On temporal relaxation used to attenuate wave reflexions in the combined DE/FE approach

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ABSTRACT

In this work we present an analysis of the temporal relaxation method used to attenuate spurious wave reflections due to size discretisation. This relaxed method is used in the combined FE/DE approach and can be generalized to bridging domain methods, with a fine discretisation close to the impacted area and a coarser mesh in the remaining media. The method is independent with the material properties and the relaxed parameter is a function of the ratio between the time step and the critical time step.

INTRODUCTION

Behaviour of reinforced concrete structure under high frequency loading poses non-negligible difficulties. Many non-linear phenomena are involved, the microstructure have a main influenced. Some authors develop multidomain approach with a local fine discretisation and another discretisation larger. That kind of model used in transient dynamics can introduce spurious reflections (Celep et al., 1983) due to the change of size discretisation. These spurious waves are not physical and the proposed method has to attenuate these as strong as possible.

For the application in a coupled method of Discrete Element (Cundall et al., 1979) and Finite element, we proposed a temporal relaxation on the kinematic constrains. This paper presents the parametric study of the relaxation through 1D and 3D model. The method uses a bridging domain (Xiao et al., 2004). The method is developed to simulate impact on reinforced concrete structure; such simulations can introduce a large range of frequency (Zineddin et al., 2007).
COUPLING METHOD

The proposed method uses a bridging domain in which the Hamiltonian is taken as a linear combination of discrete and continuum Hamiltonians (Figure 1). The bridging parameters $\alpha$ and $\beta$ are introduced respectively on FE and DE nodes. In the bridging domain, the DE degrees of freedom are linked to FE degrees of freedom through the coupling relations.

The solution minimizes the Hamiltonian in each domain by the introduction of coupling relations in the bridging domain that use Lagrange multipliers. The minimization of equation 1 gives the local equations.

$$H_{\delta}(\bar{d}, \bar{\omega}, \bar{u}, \lambda^d, \lambda^u) = H(\bar{d}, \bar{\omega}, \bar{u}) + \lambda^d(\bar{d}_b - k \bar{u}_b) + \lambda^u(\bar{\omega}_b - \bar{u}_b)$$  (1)

The time discretization is performed with the central difference method. More equations of the coupling have been presented previously (Frangin et al., 2006).

At each time step, such Lagrange algorithm strongly respects the coupling relation. The high frequency waves are so reflected at the interface between DE and FE model, because of the larger FE discretization. A length wave which can propagate in the DE parts is reflected at the interface if it is smaller than the cut-off length wave of the FE part. Some modifications have to be added to take into account these spurious reflections.

The proposed modifications consist to introduce freedom on the kinematic constraints. We propose a perturbed Lagrangian method (Belytschko et al, 2000) which consists in two points: numerical simplifications of the matrices associated to the Lagrange multipliers (Xiao et al, 2004) and temporal relaxation.
The numerical simplifications are proposed by Xiao and Belytschko to have fast and efficient computations. Due to the prevailing of the diagonal term of the matrices associated to the multipliers, they proposed to diagonalize the matrices by summing each line in the diagonal value. Such algorithm can be seen as a generalized penalty method.

The first simplification do not attenuate enough the spurious reflections, we developed a method of relaxation. The idea is to introduce freedom in the bridging domain. By divided each multiplier by the relaxation parameter, r, the coupling relation are not strictly verified at each time step. This modification provides large attenuation of the small length wave’s energy.

The proposed method is equivalent with the penalty method, where the penalty parameter is automatically computed and adapted to each degree of freedom of the bridging domain (Frangin et al, 2007).

RELAXATION CHARACTERISATION

Fundamental analysis of the relaxation

The previous paragraph defines the equations of the relaxation. The method with simplification of the Lagrange multipliers and the relaxation is equivalent to the penalty method.

The solution of the penalty method is:
\[
\min \left( H + p \left( u_{FE} - u_{DE} \right)^2 \right)
\]
(2)

With our method the penalty parameter is equal to \( \frac{\rho}{r \Delta t^2} \)

\[
\min \left( H + \frac{\rho}{r \Delta t^2} \left( u_{FE} - u_{DE} \right)^2 \right)
\]
(3)

Where H is the Hamiltonian, \( \rho \) is the mass density and \( \Delta t \) is the time step.

