Grenoble – June 27, 2011

Physics of geomaterials

at small scale

J. Carlos Santamarina
Georgia Institute of Technology

"… Coulomb… purposely ignored the fact that sand consists of individual grains

Coulomb's idea proved very useful as a working hypothesis …. but it developed into an obstacle against further progress as soon as its hypothetical character came to be forgotten by Coulomb's successors.

The way out of the difficulty lies in dropping the old fundamental principles and starting again from the elementary fact that sand consists of individual grains"

Terzaghi (1920)

Size (F=ma)

Shape

Strength: $\tau=\sigma' \tan\phi$

Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

Pores

Mixed fluids (Unsaturated Soils)

Reactive Fluids

Closing Thoughts

Particle Forces

Skeletal

Weight

Buoyant

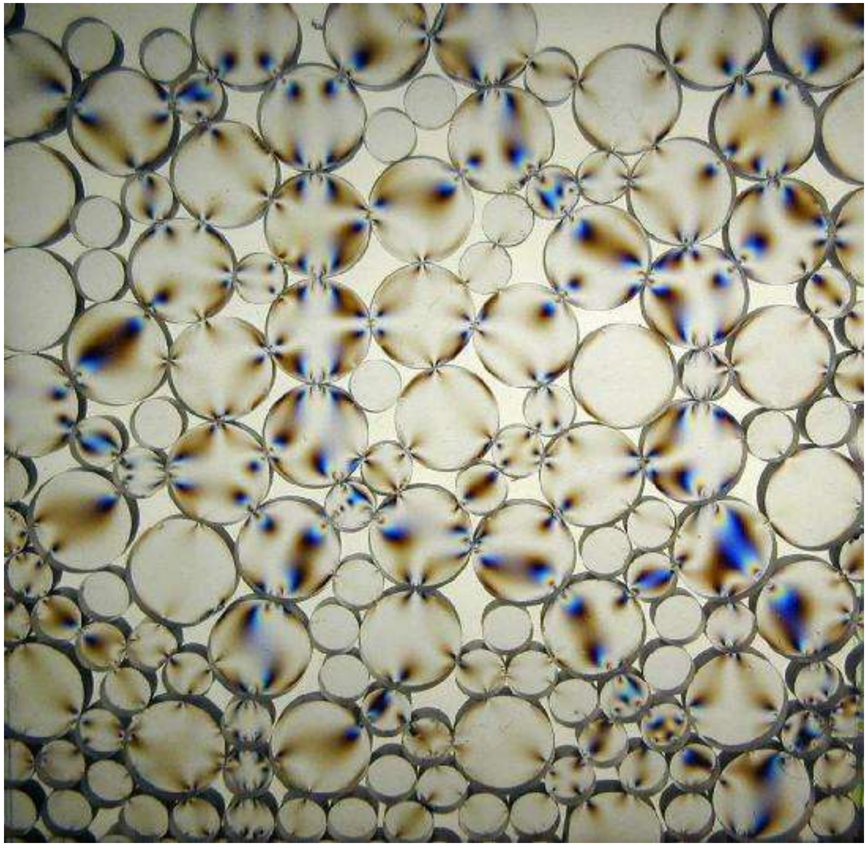
Hydrodynamic

Capillary

Electrical

- attraction
- repulsion

Cementation



Particle Forces

Skeletal

Weight

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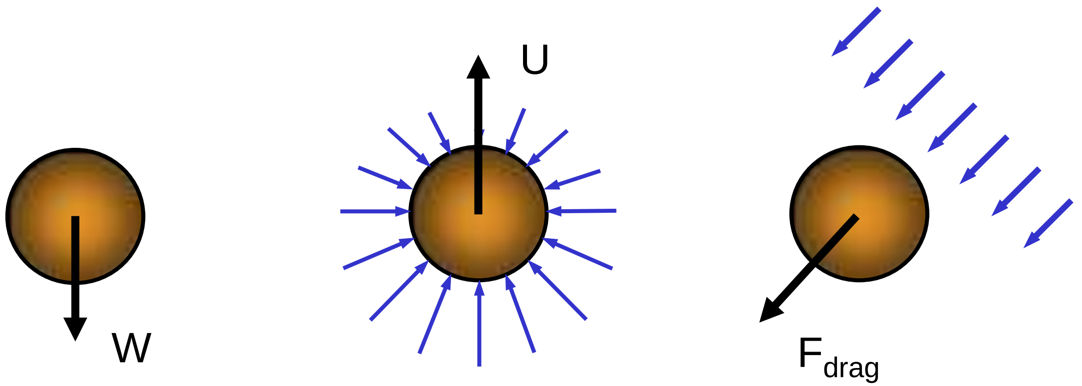
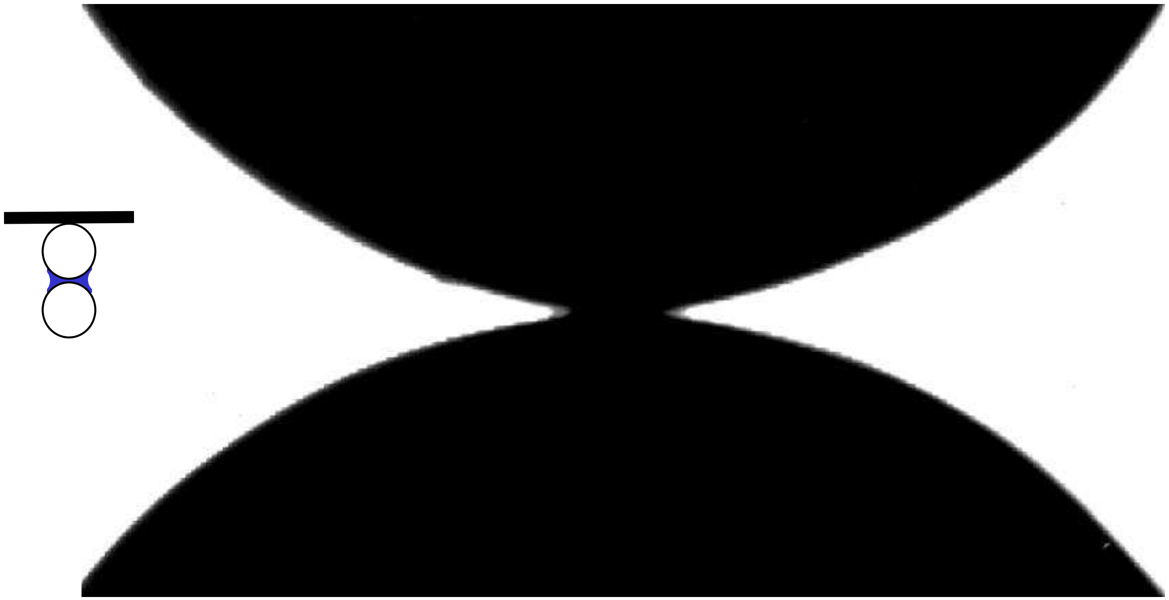
repulsion

Cementation



Particle Forces

Skeletal	
Weight	
Buoyant	
Hydrodynamic	
Capillary	
Electrical	
attraction	
repulsion	
Cementation	

