square panels is constructed. The default element for modelling concrete panels and missiles is the constant stress solid element. A one-dimensional steel reinforcement with a default element form formulation, i.e., Hughes-Liu with cross-section integration, is utilised. During impact, this model displays both bending and axial stiffness. It is difficult to find a well-accepted Lagrangian coupling method to fix the contact between concrete and rebars, especially in complicated geometries like containments and composite constructions. Flanagan-Belytschko rigidity shape ensures hourglass energy, and mesh fineness is guaranteed. The term 'Velocity Generation' is used to launch a missile with a high starting velocity. The contact between the missile and the concrete is ensured by eroding surface-to-surface and eroding nodes-to-surface for the missile and rebars. In both exchanges, Missile is always seen as the master. All degrees of freedom were restricted by all square slabs. Table 1 abstracts the keywords used for FE analysis, and a similar method utilised for the remaining FE simulations. Before developing probabilistic models, FE model validation with test results is crucial, as discussed below.

Validation

In this numerical analysis, one trial is chosen from the literature for FE validation, subjected to hard missile impact [Kojima, 1991]. Initially, numerous nose forms were intended for the numerical analysis; however, because of computing restrictions, the study was limited to the flat-nose missile design. The reason for selecting [Kojima, 1991] is that it displays configurations, reinforcement patterns, boundary limits, missile dimensions, mass, and velocity. Furthermore, the results are clearly illustrated in terms of penetration depth, damage pattern, reinforcement bar failure, impact force, and scabbing/spalling area. This vivid projection of the features aided the current study in accurately matching the numerical analysis results with test results. For FE validation, the approach of keywords is shown in Table 1. One hemispherical hard missile with a velocity of 215m/s was tested, and a specific missile mass of 2kg was struck on a 1.2m X 1.2m X 0.12m RC panel. Concrete has an unconfined compressive strength of 27MPa and a reinforcing ratio of 0.6%. The resulting conclusions were compared to the experimental research, as indicated in the figures, demonstrating the reliability of FE validation. The panel is completely perforated as seen in Figure 2. Three reinforcement bars were broken as validated numerically (Figure 3(a)). The obtained from the experiment is 101kN as well matched with a simulation which is obtained as 127kN (Figure 3(b)). Due to the perforation of the panel, an opening of 80mmX100mm is generated from the experiment, which is identical to the FE validation opening of 80mmX80mm (Figure 4 (a, b)). The rear side damage pattern of experimentation and validation is similar as shown in Figure 4. Additionally, as seen in Figure 2 and Figure 4 a, the panel is entirely perforated.

[Gangolu et al., 2022] used a similar numerical process and validated a similar panel with two other velocities of 164m/s and 95m/s. The experimental penetration of a missile at a velocity of 164 m/s is 108 mm, which is comparable to the validation value of 100 mm (Figure 5 a, b). Similarly, the experimental penetration for 95 m/s is 44 mm, which is like the validation of 45 mm. Furthermore, the damage pattern of an RC slab at 164 m/s missile velocity is compared to experimental damage, which reveals the same pattern (Figure 5 c). After several trials, the mesh size converged at 10 mm. [Ranjan et al., 2014] did a similar study (Kojima, 1991) using the CSCM model and concluded that FE models produced satisfactory results. For two tests, the mean line data in Figure 6 (a) reveals that FE results are more accurate than empirical ones. A substantial match with impact force was found in both cases, with 144.47kN (test) and 145kN (FE) for 95m/s and 117.57kN (test) and 112kN (FE) for 164m/s (Figure 6 b). The FE models based on these similar agreements with experiments are reliable. The three validations imply the trustworthiness of the numerical approach followed. However, changes in the geometry of the missile nose do not affect the computational approach or procedure used to create the probabilistic models, the current validation can be trusted even for flat nose missile shape.

The current study, as previously stated, is based on FE simulations generated in [Gangolu et al., 2022; Gangolu et al., 2022a, b] and experimental research done by [Berriaud et al., 1978]. The following are the various reasons for considering experiment studies from [Berriaud et al., 1978],

- The primary basis for selecting [Berriaud et al., 1978] is that it provides considerable experimental data. The experimental investigation included seventy-nine tests, of which fifty-five are taken into account. However, with so many test findings, the literature is deficient in the requirement of the current study.

- The current study is concerned with rigid and flat-nosed missiles, and the majority of the tests conducted by [Berriaud et al., 1978] are rigid missiles with flat noses.
- While doing tests, [Berriaud et al., 1978] considered longer length and depth of 5m and 0.6m; higher mass, velocity, and dia of 343kg, 445m/s, and 305mm. No experiments have been undertaken to date with such heavy designs and stronger missile impact. These target/missile material and geometric properties can accommodate various types of containments or protective shelters and natural disasters or accidental missiles.

The next section discusses the experimental design of both numerical and experimental investigations.

EXPERIMENTAL DESIGN

Introduction

Actual representative data encompassing the entire range of variables is required to create accurate models. With enough data, statistical uncertainty may be reduced [Yuen et al., 2006]. To generate more probability information, a substantial amount of genuine experimental data is necessary. Due to the scarcity of accessible experimental data, the current study is validated with experiments of [Berriaud et al., 1978] and the FE analysis of [Gangolu et al., 2022; Gangolu et al., 2022a]. Significant data, as is well-known in probability theory, can substantially reduce statistical uncertainty. For example, probabilistic studies such as [Stochino et al., 2022 and Zhao et al., 2022] stressed the significance of large amounts of data in minimising uncertainty and developing ideal formulae. [Berriaud et al., 1978] conducted a series of field experiments in 1976, different ranges of compressive strength, panel thickness and configuration; missile diameter, mass, and velocity were analysed. The characteristics acquired from the 79 findings include missile penetration depth, ballistic limit, and residual velocity, of which fifty-five are chosen for the current investigation. [Gangolu et al., 2022] also include 73 FE models to achieve an efficient design approach.

Selection of Range of Variables

The current study addresses the depth and compressive strength of RC slabs from a standard building to heavy constructions such as bunkers, aircraft shelters, nuclear containments, and so on, using existing literature and genuine case studies. Table 2 shows the variables that were used in this investigation. For numerical analysis. the chosen range of material and geometrical variables of RC panels are from [Balomenos and Pandey 2017], reinforcement grade and reinforcement ratio [Choi et al., 2017], missile mass and velocity [Wen and Xian, 2015]. The accepted values consider not only the current ones but also future panels. The impact of concern categories like wind-borne missiles [Suaris and Khan, 1995; Ramseyer et al., 2016], accidental missiles, and pressure released pipes are covered. Based on the realistic variables provided in Table 2, a variety of derived variables such as reinforcement spacing, and rebar diameter have been constructed (Table 2). A total of 128 combinations of tests and FE simulations were investigated in this investigation under various missile loads, striking at the centre of the panel. The instances for FE analysis are chosen using the technique described below. The D-optimal point selection strategy is used to find the optimum set of combinations by design standards [Myers and Montgomery, 1995]. Using this technique with irregular boundary conditions, any number of design instances may be chosen. Polynomial response surfaces are also suggested by this technique [LSTC, 2006b]. Lower and higher limit values are required for each parameter when creating a database. These inputs are fed into scheme-generated software, such as LS-OPT [LSTC, 2006b]. So, seventy-three combinations are generated for performing numerical analysis. A typical LS-DYNA schematic view of the RC slab with the missile is shown in Figure 7. Figure 7 depicts the configuration used, the reinforcement arrangement, the shape of the missile's nose, and other details.

