Discrete Element Modelling of a Reinforced Concrete Structure

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ABSTRACT

A 3D model using the Discrete Element Method (DEM) is proposed to study reinforced concrete structures submitted to dynamic loading. The model has been previously validated for plain concrete through quasi-static loadings, and through SHPB dynamic compression and tension tests. This paper aims at validating the introduction of steel reinforcement in the model, prior to simulations of real reinforced concrete structures submitted to dynamic loading. Lines of discrete elements represent reinforcement amidst concrete discrete elements, and the model takes into account the behaviour of concrete, of the reinforcement, and of their interface. Results of simulation of a four point beam bending test show the capabilities and limitations of the preliminary model.

1. INTRODUCTION

The design of concrete safety structures is a big challenge for engineers; for example some structures present in mountainous areas are dedicated to protection against natural hazards such as avalanches, rock falls, etc... and thus may be submitted to impact loads and high deformation. Despite their geometry which is usually massive, with an extremely high fraction of reinforcement, and of course a design satisfying usual building standards, some are found to be totally damaged. This inconsistency demands the use of a model with high predicting abilities.

In particular, it is of utter importance that the model should be able to reproduce the strain rate dependency of concrete, well understood for low strain rates (ε ≤ 101 s-1), but not beyond. Most of the numerical investigations were carried out with FE codes, which are easy to use. Nevertheless, The increasing complexity of these models, associated with the difficulties of dynamic problems, make their computational use somewhat awkward : in particular, the occurrence of cracking often has to be identified, and effects like internal friction after crack opening, or structural effects like inertia of micro-cracking have to be explicitly accounted for.

An alternative to FEM computations is the use of Discrete Element Method (Cundall [1], Cundall et al [2]). This method does not rely upon any assumption about where and how a crack or several cracks occur and propagate, as the medium is naturally discontinuous and is very well adapted to dynamic problems. Although numerous authors like Potyondy et al [3] and Meguro and Hakuno [4] have used similar two-dimensional approaches to model cohesive geomaterials, few have thus modelled concrete (Camborde et al [5]), and even fewer have modeled complete 3D concrete structures, which is now made feasible thanks to ever-increasing computing power.

Keeping in mind that the final goal is to represent 3D reinforced concrete structures, this work aims at extending the validation of a three-dimensional Distinct Element (DE) model through the simulation of a four point bending test on a reinforced concrete beam. The DE model has been fully described and validated in quasi-static problems as well as in dynamic compression and tension in Hentz et al [6]. Firstly, the model will be briefly reminded, and then the reinforcement will be introduced and the first results of four point bending test will be presented.

2. DISCRETE ELEMENT MODEL

The present numerical method uses discrete spherical elements of individual radius and mass. These elements represent a polydisperse assembly with a size distribution obtained by using a particular growing technique (Donzé [7]). Once the assembly has been set, pairs of initially interacting discrete elements are identified. These interactions have been chosen to represent as best as possible and in a simple way, the elastic and cohesive nature of a certain class of geomaterials such as concrete. To do this, elastic forces with a local rupture criterion are applied between two interacting elements.

Interaction range

The macroscopic behaviour of a material can be reproduced by means of this model by associating a simple constitutive equation to each interaction. An interaction between elements a and b of radius Ra and Rb respectively, is defined within an interaction range γ and does not necessarily imply that two elements are in contact. Then, these elements will interact if,

$$\gamma(R^a + \mathrm{R}^b) \geq \mathrm{D}^{a,b} \tag{1}$$

where Da,b is the distance between the centroids of elements a and b and γ ≥ 1. This is an important difference from classical discrete element methods which use spherical elements (Cundall et al [2]) where only contact interactions are considered γ ≥ 1. This choice was made so that the method could simulate materials other than simple granular materials in particular those which involve a matrix as found in concretes.

Interaction forces

The interaction force vector F which represents the action of element a on element b may be decomposed into a normal and a shear vector Fn and Fs respectively, so that,

$$F = F^n + F^s \tag{2}$$

Where

$$F^n = K^n (D_{eq}^{a,b} - D^{a,b})n \tag{3}$$

($D_{eq}^{a,b}$ is the equilibrium distance between the two elements a and b which was set when the interaction was created)

The shear vector force Fs is computed incrementally and was given by other authors (Hart [8]). The incremental force is given by

$$\Delta F^s = -K^s \Delta U^s \tag{4}$$

where ΔU^s is the shear displacement vector increment between the locations of the interacting points of the two elements over a timestep Δt.