The internal potential energy and the kinematic energy define the Hamiltonian. The expression becomes:

\[
\min \left( \rho V^2 + E f(u_{FE}, u_{DE}) + \frac{\rho}{r \Delta t^2} \left( u_{FE} - u_{DE} \right)^2 \right)
\]
(4)

\[ V \] is the velocity of the degrees of freedom

\[
\min \left( \rho \left( V^2 + \frac{E}{\rho} f(u_{FE}, u_{DE}) + \frac{1}{r \Delta t^2} \left( u_{FE} - u_{DE} \right)^2 \right) \right)
\]
(5)

With \( T = k t \), then \( V = \frac{du}{dt} = \frac{d}{dT} \frac{d}{dt} = k \frac{du}{dT} = k V' \)

\[
\min \left( \rho k^2 \left( V'^2 + \frac{E}{\rho k^2} f(u_{FE}, u_{DE}) + \frac{1}{r \Delta T^2} \left( u_{FE} - u_{DE} \right)^2 \right) \right)
\]
(6)
If we choose $k = \sqrt{\frac{E}{\rho}}$, we have:

$$\min \left( V^2 + f(u_{FE}, u_{DE}) + \frac{1}{r \Delta T^2} (u_{FE} - u_{DE})^2 \right)$$

(7)

The similarities between the equations (4) and (7) demonstrate the independency of the method with material celerity, $k$.

Demonstrates with the penalty method, the initial problem (4) can be transform with the new variable $T$, in an expression without any material parameter. It means that the temporal relaxation behaviour is not dependent on the material properties.

The method with Lagrange multipliers and so the influence of the multipliers on spurious waves damping depend on the factor $r \Delta T^2$. Moreover the attenuation comes from the ratio between the computational time step and the critical time step which depends on the material properties.

The best efficiency must be obtained with the same factor $r \Delta T^2$. If at $\Delta T = \Delta t_{critique}$, we have

$r = r_{optimal}$, then at $\Delta T = \alpha \Delta t_{critique}$, the optimized $r$ is $r = \frac{r_{optimal}}{\alpha^2}$. The relaxation varies with the square of the ratio of computational time step on the critical time step (Figure 3).

Numerical analysis of the relaxation

Two models have been developed. The first, in 1D uses to compare the different models and simplifications available by the approach (Frangin et al., 2007), in the way to find an efficient model. The second is implanted in SDEC (Donzé & Magnier, 1995) which is a 3D discrete element code. The 3D model is not adapted for quantify the best method due to the main dispersion and reflections of the waves on each surfaces and along the propagation inside the structure. Nevertheless, comparisons have been realized between the both models with regular arrangement that confirms the previous results: independence with the material properties and evolution with the square of the ratio between the computational time step and the critical one.

The figures 2 and 3 present simulations in respectively the 1D and the 3D model. The 1D simulation shows the evolution with the number of FE layers (Figure 2). The 3D simulation (Figure 3) realized with three FE layers shows the evolution of the efficiency if the relaxation method with the time step. Moreover the results in 1D and 3D are comparable at the critical time step with 3FE layers, the optimal relaxation parameter is equal to 35 in both model. Both simulations were realized with different material properties.
Figure 2. Evolution of the percentage of attenuation of the energy of the spurious reflection wave with the number of FE layer at the critical time step in the 1D model.

Figure 2. Evolution of the percentage of attenuation of the energy of the spurious reflection wave with the time step in 3D model.
Moreover, the low frequencies are not strongly modified with the relaxation, less than 4% for a simulation with the computational time step equals to the critical time step. The low frequencies are more and more perturbed when the time step comes smaller. An efficient simulation should use a time step equals to the critical time to have faster computation and to improve the results.

CONCLUSION

The proposed temporal relaxation method was simulated with a combined Discrete Element / Finite Element Model. It is numerically verified that the method reduce the reflection due to the high frequencies wave that are defined in the DE region and over the cut-out frequency of the FE model. This method is independent with the materials properties but the optimal relaxation parameter is a function of the square of the ratio between the computational time step and the critical time step. Before running a full 3D model, the parameter can be calculated in a 1D simulation.

REFERENCES


