1/

(a) $\begin{cases} M\ddot{q} + C\dot{q} + Kq = f e^{i\omega t} \\ \text{réponse harmonique forcée } \quad q = z e^{i\omega t} \end{cases} \Rightarrow$

(b) $(K - \omega^2 M + i\omega C) z = f$

Développons z sur la base des $\{x^{(i)}, i : 1 \to N\}$:

(c) $z = \sum_{i=1}^{N} \alpha_i x^{(i)}$

D'où :

$$\sum_{i=1}^{N} \alpha_i (K x^{(i)} - \omega^2 M x^{(i)} + i\omega C x^{(i)}) = f$$

$x^{(j)T}$

$$\alpha_j \omega_{0j}^2 - \alpha_j \omega^2 + i\omega \underbrace{\sum_{i=1}^{N} \alpha_i x^{(j)T} C x^{(i)}}_{} = x^{(j)T} f$$

$\alpha_j (\omega_{0j}^2 - \omega^2 + i\omega\, 2\xi_j \omega_{0j}) = x^{(j)T} f$ pour amortissement faible

et donc : $z = \sum_{j=1}^{N} \alpha_j x^{(j)} = \left(\sum_{j=1}^{N} x^{(j)} \dfrac{x^{(j)T}}{(\quad)} \right) f = (K - \omega^2 M + i\omega C)^{-1} f$

$$a_{k\ell}(\omega) = \sum_{j=1}^{N} \frac{x_k^{(j)} x_\ell^{(j)}}{\omega_{0j}^2 - \omega^2 + 2i\xi_j \omega\omega_{0j}}$$

2/ $N = 2$

$$a_{kk}(\omega) = \frac{x_k^{(1)2}}{\omega_{01}^2 - \omega^2 + 2i\xi_1 \omega\omega_{01}} + \frac{x_k^{(2)2}}{\omega_{02}^2 - \omega^2 + 2i\xi_2 \omega\omega_{02}}$$

Si ω_{01} et ω_{02} sont bien séparés :

$$a_{kk}(\omega_{01}) \simeq \frac{x_k^{(1)2}}{2i\xi_1 \omega_{01}^2}, \quad a_{kk}(\omega_{02}) \simeq \frac{x_k^{(2)2}}{2i\xi_2 \omega_{02}^2}$$

et dans un voisinage de ω_{01} (1) $\quad \dfrac{1}{\omega_0^2 - \omega^2 + 2i\xi\omega\omega_0} - \left(-\dfrac{i}{4\xi\omega_0^2} \right) = \dfrac{i}{4\xi\omega_0^2} \dfrac{\omega_0^2 - \omega^2 - i2\xi\omega_0^2}{\omega_0^2 - \omega^2 + i2\xi\omega_0^2}$

$a_{kk}(\omega) \simeq x_k^{(1)2} \dfrac{\omega_{01}^2 - \omega^2 - 2i\xi_1\omega\omega_{01}}{(\omega_{01}^2 - \omega^2)^2 + (2\xi_1\omega\omega_{01})^2} \simeq$ cercle centre $\left(0, -\dfrac{x_k^{(1)2}}{4\xi_1 \omega_{01}^2} \right)$, rayon $\dfrac{x_k^{(1)2}}{4\xi_1 \omega_{01}^2}$

$$|a_{kk}(\omega)| \simeq \frac{x_k^{(1)2}}{2\xi_1 \omega_{01}\omega}$$

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(1) $\omega_0 = \omega_{01}(1+\xi) \Rightarrow a_{kk}(\omega) = -\dfrac{x_k^{(1)2}}{2\omega_{01}^2} \dfrac{2i}{\xi_1 - i\xi_1} \Rightarrow \left| a_{kk} - \left(-\dfrac{i x_k^{(1)2}}{4\omega_{01}^2 \xi_1} \right) \right| = \dfrac{x_k^{(1)2}}{4\omega_{01}^2} \left| \dfrac{-2}{\xi - i\xi_1} + \dfrac{i}{\xi_1} \right| = \dfrac{x_k^{(1)2}}{4\omega_{01}^2} \dfrac{1}{\xi_1}$