D-optimal designs are straightforward optimizations based on the model to be fitted and an optimality criterion of choice. Maximizing |X'X|, the determinant of the information matrix X'X, serves as the optimality criterion for creating D-optimal designs (Aguiar et al., 1995). To estimate the parameters with the same precision as an ideal design, a non-optimal design needs more experimental run. D-optimal experiments can lower the costs of experimentation".

Pros of D-Optimal Point Selection

- For a pre-specified model, this optimality criterion leads to the generalised variance of the parameter estimations being minimised. Consequently, a particular D-optimal design's "optimality" depends on the model. In other words, before a computer can produce the precise treatment combinations, the researcher must describe a model for the design.
- The computer method selects the best set of design runs from a candidate set of potential design treatment runs given the total number of treatments runs for the experiment or simulation and the supplied model.
- Typically, this candidate set of treatment runs includes all feasible pairings of the different factor levels that one desires to use in the experiment or simulation.

Cons of D-Optimal Point Selection

Some precisely developed treatment combinations from D-optimal point selection are occasionally invalid according to the design principles recommended by codes.

- Consider a scenario where a panel configuration of 3m X 3m X 0.5m with a compressive strength of 55MPa and reinforcement ratio of 3% is subjected to 10kg mass and 10m/s velocity because of various combinations of D-optimal point selection. It is obvious that because of the structural target's higher resistance and smaller missile demand, there won't be any damage based on chosen quantity of interests like penetration or perforation. The generated cases in these situations are useless for simulating and developing probabilistic models.
- Similarly, it is generally recognised according to design techniques that the structural configuration should be compatible with the reinforcement ratio i.e., higher geometry with a lower reinforcement ratio or vice versa. A realistic structure with these design concepts does not imply odd combinations, but there is a chance that they could arise from a D-optimal selection scheme and these cases are useless for evaluating probabilistic models. However, this problem can be resolved by differentiating the chosen variables into basic and derived variables.

PROBABILISTIC MODELS OF LOCAL DAMAGE EFFECTS OF HARD MISSILE IMPACT

The present formulas for hard missile local damage effects are deterministic, based on experimental data and simplified mechanical principles. As a result, the inherent uncertainties in the deterministic model are not accounted for, resulting in a biased estimate. The reason for the biased estimate of deterministic models is their mismatch with the experimental results. Stochino et al., 2022 studied probabilistic models of industrial tanks under blast loads. The investigation demonstrates the significant difference between 27 models of UFC 3-340-02 code (deterministic) and experimental results from [Ameijeiras and Godoy, 2016, Chen et al., 2016, Jiang et al., 2020, Duong et al., 2012 a, b]. In this study, the correction terms are introduced in the probabilistic model to correct the bias. Likewise, studies like [Sharma et al., 2014, Zhao et al., 2022 and Gardoni et al., 2002] demonstrate the biasedness of deterministic models. Even if contemporary structural engineering practice has developed safe design, for the sake of impeccable provisions, the design must be unbiased and account for the inherent uncertainties. The current study is based on a Bayesian framework for evaluating multivariate probabilistic models for structural members that account for all the major uncertainties such as measurement errors, statistical uncertainty, and bias of the model due to an inaccurate model [Gardoni et al., 2002]. Due to a considerable difference between test findings and empirical studies for penetration depth, perforation limit, and ballistic and residual velocity of the missile, this research builds innovative probabilistic models rather than changing current models. The current work considers the overall methodology described in [Gardoni et al., 2002] in producing new formulations. This technique catches and comprehends all the underlying physical phenomena, resulting in an enhanced model. The probabilistic models were created by including necessary correction terms in deterministic models [Gardoni et al., 2002]. Numerous research that used the same method produced precise probabilistic models. However, because of the novelistic approach, the current research cannot be evaluated using deterministic models. The FE simulations have no mistake or uncertainty in measurement errors, while experimental research may. Measurement errors in laboratory and field tests are well-known to occur. Due to unknowable imperfections in geometrical and material qualities, design codes always recommend a certain level of safety. The assumption is that FE simulations are more accurate than testing scenarios in terms of measurement errors. These test-related measurement flaws can eventually be removed from FE simulations [Sharma et al., 2014]. In this way, measurement errors eliminate some of the epistemic uncertainties of the model. So, a certain deviation in the error of the probabilistic models can be eliminated through FE models. To eliminate significant errors the sample data is combined with FE results [Gangolu et al., 2022] as well as experimental ones [Berriaud et al., 1978]. A large amount of data is required to correctly execute the risk-based design paradigm [Haldar and Mahadevan, 2000]. Statistical experts have always recommended that a larger sample size be used to lower the model's uncertainty [Riley et al., 2021]. However, Stochino et al., 2022 investigated probabilistic models of industrial tanks subjected to blast loads using 27 models drawn from diverse studies for a single parameter, maximum deflection [Ameijeiras and Godoy, 2016, Chen et al., 2016, Jiang et al., 2020, Duong et al., 2012 a, b]. Kishore et al., 2022 used 43 beams and 48 columns to test probabilistic models subjected to blast loads at three different deflection performance levels [Kishore et al., 2022]. They investigated 14 scenarios for creating probabilistic equations at performance level 2. In the current study, a total of 128 samples, including experimental and numerical investigations, are considered, with 59 models utilised to analyse missile penetration depth, perforation limit, and ballistic limit of the missile. And rest are utilised for assessing the residual velocity of the missile. The generic technique for probabilistic models provided by [Gardoni et al., 2002] is written as,

$$\ln\left[P_i\left(x,\Theta_{P_i}\right)\right]=\ln\left[p_i\left(x\right)\right]+\gamma_{P_i}\left(x,\theta_{P_i}\right)+\sigma_{P_i}e_{P_i} \qquad (1)$$

where P_i = Probabilistic Model for penetration depth, perforation limit of the concrete, ballistic limit and residual velocity of missile and; p_i = Deterministic model for penetration depth, perforation limit of the concrete, ballistic limit and residual velocity of the missile; but this term will be nullified due to novel formulations development; $\gamma_{P_i}\left(x,\theta_{P_i}\right)$ = correction term for bias inherent in the model defined as,

$$\gamma_{P_i}\left(x,\theta_{P_i}\right)=\sum_{j=1}^{n}\theta_{P_{i,j}}h_{P_{i,j}}\left(x\right) \qquad (2)$$

where $h_{P_{i,j}}(x)$, j = 1,…,n = explanatory function (or regressors) defined as functions of x, and x is defined as a set of measurable variables such as member configurations, material property constants, executed boundary constraints, $\theta_{P_{i,j}}$, j = 1, …,n are parameters associated with explanatory functions, $\sigma_{P_i}e_{P_i}$= model error (additivity assumption), σ_{P_i} =standard deviation of the model error, which is assumed to be independent of x (homoscedasticity assumption). e_{P_i}=

random variable with zero mean and unit variance. The natural logarithm is used in Eq. (1) to stabilize the model variance to satisfy the additivity assumption, homoscedasticity assumption (i.e., σ_{Pi} is independent of x) and the normality assumptions (i.e., e_{Pi} follows a normal distribution). In the transformed logarithmic space, the homoskedasticity assumption should hold, at least to an extent. In the original space, however, the model error can vary with x (i.e., can be heteroskedastic.). The assumption of homoskedasticity is useful for model calibration but is not strictly necessary. However, adding additional parameters to model the model error's dependence on x would increase the uncertainty in model predictions and computational costs. As a result, when a suitable transformation is found, it is advantageous to transform the model and data to a new space where the homoskedasticity assumption is satisfied at least to an extent (Tabandeh et al., 2020). Out of various assumptions in the Bayesian linear regression model, homoscedasticity i.e., a constant standard deviation of the chosen cases is one of the assumptions. The study (Stochino et al., 2022) claims the validity of their experimental data's homoscedasticity assumption, as does the current study.