Particle Forces

Skeletal

Weight

Buoyant

Hydrodynamic

Capillary

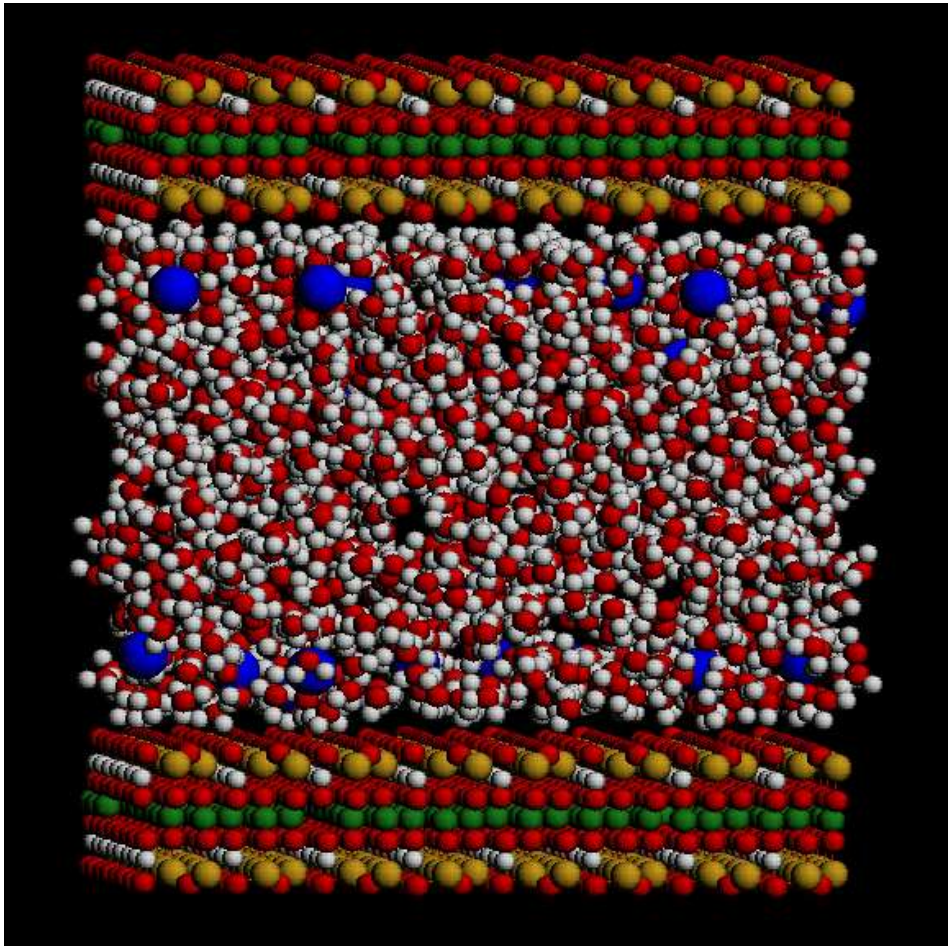
Electrical

attraction

repulsion

