

# Biomechanical aspects of soft tissues

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Crée comme Dieu  
Ordonne comme un roi  
Travaille comme un esclave  
*Constantin Brancusi*



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