In general, the current study deals with aleatory and epistemic uncertainty while examining any model for estimating purposes. In the present study, current models account for either inherent errors that cannot be observed by a spectator or modes of observation (aleatory). In the above-detailed formulations, this kind of uncertainty is accounted for by variables x and partially by error terms e_{P_i}. The obtained errors may ascend due to deficiency of knowledge, simplified assumptions, measurement errors and limited availability of data. As discussed, prior this error is present in the model parameters Θ_{P_i} and partially in error terms e_{P_i}. However, as stated in [Sharma, 2012], the aleatory uncertainties cannot be reducible but epistemic errors are by better-quality models, appropriate measurements, and data from additional samples. The uncertainties that arise from the deterministic model (p_i) will be eliminated because of not taking them into account. The missing variables and remaining error owing to inexact model form are represented by the model error term $(\sigma_{P_i} e_{P_i})$ in equation 1. Because missing variables are inherently random, the component of (e_{P_i}) that indicates their influence has aleatory uncertainty, but the component representing the inexact model form $\left(\gamma_{P_i}(x, \theta_{P_i})\right)$ has epistemic uncertainty. Distinguishing between the two uncertainty components of (e_{P_i}) is challenging (Gardoni et al., 2002). The uncertainties caused by measurement errors and observation sample size (n) are epistemic. Nevertheless, by using FE simulations measurement errors and collecting many samples, these uncertainties are greatly reduced. Regardless of how much the study strives to minimise all uncertainties, there will be some inaccuracy in the models, but the purpose of this research is to reduce most of the uncertainty.

The proposed approach employs Bayesian linear regression, which can quantify the model's epistemic uncertainty. The Bayesian approach's role in a model is to forecast unknown parameters (Guo et al., 2022). In Bayesian model updating, uncertainties are represented by a prior distribution over the model inputs and then updated to a posterior distribution using the likelihood function that quantifies the discrepancy between the model predictions and observations (Cheung and Beck, 2009). This study presents a probabilistic model based on a mix of data from existing tested specimens and numerical models. The variables and model form are defined, and the suggested model is assessed using the collected samples. Bayesian linear regression determines model uncertainties using random variables; in fact, the parameters of the probability distribution function are regarded as random variables in this approach and have a probability distribution function. The update of the model based on fresh data is another distinguishing aspect of the Bayesian approach. New data cause the probability distributions of these explanatory variables to be updated. This distinguishing trait enables the model's cognitive uncertainty to be reduced through more additional data [Hassanzadeh et al., 2022]. Furthermore, using this strategy gives a better knowledge of the most influential parameters. However, the concept of symbolic regression and Bayesian linear regression process is same. The basic goal of this method is to reduce the large formula to a small one without affecting the accuracy of the model. In the current study, a similar technique is used, and the derived formula is reduced to a simple and easy equation by monitoring the model's COV. This analysis is described in further detail in [Box and Tiao, 2011; Asem and Gardoni, 2021; Hassanzadeh et al., 2022; Taheri and Mohammadi, 2022] and the resulting regression models account for most of the uncertainties, bias, and high correlation with experimental studies.

Dataset Used in the Original Probabilistic Model

The dataset used to evaluate the original model in [Gangolu et al., 2022; Gangolu et al., 2022a] consists of 55 different combinations of panel material and geometrical features, missile configuration, and velocity. These combinations were created with LS-OPT by specifying the geometrical parameters of certain variables and the number of combinations needed. The preliminary issue in running a greater number of numerical cases is lower computational configuration. Depending on the system configuration, each simulation took between 2hr – 10hr or maybe more than 24hr in some cases. Eventually, the ability to run many simulations is lacking. Various levels of numerical impact scenarios are captured. The sample size of the dataset used in model evaluation affects the precision of parameter estimations in probabilistic models (Sharma et al., 2014). A smaller sample or data size implies higher uncertainty in the model parameter estimation (Gardoni and Murphy, 2013). The prior models (Gangolu et al., 2022; Gangolu et al., 2022a) did not account for the influence of reinforcement ratio and moment-bearing capacity for the formulation of penetration depth. In high-strain loadings, the reinforcement opposes with a greater strain rate eventually the influence is evident. In the present research, the depth and length of the panels are enlarged (Gangolu et al., 2022; Gangolu et al., 2022a) due to the inclusion of experimental studies (Berriaud et al., 1978). However, the current analysis considers 128 different experimental (Berriaud et al., 1978) and numerical combinations (Gangolu et al., 2022; Gangolu et al., 2022a). Statisticians believe that 128 combinations as a good sample size for constructing a probabilistic strategy. Including more data for the model, evaluation helps to overcome the various limitations resulting (Gangolu et al., 2022; Gangolu et al., 2022a). Statisticians believe that more than 100 combinations are a good sample size for constructing a probabilistic strategy (Riley et al., 2021). The newly collected data widen the ranges of the data, and statistical uncertainty, which is epistemic, can be minimised by utilising more data for parameter estimates (Gardoni et al., 2002).

Model Correction

Explanatory terms that are prominent are selected above those that could be able to capture the reasonable phenomena of experimental and FE models. These parametric functions are based on existing equations and intuitions about fundamental panel properties. Table 3 lists the functions that were chosen. These explanatory terms are dimensionless, and the specifics of variables are as follows: the first explanatory function is chosen to allow for constant potential bias. The second function represents the effect of missile kinetic energy on concrete internal resistance. The third explanatory function considers the reinforcement ratio. The fourth function helps to increase the system's moment-carrying capability. The fifth function considers the panel's length-to-depth ratio. The missile slenderness ratio is captured by the sixth function. The seventh explanatory function considers the outcome of the desired natural frequency.

Model Assessment

A non-informative prior has been selected [Box and Tiao, 2011]. A progressive deletion process for lowering the number of terms in $\gamma_{P_i}(x, \theta_{P_i})$ is given to strike a balance between model simplicity (few corrective terms) and model correctness (small σ). In summary, the study deletes each term when the COV $\theta_{P_{i,j}}$ is greater than σ. Because the logarithmic adjustment in

Eq. 1, is almost equivalent to the predicted model's COV. In general, adding a term with a COV substantially bigger than σ is not likely to increase the model's accuracy. The progressive term deletion approach for the selected parameters is summarised in Figure 8 (a, b, c). The figure depicts the posterior COV of the model parameters (red solid dots) and the posterior mean of the model standard deviation for each step (black squares). The cross mark on the red solid dots signifies the deletion of the explanatory function with the highest COV.