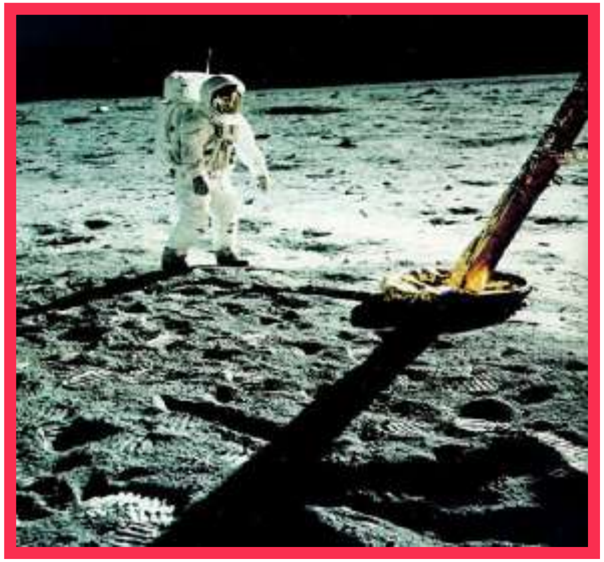
Cementation

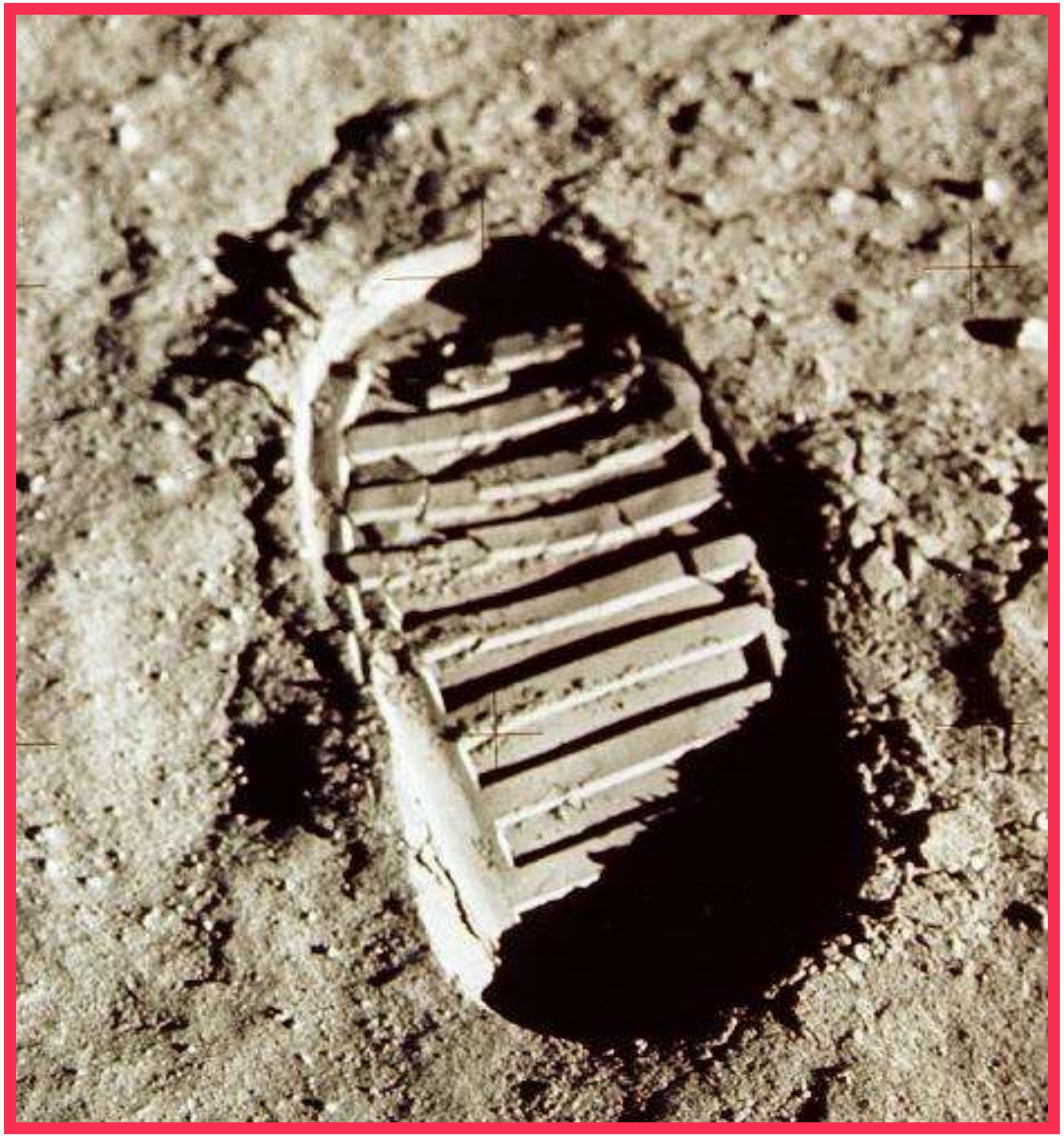
Laponite $1200\ H_2O$ $24\ Na^+$



(N. Skipper - UCL)