Elastic properties

The strain energy stored in a given interaction cannot be assumed to be independent of the size of the interacting elements. Therefore interaction stiffnesses are not identical over the sample, but follow a certain distribution, which is another important particularity of the SDEC model. The macroscopic elastic properties, here Poisson's ratio ν and Young's modulus E, are thus considered **to be the input parameters of the model.**

"Macro-micro" relations are then needed to deduce the local stiffnesses from the macroscopic elastic properties and from the size of the interacting elements. Compression tests have been run with one given sample and values linking Poisson's ratio ν , and Young's modulus E to the dimensionless values of $\frac{K^s}{K^n}$ were obtained. To fit these values, relations based on the best-fit model (Liao [8]) are used.

Strength properties

A modified Mohr-Coulomb rupture criterion is used (see figure 1, where Aint is the interaction surface : $A_{\text{int}} = \pi.\min\left(R^a, R^b\right)^2$). When the shear criterion is reached, the shear force is limited to this value ; when the tensile criterion is reached, the interaction undergo a softening behaviour (see figure 2, where β is the softening factor).

This model has been developed to take into account the fact that damage of concrete is mainly due to micro-cracks opening in mode I. Moreover, heterogeneity of concrete is reproduced through the use of A_{int} which induces a strength properties distribution over the sample.

The model is enriched with a local strain rate dependence $T = f(\dot{\varepsilon})$ based on the CEB formulation. Of course this dependence is of no importance in the case of quasi-static simulations.

A detailed description of the model can be found in (Hentz et al [6]).

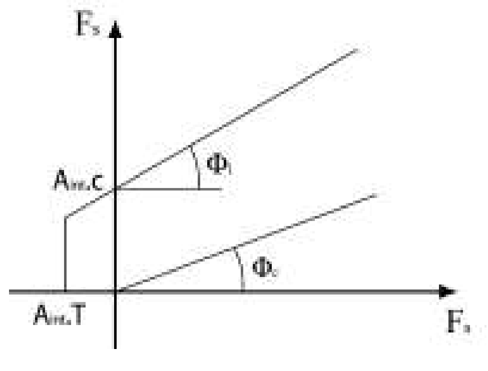

Figure 1 : Rupture criterion for a cohesive interaction (above), and a contact interaction (below)

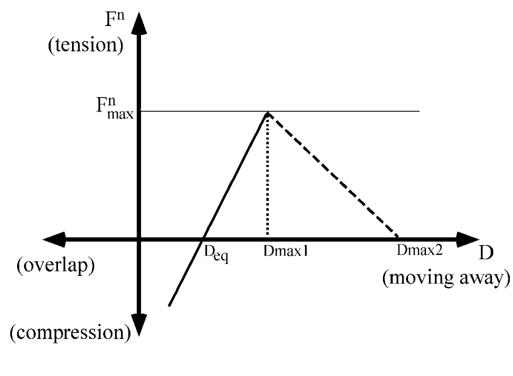

Figure 2 : Softening behaviour of the normal force

Parameters calibration

Calibration of the model parameters is necessary to adjust the properties of the material represented by the assembly of discrete elements to the real geomaterial properties, a particular type of concrete. For this purpose, a quasi-static uniaxial compression/traction procedure has been established. This procedure allows the user to determine for a single assembly the values of the local parameters T, c, Φ_i, β and γ to obtain the macroscopic behaviour characterized by the Young's modulus, the Poisson's ratio, the tensile and compressive strengths, as well as the fracture energy. As far as the macroscopic elastic properties are concerned, it appeared that the “macro-micro” relationships discussed in section “Elastic Properties” give only a good approximation of the macroscopic elastic properties, because of the random aspect of the generation of the assembly. To solve this problem, the procedure is the following :

1. A compact, polydisperse discrete element assembly is generated.
2. An elastic compression test is run with elastic local parameters given by the “macro-micro” relations.
3. A correction is applied according to an energy-based criterion. Compressive and tensile rupture axial tests are simulated to deduce the remaining local parameters.