In general, sensitivity analysis is used to determine the system's most sensitive material and geometric properties (Choe et al., 2007). However, the current study has not performed a sensitivity analysis of all the parameters. But the COV of the probabilistic model is measured after each explanatory function elimination. The probabilistic model is fixed based on the COV and the simplicity of the equations. Furthermore, as a structural engineer, most of the parameters and explanatory functions chosen are appropriate based on literature, experience, and intuitions. However, as it is well known, global sensitivity analysis aids in the retention of the most sensitive parameters. For example, (Choe et al., 2007) evaluated the seismic sensitivity of bridge columns. The diameter of the transverse reinforcement ratio is the most important characteristic of Deformation and Shear Failure Modes. Nonetheless, concrete compressive strength, missile mass, velocity, and diameter may be the most sensitive parameters for the current investigation (postulation).

Sensitivity analyses are also used to identify the critical inputs that influence output variability (Allaire et al., 2014). The quantification of system sensitivity provides insight into which factors contribute to the uncertainty in the outcome of specific scenario analysis. Sensitivity analysis, for example, identifies which modelling assumptions, uncertain model inputs, and/or uncertain scenario parameters are most essential. Sensitivity analysis is crucial for guiding future research efforts aimed at lowering output variability, in addition to assisting with better decision-making through knowledge of uncertainties. This is especially critical when the unpredictability is so great that model outputs are meaningless for decision-making. When the difference between the outcomes of two policy alternatives is not statistically significant due to large uncertainty then the concept of sensitivity analysis is more suitable (Allaire and Willcox, 2014). However, the uncertainty in the current study is not considerable due to the large number of samples used, and this study did not use sensitivity analysis to evaluate the uncertainty of the outcome. The study, however, suggests that the posterior mean (σ) and COV of each $\theta_{P_{i,j}}$ stepwise deletion is sufficient to eliminate superfluous functions.

Parameter Estimation of Updated Models

The present part generates probabilistic models for penetration depth (x), perforation limit (h_p), the ballistic limit of the missile (V_{bl}), and residual velocity of the missile utilising the results of FE simulations [Gangolu et al., 2022; Gangolu et al., 2022a] and experimental test data [Berriaud et al., 1978]. (V_r). These specifications are designed for RC panels that will be subjected to a hard missile strike at the centre. Most of the points are in the interval of confidence bounds as seen in Figure 9. The dashed lines in the figures signify the $\pm\sigma_{P_i}$ of the model and is represented logarithmically. However, the assumption of homoscedasticity is valid by looking into the probabilistic plots for four parameters (Figure 9 a, b, c, d). Most of the points are almost near to mean line and the case of significant variation of the data points suggests that the model is not in the category of homoscedasticity. The homoscedasticity assumption is valid for majority of the data points for both experimental and numerical models.

Probabilistic Model for Penetration Depth (x)

The proposed formulation to evaluate the probabilistic model of penetration depth (x) for an RC panel subjected to hard missile impact is,

$$\frac{x}{d} = \exp\left(-1.34 + 0.108\frac{MV_0^2}{d^3 f_c'} + 0.22\frac{MV_0^2}{M_c} + 0.011\frac{L_m}{d} - 0.79\frac{MV_0}{TLHf_c'}\right) \quad (3)$$

The influential functions are J_2, J_4, J_6, and J_7 and the points are near to mean line Figure 9 a. The attained COV of the probabilistic model is 0.48. And the posterior statistics of the penetration depth of the missile are presented in Table 4.

Probabilistic Model for Perforation Limit (h_p)

The proposed formulation to evaluate the probabilistic model for the perforation limit of concrete target i.e., the minimum required thickness of the panel to avoid complete penetration after missile impact is,

$$\frac{h_p}{d} = \left(-3.41 + 10.03\left(\frac{x + 0.05}{d}\right)\right) \quad (4)$$

Where the COV of the probabilistic model is 0.28 and the points are near to mean line Figure 9 b. And the posterior statistics of the perforation limit of the target are presented in Table 5. A factor of safety is considered with an excess thickness of 50mm for estimating this formulation. In the case of double wall containment structures and protective structures, this model helps arrest missiles on the surface. However, the designer could frame guidelines based on penetration depth or place a steel liner to avoid complete passage. This solution could be useful for economical codal provisions for construction purposes.

Probabilistic Model for Ballistic Limit of Missile (V_{bl})

In designing containment or protective structures subject to missile impact, it is mandatory to ensure minimal velocity required to avoid complete penetration of the missile to the rear side of the target. Contemporary research is very less focused on the formulation for a ballistic limit of the missile. The current analysis developed a reliable and economic probabilistic equation (standard normal distribution) to estimate the ballistic limit of the missile with an RC panel is,

$$V_{bl} = \exp\left(4.68 + 0.07\frac{MV_o^{\,2}}{d^3 f_c^{'}} + 0.23\frac{MV_o^{\,2}}{M_c} - 0.24\frac{L_m}{d}\right) \qquad (5)$$

The influential functions for this model are J_2, J_4, and J_6 and the obtained COV of the probabilistic model is 0.32. The obtained points are nearby as shown in Figure 9 c. And the posterior statistics of the penetration depth of the missile are presented in Table 6.

Probabilistic Model for Residual Velocity of Missile (V_r)

Most of the containments and bunkers are constructed double layered to have additional safety from external hazards like missile loadings. In such cases, the current parameter does have a momentous role in design criteria. The developed probabilistic equation (standard normal distribution) to estimate the residual velocity of the missile (V_r) after impacting the target is,

$$\frac{V_r}{V_0} = \exp\left(-0.94 + 0.0261\frac{MV_0^2}{Hd^2 f_c'} - 0.07\frac{MV_0^2}{M_c} - 0.0036\frac{L_m}{d}\right) \quad (6)$$

The influential functions for this model are J_2, J_4, and J_6 and the obtained COV of the probabilistic model is 0.63. The obtained points are near to mean line as seen in Figure 9 d. And the posterior statistics of the penetration depth of the missile are presented in Table 7. Since the influence of panel depth is significant the current study modified the J_2 function with the depth parameter. Where, x – penetration depth of the missile into the concrete target, d – diameter of the missile (m), M – Mass of the missile (kg), V_0 – Velocity of the missile (m/s), f_c' – Compressive strength of concrete (N/m^2), A_s – Area of Steel, b – width of the panel, H – depth of the panel, $1/T$ – frequency of the panel, M_c – Moment carrying capacity of the target, h_p – perforation limit of the concrete target (m), L_m – Length of the missile (m), V_{bl} – Ballistic limit of the missile (m/s), V_r – Residual Velocity of Missile.

Figure 9 appears to show that the homoskedasticity assumption has divergence with higher values of the respective parameters. However, for developed models, the homoscedastic assumption is satisfied to a large extent. One possible reason for not fully meeting this assumption is a lack of data from the higher range of variables. Only a few numerical models are carried out with higher structural configurations and higher missile demand, and no experimental tests have been performed. Based on these plots (Figure 9) the current study is more suitable to lesser range of hard missiles such as wind-borne missiles, and impacts due to pressure pipes debris, iron rods, hard logs and so on. In the future, significant data with higher structural and missile configurations can be considered for developing local missile impact scenarios, which are desperately needed considering the global disasters.