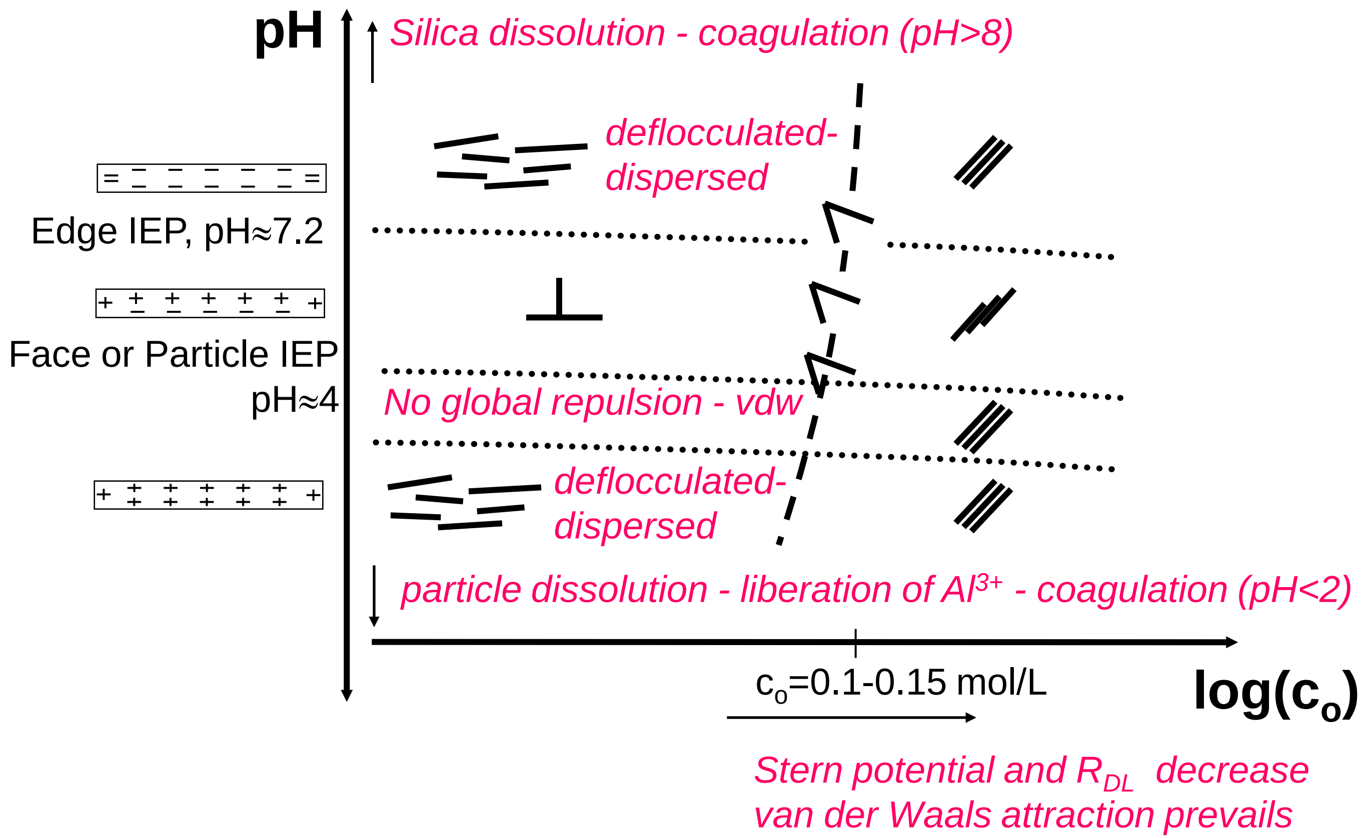
Footprints at 1/6 g





www.hq.nasa.gov

Fabric map - Kaolinite



Particle Forces

Skeletal

Weight

Buoyant

Hydrodynamic

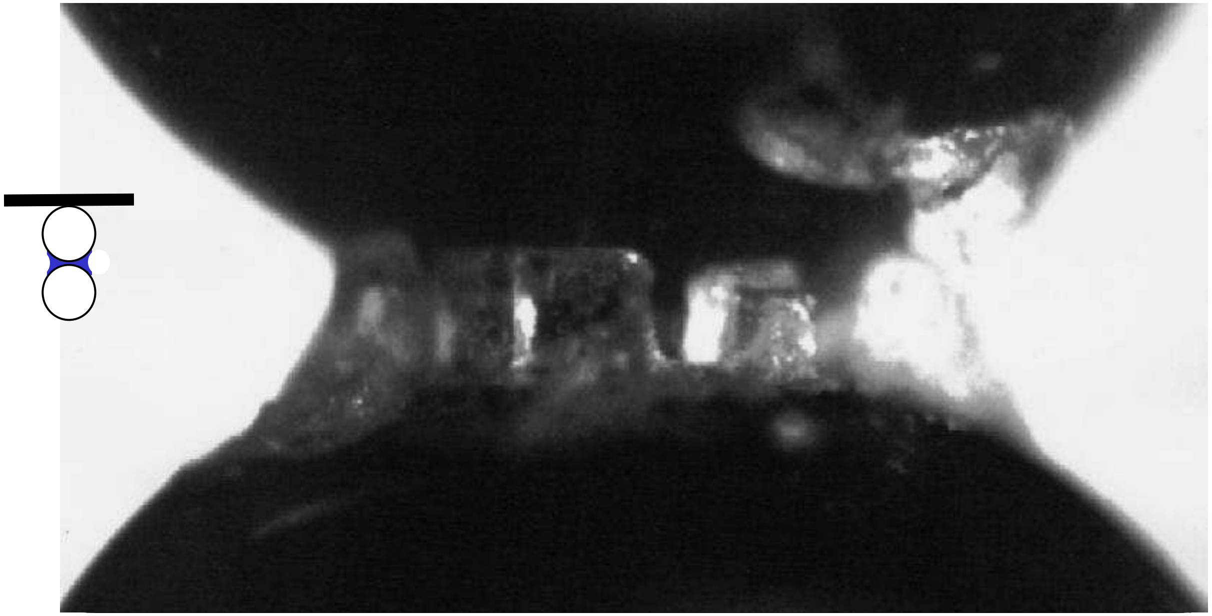
Capillary

Electrical

attraction

repulsion

Cementation



(passive)

Particle Forces – Spherical Particles

Skeletal	$\underline{N} = \sigma' d^2$	boundary-determined
Weight	$W = (\pi G_s \gamma_w / 6) d^3$	
Buoyant	$U = Vol \cdot \gamma_w = (\pi \gamma_w / 6) d^3$	particle-level
Hydrodynamic	$F_{drag} = 3\pi \mu v d$	
Capillary	$F_{cap} = \pi T_s d$	
Electrical		
attraction	$Att = \frac{A_h}{24t^2} d$	contact-level
repulsion	$Rep = 0.0024 \sqrt{c_o}\, e^{-10^8 t \sqrt{c_o}} d$	
Cementation	$T = \pi \sigma_{ten} t d$	

