Choices: exercises 1+2, or 1+3, or 1+4, or 5+6, or 5+7, or 5+8

**1.The elastic-viscoelastic analogy and the correspondence principle for beam bending** Trick to solve bending problems on viscoelastic beams using standard (elastic) strength of materials

**2.Direct solution of viscoelastic beam bending using Laplace-Carson transforms**

Hypotheses: a) time-translation invariance; b) no difference in material response in tension and compression

2.1 Determination of the neutral axis 2.2 Moment-curvature relation 2.3 Application to a viscoelastic beam of the Maxwell type: step and ramp loadings 2.4 Consider the case of a heterogeneous beam

3. **An elastic beam resting on a visco-elastic foundation: the rail-track problem** Note: this exercise uses the correspondence principle of exercise 1.

3.1 Consider first an elastic soil and find the vertical displacement when the beam is subjected to an arbitrary nonstationary load 3.2 Using the correspondence principle, find the vertical displacement and soil pressure for a visco-elastic soil of the Kelvin type under a stationary load 3.3 Same question for a visco-elastic soil of the Burgers type under a stationary load

4. **An elastic beam resting on an elastic foundation**

4.1 The beam is subjected to a stationary pointwise loading 4.2 The beam is subjected to a stationary distributed load over a finite length

5.**The elastic-viscoelastic analogy and the correspondence principle for 3D viscoelasticity** Trick to solve problems on viscoelastic bodies using elastic information

5.1 Obtain the formal bulk (volumetric) and shear operators, Young’s and Poisson’s operators for general isotropic elasticity and viscoelasticity 5.2 The volumetric response is often assumed to be instantaneous. For the shear response, consider the particular cases of a Maxwell type of response and next of a Kelvin type of response.

6. **The Boussinesq problem over a visco-elastic half-space**

Starting from the known stress components due to a concentrated vertical load applied at the surface of an elastic half-space, obtain the stress components for a visco-elastic half-space of the Kelvin type in shear, using the correspondence principle for 3D viscoelasticity of exercice 5.

7. **A visco-elastic thick-walled tube under internal pressure**

Consider a thick-walled tube of infinite length along its axis, and internal radius a and external radius b. Starting from the known stress, strain and displacement components due to an internal pressure applied to an elastic tube, obtain stress, strain and displacement components for a visco-elastic half-space of the Maxwell type in shear, using the correspondence principle for 3D viscoelasticity of exercice 5.

8. **A visco-elastic thick-walled tube under internal pressure reinforced by an elastic sheath**

Consider a thick-walled tube of infinite length along its axis, and internal radius a and external radius b reinforced on its external side by an elastic sheath of thickness h. 8.1 Consider first the tube to be elastic. Obtain the stress, strain and displacement components due to an internal pressure . 8.2 Consider now the tube to be viscoelastic. Obtain the stress, strain and displacement components for a visco-elastic tube of the Maxwell type in shear, using the correspondence principle for 3D viscoelasticity of exercice 5.

**1.The elastic-viscoelastic analogy and the correspondence principle for beam bending** Trick to solve bending problems on viscoelastic beams using standard (elastic) strength of materials

1.1 Solve the bending pb as if the beam was elastic with constant Young’s modulus Ee 1.2 Write exactly the same relations in the Laplace-Carson transform domain replacing Ee by E\*(p) 1.3 Obtain the solution in time domain by inverse Laplace-Carson transform

**2.Direct solution of viscoelastic beam bending using Laplace-Carson transforms**

Hypotheses: a) time-translation invariance; b) no difference in material response in tension and compression

2.1 Kinematics: Consider beam bending governed by the Navier-Bernoulli model. Define for the strain 2.2 Obtain the stress for a linear visco-elastic beam assuming time translation invariance 2.3 Statics: for pure bending, obtain the position of the neutral axis 2.4 Obtain the moment-curvature relation 2.5 Obtain the curvature by inverse Laplace-Carson transform 2.6 Application to a viscoelastic beam of the Maxwell type: step and ramp loadings

**1.The elastic-viscoelastic analogy and the correspondence principle for beam bending** Trick to solve bending problems on viscoelastic beams using standard (elastic) strength of materials

1.1 Solve the bending pb as if the beam was elastic with constant Young’s modulus Ee 1.2 Write exactly the same relations in the Laplace-Carson transform domain replacing Ee by E\*(p) 1.3 Obtain the solution in time domain by inverse Laplace-Carson transform

3. **An elastic beam resting on a visco-elastic foundation: the rail-track problem** Note: this exercise uses the correspondence principle of exercise 1.