CREDIBILITY OF PROPOSED FORMULAE

The current study has compared the proposed formulae with existing experimental results. The chosen material and geometrical properties of the RC slab are shown in Figure 10. From this study, it is very evident that the obtained probabilistic models are like test results from Table 8 and Figure 10 (a, b, c).

CONCLUSION

The current research is concerned with four innovative probabilistic models created for local hard missile impacts. The criteria of the investigation are missile penetration depth, target perforation limit, missile ballistic limit, and missile residual velocity. These formulations are created using 128 models by combining experimental and finite element (FE) analysis data. The previously created probabilistic models comprise 55 FE models; however, due to continued severe threats, these formulations are expected to be upgraded for improved performance of missile-protected structures. A large dataset reduces uncertainty and makes equations more efficient. The probabilistic equations developed are based on Gardoni's probabilistic approach and Bayesian inference. The influence of missile’s kinetic energy, panel’s internal energy, moment carrying capacity, missile dimensions, and panel’s frequency are noteworthy for the chosen four parameters. The previous developed models did not take moment carrying capacity, missile dimensions, and panel’s frequency into consideration, while the new penetration depth formulation does. COV for these models is within acceptable limits, demonstrating the dependability of produced models. The posterior statistics and standard deviation for each model is also presented in the current study. The credibility of the current study was assessed using test results, which revealed an excellent match. Material modelling, dimensional inaccuracies, statistical uncertainty, strain rate effect, boundary condition, and other aleatoric and epistemic uncertainties are all considered in these probabilistic models. Analysing these complicated setups with current programmes and experimental testing may have constraints of their own. However, the current formulas are readily available for usage. However, for future versions of the work, the range of variables can be updated with more statistical data, and a greater number of simulations with a higher range of missile velocity and configuration can be undertaken. The current study accommodates wind-borne missiles, and impacts due to pressure pipes debris, iron rods, hard logs and so on. This procedure may be extended to include blast loadings, different nose shapes, impact, and so on.

Conflict of Interest

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

- Design and validation details
- FE simulation details
- Probabilistic Analysis details

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Table 1 Keywords used for FE Validation

Material	**Keyword**
Missile	Rigid Material, MAT_20
Concrete	Winfrith Concrete MAT_084 with erosion of 10%
Rebar’s	Plastic Kinematic, MAT_003 failure strain of 20%
Contact of Missile and Concrete	Eroding Surface to Surface Missile – Master & Concrete – Slave
Contact of Missile and Beam elements	Eroding Nodes to Surface Missile – Master & Beam elements – Slave

Table 2 Range of Basic and Derived Variables for RC Panels

	Variable	Symbol	Range of Panels
Range of Basic Variables	Length of Panel (m)	L_s	1.46 – 5
	Thickness of Concrete Panel (m)	t_c	0.2 – 0.6
	Longitudinal Reinforcement Ratio (%)	ρ_l	1 – 3
	Transverse Reinforcement Ratio (%)	ρ_t	1 – 3
	Compressive strength of concrete (MPa)	f_c'	25 - 55
	Yield strength of reinforcement (MPa)	f_y	250 – 550
	The boundary condition of the panel	B_c	Fixed Constraint
	Missile Mass(kg)	M	10 – 2500
	Missile Velocity (m/s)	V_0	10 - 1000
Range of Derived Variables	Dia of Longitudinal bar (mm)	d_l	8, 10, 12, 14, 16, 18, 20, 25
	Dia of Transverse bar (mm)	d_t	8, 10, 12, 14, 16, 18, 20, 25
	Spacing of Longitudinal bars	s_l	$\left(\frac{\pi}{4}.d_l^2\right).1000 / \left(\frac{\rho_l.B.D}{100}\right)$
	Spacing of Transverse bars	s_t	$\left(\frac{\pi}{4}.d_t^2\right).1000 / \left(\frac{\rho_s.B.D}{100}\right)$

Table 3 List of Explanatory Functions for RC Probabilistic Models

Variable	Expression	Representation of Explanatory Function
$J_1(x)$	*1*	Constant Bias
$J_2(x)$	$\left(\frac{MV_0^2}{d^3 f_c'}\right)$	Missile Energy to Target Resistance
$J_3(x)$	$\left(\frac{A_s}{b.H}\right)$	Reinforcement Ratio
$J_4(x)$	$\left(\frac{MV_0^2}{M_c}\right)$	Moment Carrying Capacity
$J_5(x)$	$\left(\frac{b}{H}\right)$	Length to Depth Ratio of Panel
$J_6(x)$	$\left(\frac{L_m}{d}\right)$	Slenderness Ratio of Missile
$J_7(x)$	$\left(\frac{MV_0}{TLHf_c'}\right)$	Frequency Ratio

Table 4 Posterior Statistics of Parameters in Selected Penetration Depth of Missile Model

Parameter	Mean	Standard Deviation	Correlation Coefficient				
			Θ_1	Θ_2	Θ_4	Θ_6	Θ_7
Θ_1	-1.3407	0.02	1.000				
Θ_2	0.108	0.0022	0.1342	1.000			
Θ_4	0.2206	0.0169	-0.4979	-0.2045	1.000		
Θ_6	0.0112	0.0079	-0.7214	-0.6319	0.3568	1.000	
Θ_7	-0.7968	0.1801	0.0121	0.0489	-0.7308	-0.0204	1.000

Table 5 Posterior Statistics of Parameters in Selected Perforation Limit of Target Model

Parameter	Mean	Standard Deviation	Correlation Coefficient	
			Θ_1	Θ_2
Θ_1	-3.4114	0.0519	1.000	
Θ_2	10.0356	0.0441	-0.7443	1.000

Table 6 Posterior Statistics of Parameters in Selected Ballistic Limit of Missile Model

Parameter	Mean	Standard Deviation	Correlation Coefficient			
			Θ_1	Θ_2	Θ_4	Θ_6
Θ_1	4.6832	12.6378	1.0000			
Θ_2	0.0707	1.3663	0.1338	1.0000		
Θ_4	0.2352	7.3094	-0.7166	-0.2475	1.0000	
Θ_6	-0.2423	4.9679	-0.7213	-0.6318	0.5011	1.0000

Table 7 Posterior Statistics of Parameters in Selected Residual Velocity of the Missile Model

Parameter	Mean	Standard Deviation	Correlation Coefficient			
			Θ_1	Θ_2	Θ_4	Θ_6
Θ_1	-0.03178	12.2751	1.0000			
Θ_2	0.1407	3.7651	-0.0197	1.0000		
Θ_4	6.6115	290.8881	-0.0731	-0.9704	1.0000	
Θ_6	-0.0068	0.4083	-0.4059	-0.1531	0.1623	1.0000

