

Biomechanical aspects of soft tissues

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Crée comme Dieu
Ordonne comme un roi
Travaille comme un esclave
Constantin Brancusi

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