1_1 Energie cinétique :

$$T = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2$$

1_2. Energie potentielle :

$$V_{int} = \frac{1}{2} k_1 \delta q_1^2 + \frac{1}{2} k_2 (\delta q_2 - \delta q_1)^2 + \frac{1}{2} k_3 (\delta q_3 - \delta q_2)^2$$

$$= \frac{1}{2} (k_1 + k_2) \delta q_1^2 + \frac{1}{2} (k_2 + k_3) \delta q_2^2 + \frac{1}{2} k_3 \delta q_3^2 - k_2 \delta q_1 \delta q_2 - k_3 \delta q_2 \delta q_3$$

1_3 Travail virtuel des forces visqueuses :

$$\delta W_{nc,v} = -c_1 \dot{q}_1 \delta q_1 - c_2 (\dot{q}_2 - \dot{q}_1)(\delta q_2 - \delta q_1) - c_3 (\dot{q}_3 - \dot{q}_2)(\delta q_3 - \delta q_2)$$

$$= -(c_1 + c_2) \dot{q}_1 \delta q_1 - (c_2 + c_3) \dot{q}_2 \delta q_2 - c_3 \dot{q}_3 \delta q_3 + c_2 \dot{q}_2 \delta q_1 + c_2 \dot{q}_1 \delta q_2 + c_3 \dot{q}_2 \delta q_3 + c_3 \dot{q}_3 \delta q_2$$

$$\delta W_{nc,ext} = Q_1^{ext} \delta q_1 + Q_2^{ext} \delta q_2 + Q_3^{ext} \delta q_3$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = Q_{nc}$$

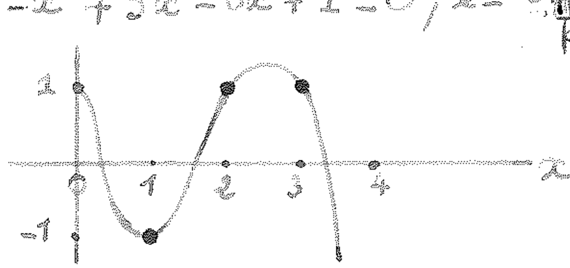
2.

$$M \ddot{q} + C \dot{q} + K q = Q^{ext}$$

$$M = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}, \quad K = \begin{pmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{pmatrix}.$$

3. Valeurs propres solutions de

$$-x^3 + 5x^2 - 6x + 1 = 0, \quad x = \omega^2 \frac{m}{k}$$



Vecteurs propres $q = [1/(2-x), 1, 1/(1-x)]^T$

$$\begin{cases} x \in [0,1] & + & + & + \\ x \in [1,2] & + & + & - \\ x \in [3,4] & - & + & - \end{cases}$$