Table 8 Credibility of Proposed Models with Experimentation

Parameters	Dimension of Panel	Compressive Strength, MPa	Mass of Missile, kg	Velocity of Missile, m/s	Dia of Missile (m)	Experiment	Predicted Formula
Penetration Depth	5m X 5m X 0.4m	33.5	160	133	0.305	0.15m	0.13m
	5m X 5m X 0.4m	36	240	72	0.2	0.09m	0.1m
	1.46m X 1.46m X 0.26m	40.5	35	220	0.3	0.15m	0.15m
Ballistic Limit of Missile	5m X 5m X 0.4m	40	160	108	0.305	108m/s	107.1m/s
	5m X 5m X 0.4m	33.5	192	110	0.305	110m/s	113.6m/s
	1.46m X 1.46m X 0.416m	50	103	187	0.3	187m/s	180m/s
Residual Velocity of the Missile	1.46m X 1.46m X 0.26m	45	51	114	0.2	42m/s	42.6m/s
	1.46m X 1.46m X 0.26m	40.5	295	29.7	0.1	12.9m/s	10.05m/s
	5m X 5m X 0.4m	36	300	89	0.305	37m/s	34.6m/s

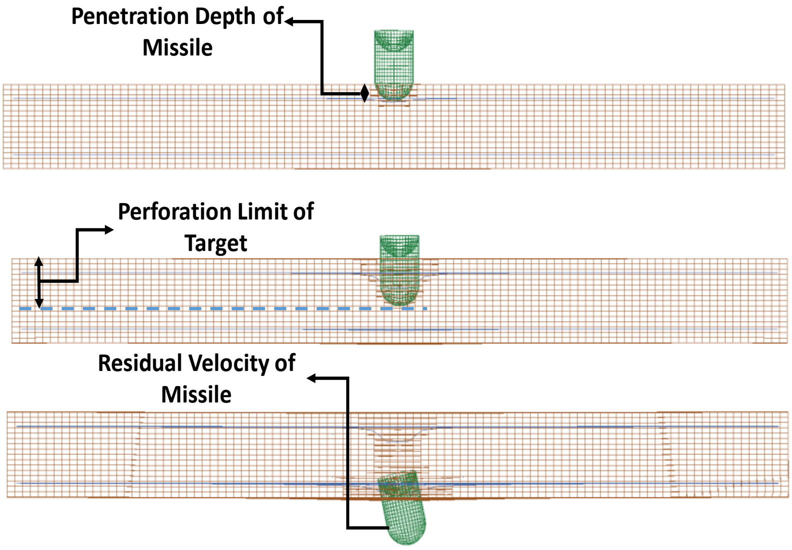

Figure 1 Local Missile Impact Effects upon RC Panel

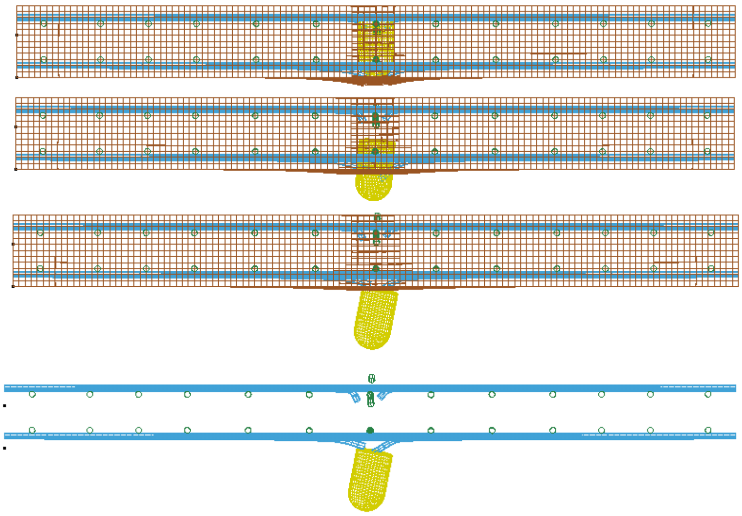

Figure 2 Various Stages of Missile Impacting the Panel with a Velocity of 215m/s

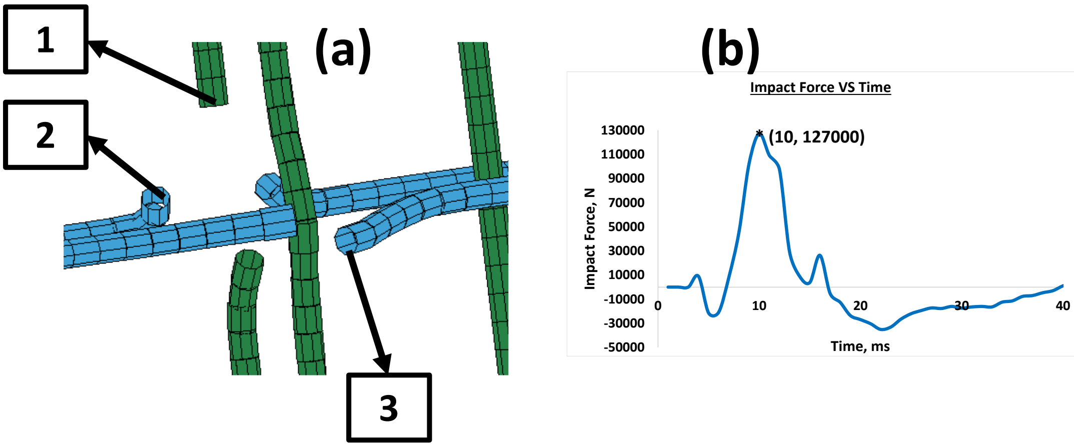

Figure 3 (a) Elimination of Three Reinforcement Bars, Numerically, (b) Reaction force of the Panel due to Missile Impact