An important point is that element rotations have to be inhibited to allow the ratio compressive over tensile strengths to be equal to 10. Nevertheless, this inhibition has proved relevant when dealing with traction /compression loadings, which mobilize a very low level of rotations (Hentz et al [6]), whereas in the case of long structures submitted to bending, this results in extremely high forces, at non-physical level. Then in the following simulations, rotations are freed.

3. FOUR POINT BENDING TEST OF A REINFORCED CONCRETE BEAM

The experimental data set

Four point bending tests were carried out by students of Grenoble University I, using rectangular cross-section beams; height 12cm, width 6cm and length 1.6m. (see figure 3). Reinforcement is constituted of 2 longitudinal steel bars (diameter 6mm) in the lower part of the beam (see figure 4), and of transversal bars (diameter 6mm) in the A and C zones. Concrete has the following properties: Young's modulus E=30GPa, density ρ = 2500kg.m-3, compressive strength σc = 30MPa and tensile strength σt= 3MPa. Steel has the following properties: Young's modulus E=210GPa, density ρ = 7800kg.m-3, yield strength σy = 500Mpa. The tests were displacement-controlled; figure 5 shows the data, in terms of total force versus jack displacement. Three phases can be distinguished: firstly, an elastic part, ending with the micro-cracking of concrete, secondly a less rigid behaviour, due to the development of cracks in concrete, and to the carrying of the load by the bars, and finally a phase (not always very clear on the experimental results) where the steel bars undergo plastic flow.

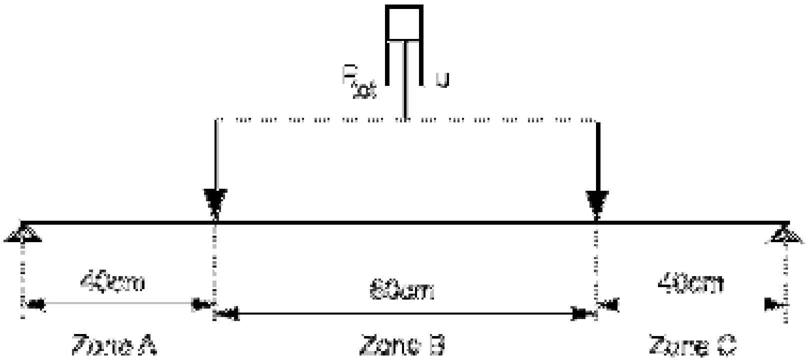

Figure 3: Four point bending test setup

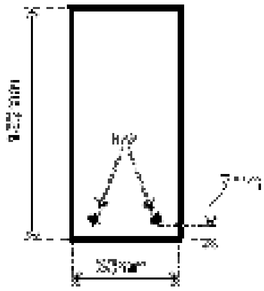

Figure 4 : Beam cross-section

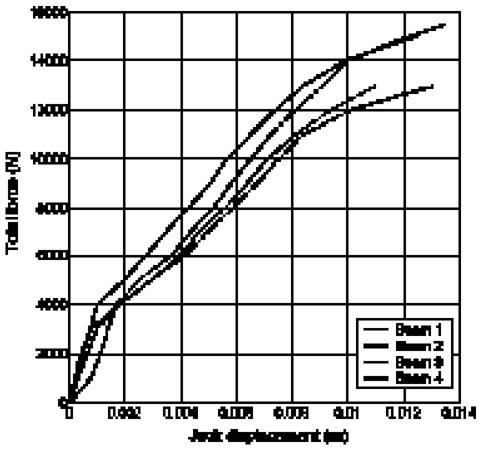

Figure 5 : Experimental data

The numerical setup

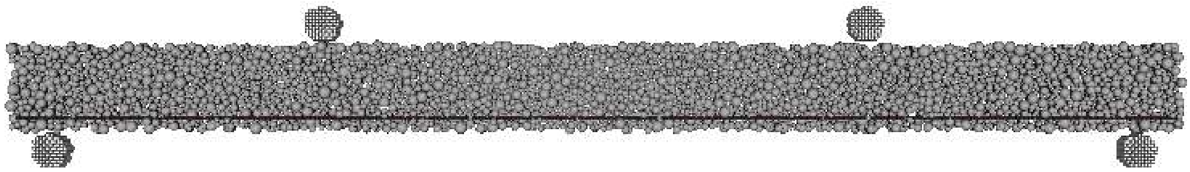

Figure 6: Four point bending test setup ; beam section

Reinforcement bars were modeled using lines of spheres, of diameter equal to 6mm (Figure 6 shows a section of the beam parallel to a reinforcement bar). The transversal bars are not modeled. Three classes of interactions may be defined: concrete interactions follow the previously defined behaviour, and their parameters were calibrated using the already discussed procedure. Steel interaction parameters are calibrated so the exhibited behaviour is elastic-perfectly plastic. Concrete/steel interactions parameters are identical to steel ones, which is consistent with pull-out tests observations. Table 1 shows local parameters. Roughly 9000 elements were used. Cylinders were used to apply the boundary conditions.