Force Balance: Capillary Force

Skeletal

Weight

Buoyant

Hydrodynamic

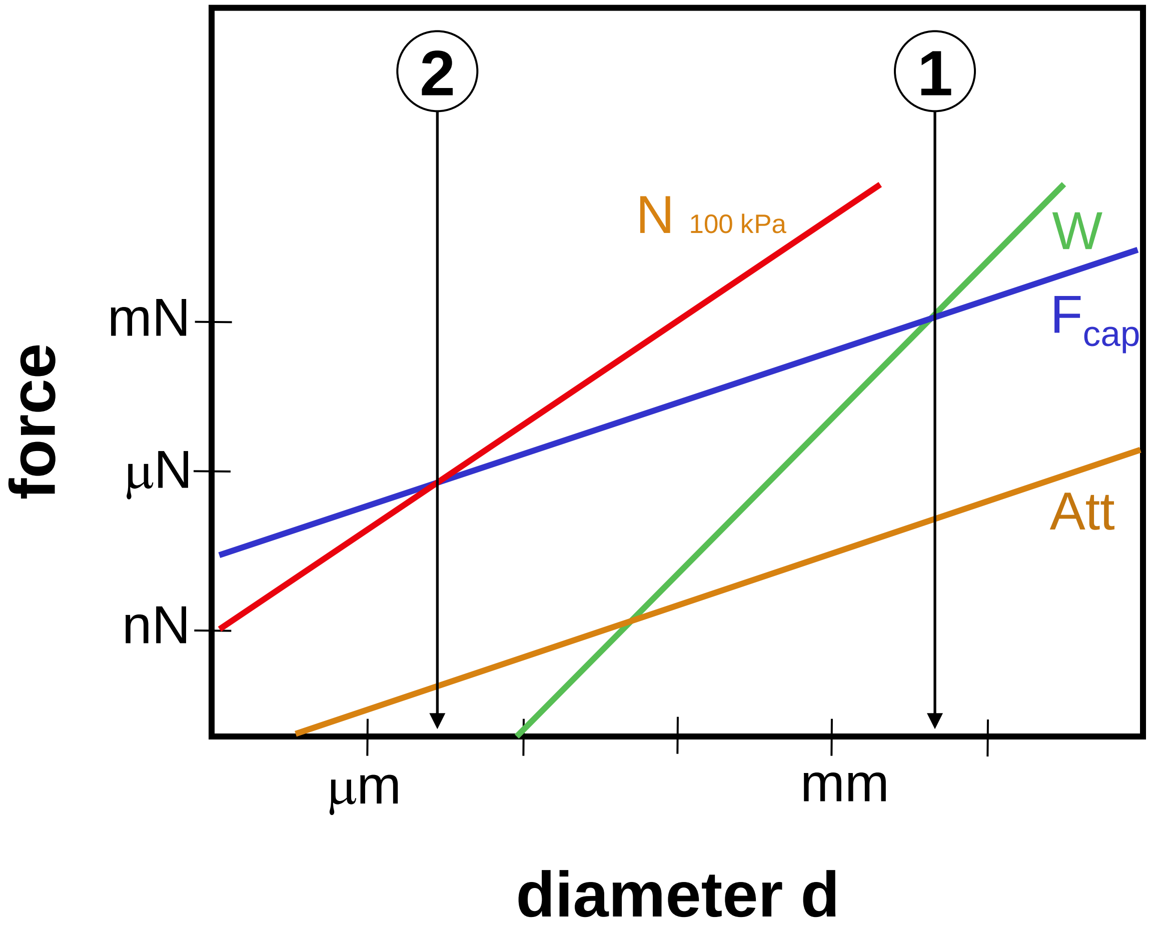
Capillary

Electrical

attraction

repulsion

Cementation



Particle Forces - Balance

Skeletal

Weight

Buoyant

Hydrodynamic

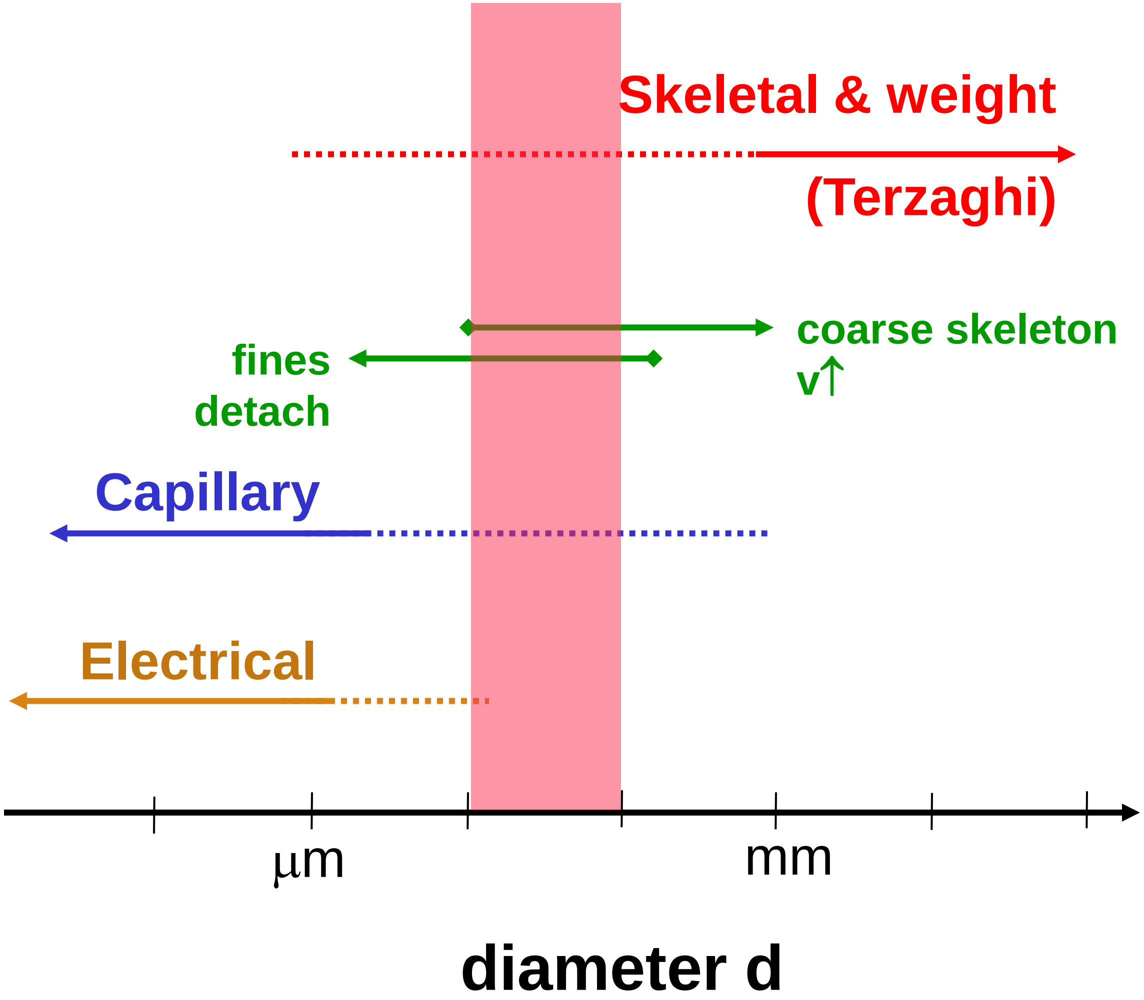
Capillary

Electrical