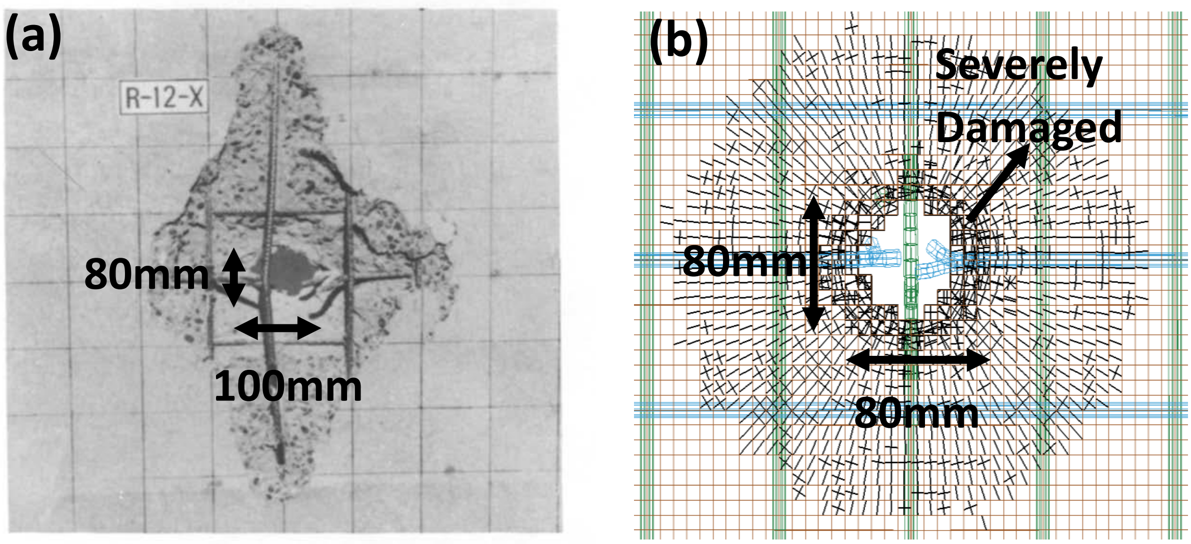

Figure 4 Panel after Impact Scenario (a) Experiment (b) Numerical

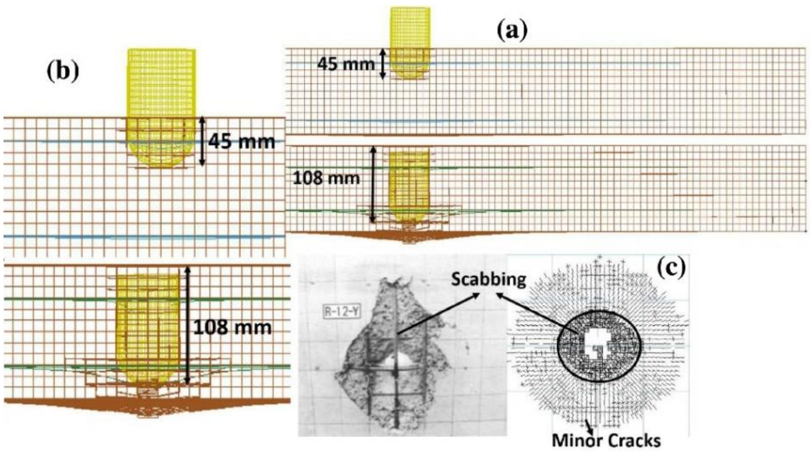

Figure 5 FE Validation outcomes for two chosen cases [Gangolu et al., 2022]

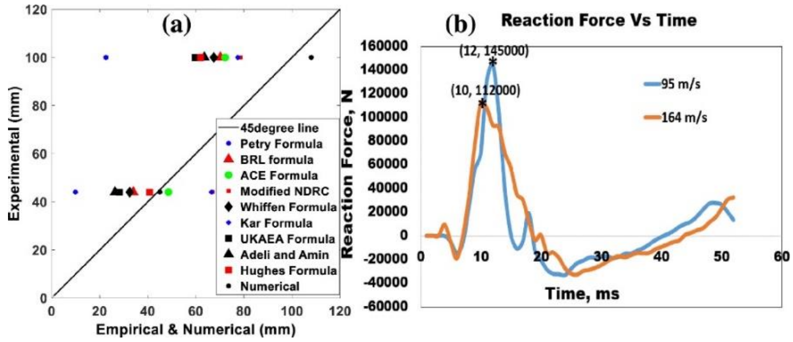

Figure 6 (a) Comparison of experiments with empirical & FE model, (b) Reaction force of FE models [Gangolu et al., 2022]

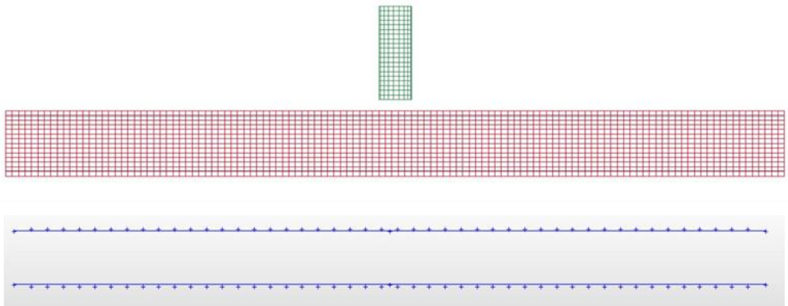

Figure 7 Typical Finite Element Modelling of RC panel with a missile (LS-DYNA)

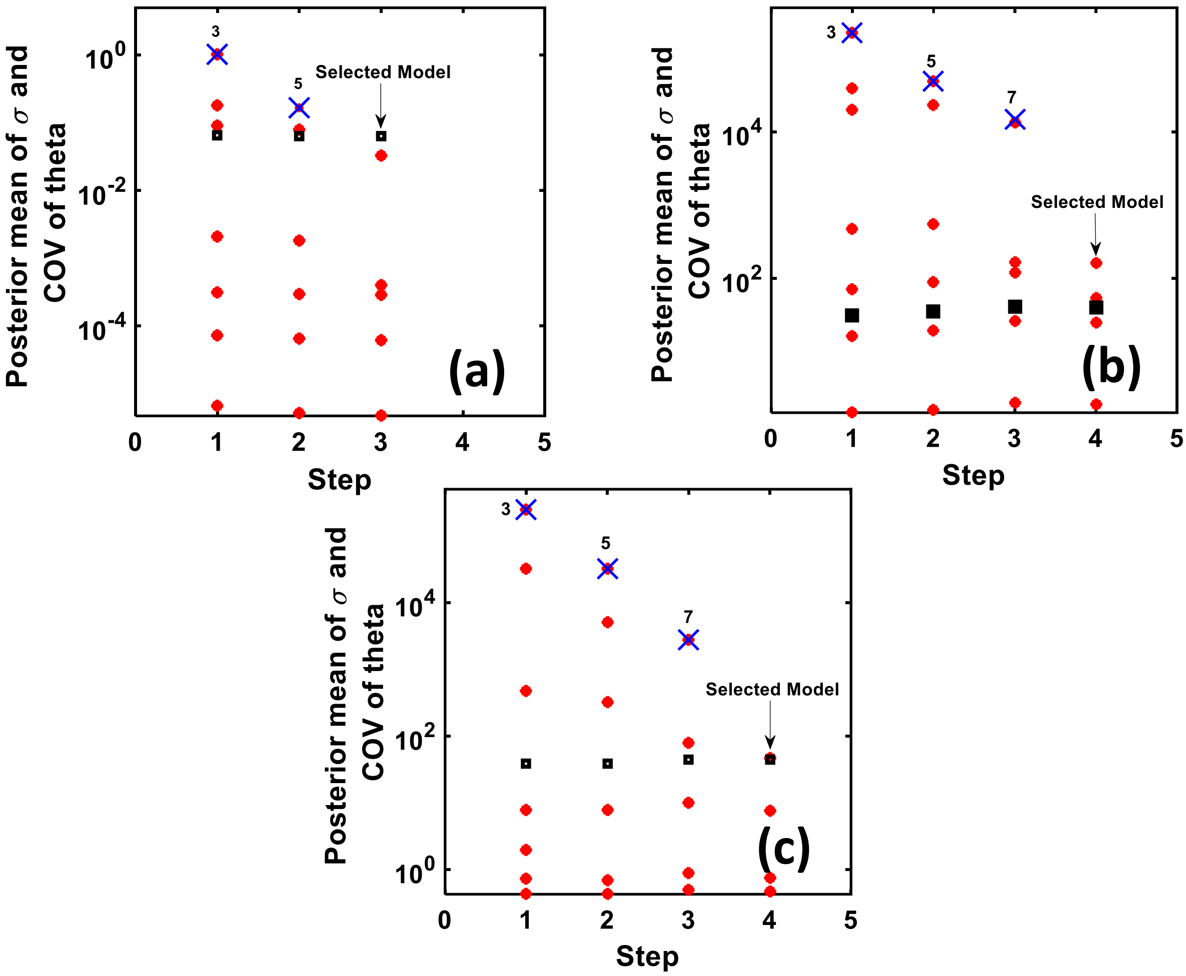

Figure 8 Stepwise Deletion Process of (a) Penetration Depth, (b) Ballistic Limit of the Missile and (c) Residual Velocity of the Missile