Parameter	E (Gpa)	ν	γ	Φ_i (°)	C (Mpa)	T (Mpa)	β
Concrete	30	0.2	1.4	20	3	1	120
Steel	210	0.25	1.05	0	250	500	10000

Table 1 : Model parameters

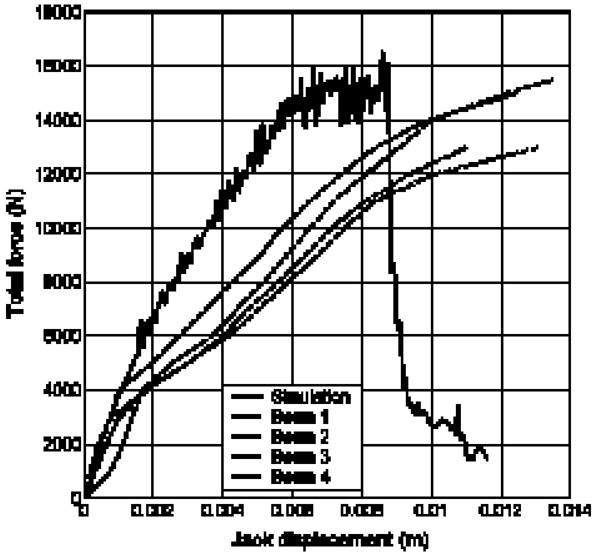

Figure 7 : Comparison between numerical and experimental results**.**

Results

Figure 7 shows the total force versus the jack displacement, and the comparison between numerical and experimental results. The three phases already described can be clearly distinguished : the elastic phase is very well fitted. On the other hand, the second phase begins slightly too late, and its stiffness is too high. This is due to the fact that a high value of local softening had to be used to counterbalance the free rotations : as soon as concrete has fully cracked (which occurs much too early when low softening is used), the bars alone are supposed to carry the load, which they do not when rotations are free. When high softening is used, cracking of concrete occurs late enough so yielding of steel bar appears as a third phase. This said, these results may be explained as well by observed experimental inaccuracies.

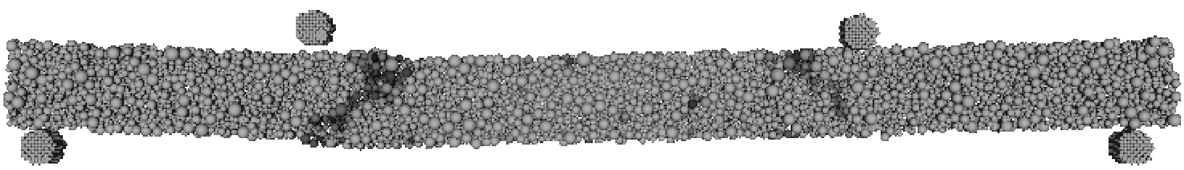

Figure 8 : Damage state of the beam after rupture (the darker, the higher the damage)

Figure 8 shows the damage field in the beam after rupture. Cracking starts on the lower face of the beam, right below one loading point, and propagates towards the inner side of the beam, which is very consistent with experimental results.

4. CONCLUSION

A 3D discrete element model has been proposed to ultimately simulate impacts on reinforced concrete structures. Previously validated for plain concrete in quasi-static and in dynamic loadings.

A reinforcement model has been introduced, and validated through the simulation of four point bending tests. The comparison with experimental results is relatively good ; The fact that rotations are free make the model overestimate somewhat the beam strength : the model should be improved in this matter. Nevertheless, the different test phases are well distinguished, and the beam behaviour is quite well reproduced, which validates the reinforcement model. Moreover, the DE model allows us to investigate the inner damage of the material.

Now the model has been validated through a wide range of tests, impacts on real reinforced concrete structures will be carried out.

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