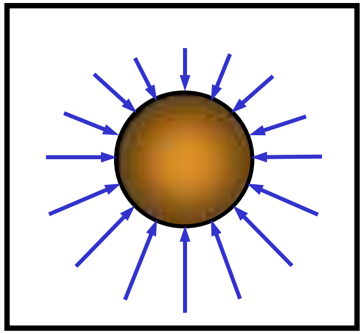
attraction

repulsion

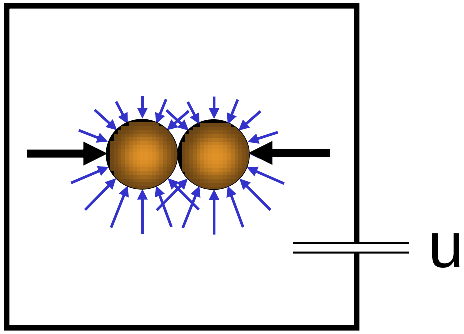
Cementation



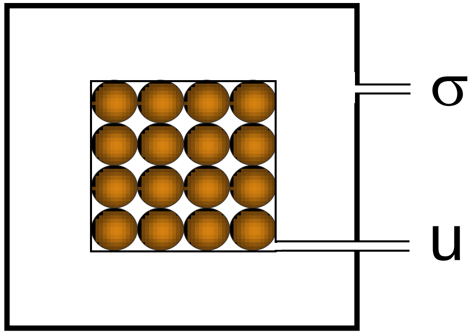
Effective stress: boundary determined



Archimedes buoyancy force
- ***NOT** affected by u*
- ***depends on du/dz***



Skeletal force (effective stress)