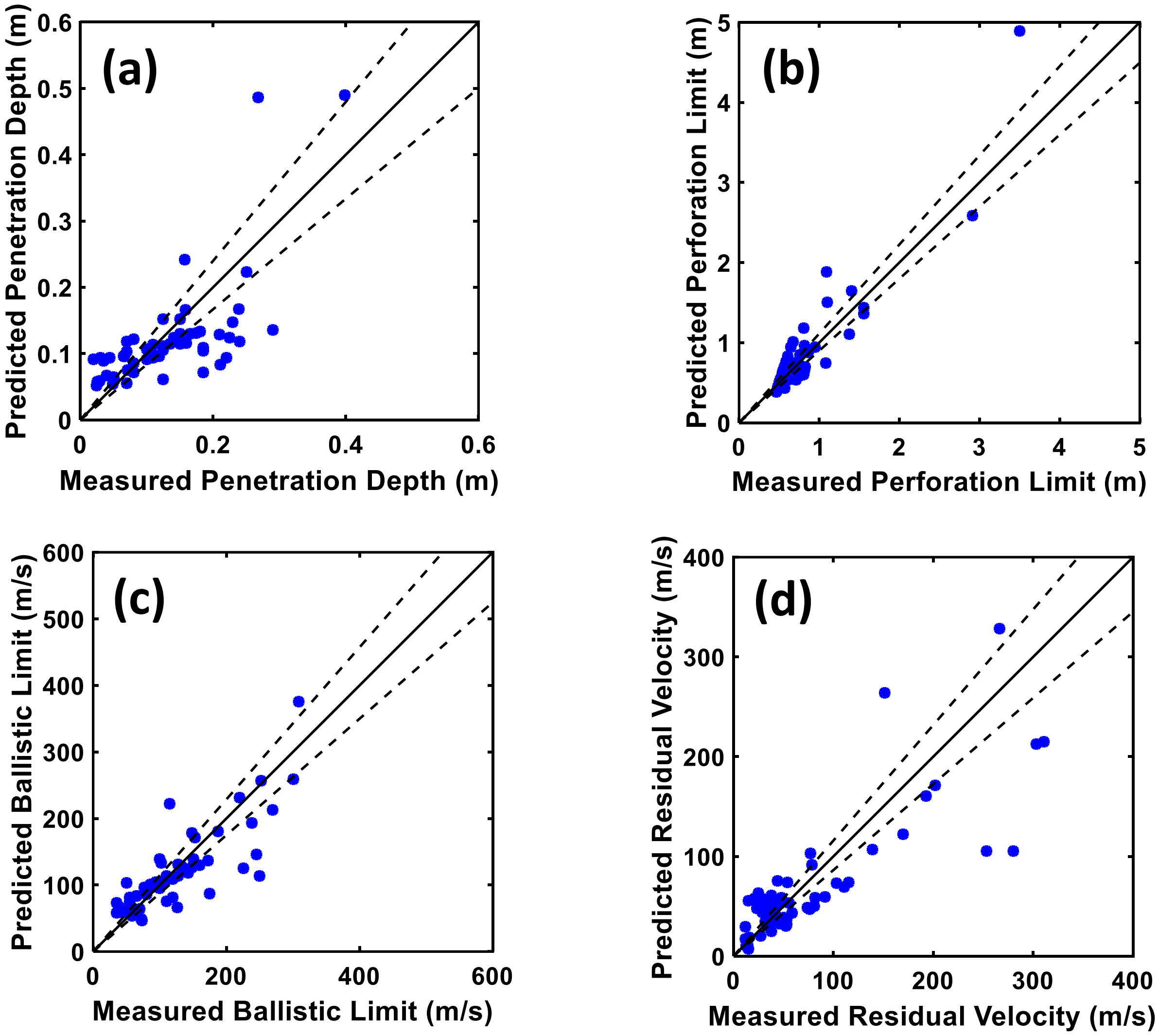

Figure 9 Probabilistic Models (a) Penetration Depth, (b) Perforation Limit, (c) Ballistic Limit of Missile and (d) Residual Velocity of Missile

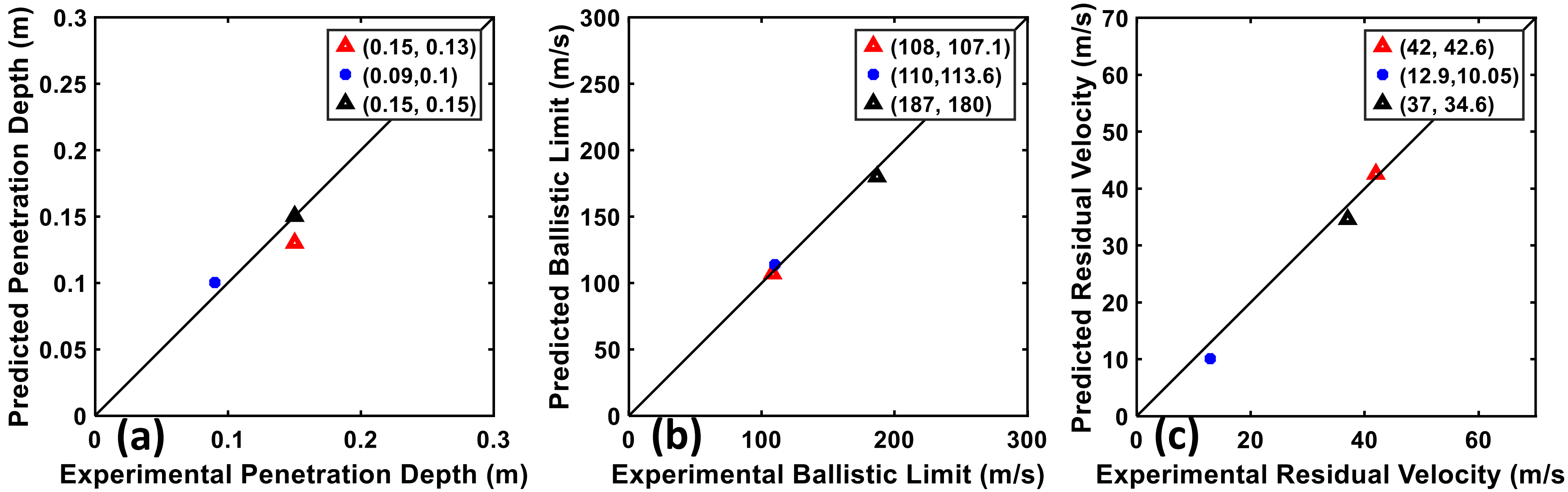

Figure 10 Credibility of Test results (a) Penetration Depth (b) Ballistic Limit of Missile (c) Residual Velocity of Missile