1. angles θ_1, θ_2.

2. $T = \frac{1}{2} m l^2 (\dot\theta_1^2 + \dot\theta_2^2)$ kinetic energy

Position $A = \begin{pmatrix} a \sin\theta_1 \\ a(1-\cos\theta_1) \end{pmatrix}$; $B = \begin{pmatrix} a\sin\theta_2 + L \\ a(1-\cos\theta_2) \end{pmatrix}$; $AB = \begin{pmatrix} L + a(\sin\theta_2 - \sin\theta_1) \\ -a(\cos\theta_2 - \cos\theta_1) \end{pmatrix}$

$|AB|^2 = L^2 + 2a \times 2L \cos\frac{\theta_2+\theta_1}{2} \sin\frac{\theta_2-\theta_1}{2} + (2a)^2 \sin^2\left(\frac{\theta_2-\theta_1}{2}\right)$

$\simeq \left(L + 2a \sin\frac{\theta_2-\theta_1}{2}\right)^2 + O(\theta_2-\theta_1)^3$

$|AB| - L = 2a \sin\frac{\theta_2-\theta_1}{2} \simeq a(\theta_2-\theta_1)$ elongation

$V_{int} = \frac{1}{2} k a^2 (\theta_2-\theta_1)^2$; $V_{ext} = mgl(1-\cos\theta_1) + mgl(1-\cos\theta_2)$

$V = V_{int} - V_{ext}$

3. $\frac{\partial V}{\partial \theta_1} = ka^2(\theta_1-\theta_2) + mgl \sin\theta_1$; $\frac{\partial V}{\partial \theta_2} = ka^2(\theta_2-\theta_1) + mgl\sin\theta_2$

$\frac{\partial V}{\partial \theta_1} = \frac{\partial V}{\partial \theta_2} = 0$ for $\theta_1 = \theta_2 = 0$ and $V \geq 0 \Rightarrow$ equilibrium + stability

4. Linearization

$V \simeq \frac{1}{2} \Theta^T \left.\frac{\partial^2 V}{\partial\Theta \partial\Theta}\right|_{\Theta=0} \Theta + \frac{1}{2} mgl\, \Theta^2$; $\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$

$M\ddot\Theta + K\Theta = 0$

$M = \begin{pmatrix} ml^2 & 0 \\ 0 & ml^2 \end{pmatrix}$; $K = \begin{pmatrix} ka^2 + mgl & -ka^2 \\ -ka^2 & ka^2 + mgl \end{pmatrix}$ ← geometrical stiffness

5. Eigenmodes

$\omega_1^2 = \frac{g}{l}$; $\frac{\theta_1}{\theta_2} = 1$ (~ single pendulum)

$\omega_2^2 = \frac{g}{l} + \frac{2ka^2}{ml^2} \Rightarrow \frac{\theta_1}{\theta_2} = -1$