3.1 Consider first an elastic soil and find the vertical displacement when the beam is subjected to an arbitrary nonstationary load 3.2 Consider that the applied load can be expanded in Fourier series, and obtain the vertical displacement as if the soil was elastic 3.3 Using the correspondence principle, find the vertical displacement and soil pressure for a visco-elastic soil of the Kelvin type under a stationary load , first in the Laplace-Carson domain, and, next, in the time domain 3.4 Same question for a visco-elastic soil of the Burgers type under a stationary load

**1.The elastic-viscoelastic analogy and the correspondence principle for beam bending** Trick to solve bending problems on viscoelastic beams using standard (elastic) strength of materials

1.1 Solve the bending pb as if the beam was elastic with constant Young’s modulus Ee 1.2 Write exactly the same relations in the Laplace-Carson transform domain replacing Ee by E\*(p) 1.3 Obtain the solution in time domain by inverse Laplace-Carson transform

4. **An elastic beam resting on an elastic foundation**

4.1 The beam is subjected to a stationary pointwise loading 4.2 consider the symmetries of the vertical displacement and its derivatives 4.3 obtain the solution (vertical displacement ) of the homogeneous equation satisfied by the displacement 4.4. obtain the solution under a point load at x=0, next at x=a 4.5 obtain the solution for a uniform load over a finite interval

5.**The elastic-viscoelastic analogy and the correspondence principle for isotropic 3D viscoelasticity** Trick to solve problems on viscoelastic bodies using elastic information

5.1 Start from the linear differential equation of viscoelasticity for scalars, defining accurately the time derivative 5.2 Define the isotropic-deviatoric decomposition of the stress and strain tensors 5.3 Obtain the formal bulk (volumetric) and shear operators, Young’s and Poisson’s operators for general isotropic elasticity 5.4 Same question for viscoelasticity 5.5 The volumetric response is often assumed to be instantaneous. For the shear response, consider the particular cases of a Maxwell type of response and next of a Kelvin type of response.

6. **The Boussinesq problem over a visco-elastic half-space** This exercice uses the correspondence principle for 3D viscoelasticity of exercice 5.

6.1 Start from the known stress components due to a concentrated vertical load applied at the surface of an elastic half-space 6.2 Obtain the Poisson’s ratio for a visco-elastic half-space of the Kelvin type in shear, the correspondence principle for 3D viscoelasticity of exercice 5. 6.3 Deduce the stress components 6.4 Scrutinize the cas of an instantaneous load

5.**The elastic-viscoelastic analogy and the correspondence principle for isotropic 3D viscoelasticity** Trick to solve problems on viscoelastic bodies using elastic information

5.1 Start from the linear differential equation of viscoelasticity for scalars, defining accurately the time derivative 5.2 Define the isotropic-deviatoric decomposition of the stress and strain tensors 5.3 Obtain the formal bulk (volumetric) and shear operators, Young’s and Poisson’s operators for general isotropic elasticity 5.4 Same question for viscoelasticity 5.5 The volumetric response is often assumed to be instantaneous. For the shear response, consider the particular cases of a Maxwell type of response and next of a Kelvin type of response.

7. **A visco-elastic thick-walled tube under internal pressure** This exercice uses the correspondence principle for 3D viscoelasticity of exercice 5.

7.1 For a visco-elastic half-space of the Maxwell type in shear, using the correspondence principle for 3D viscoelasticity of exercice 5, define the Laplace-Carson of the Young’s modulus and Poisson’s ratio. The volumetric response is assumed to be instantaneous. 7.2 Consider a thick-walled tube of infinite length along its axis, and internal radius a and external radius b. Starting from the known stress, strain and displacement components due to an internal pressure applied to an elastic tube, obtain the stress, strain and displacement components for the visco-elastic tube.

5.**The elastic-viscoelastic analogy and the correspondence principle for isotropic 3D viscoelasticity** Trick to solve problems on viscoelastic bodies using elastic information

5.1 Start from the linear differential equation of viscoelasticity for scalars, defining accurately the time derivative 5.2 Define the isotropic-deviatoric decomposition of the stress and strain tensors 5.3 Obtain the formal bulk (volumetric) and shear operators, Young’s and Poisson’s operators for general isotropic elasticity 5.4 Same question for viscoelasticity 5.5 The volumetric response is often assumed to be instantaneous. For the shear response, consider the particular cases of a Maxwell type of response and next of a Kelvin type of response.

8. **A visco-elastic thick-walled tube under internal pressure reinforced by an elastic sheath** This exercice uses the correspondence principle for 3D viscoelasticity of exercice 5.

Consider a thick-walled tube of infinite length along its axis, and internal radius a and external radius b reinforced on its external side by an elastic sheath of thickness h. 8.1 Consider first the tube to be elastic. Obtain the stress, strain and displacement components due to an internal pressure . 8.2 Consider now the tube to be viscoelastic. Obtain the stress, strain and displacement components for a visco-elastic tube of the Maxwell type in shear and an incompressible elastic volumetric response, using the correspondence principle for 3D viscoelasticity of exercice 5. 8.3 Specialize to an instantaneous applied internal pressure