- ***NOT** affected by u*



Effective stress:
- ***established at the boundary***
- ***In the field? seepage force***

Size (F=ma)

Shape

Strength: $\tau=\sigma' \tan\phi$

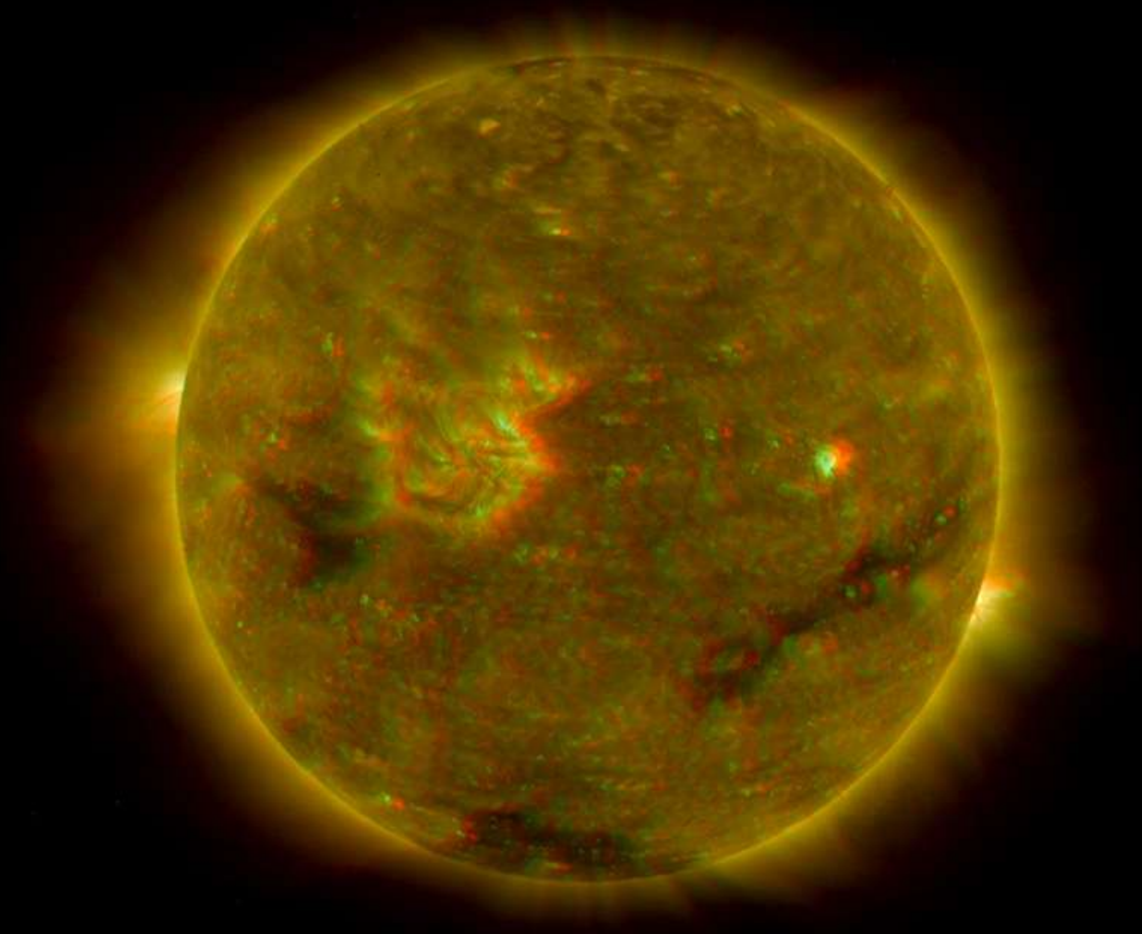
Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

Pores

Mixed fluids (Unsaturated Soils)

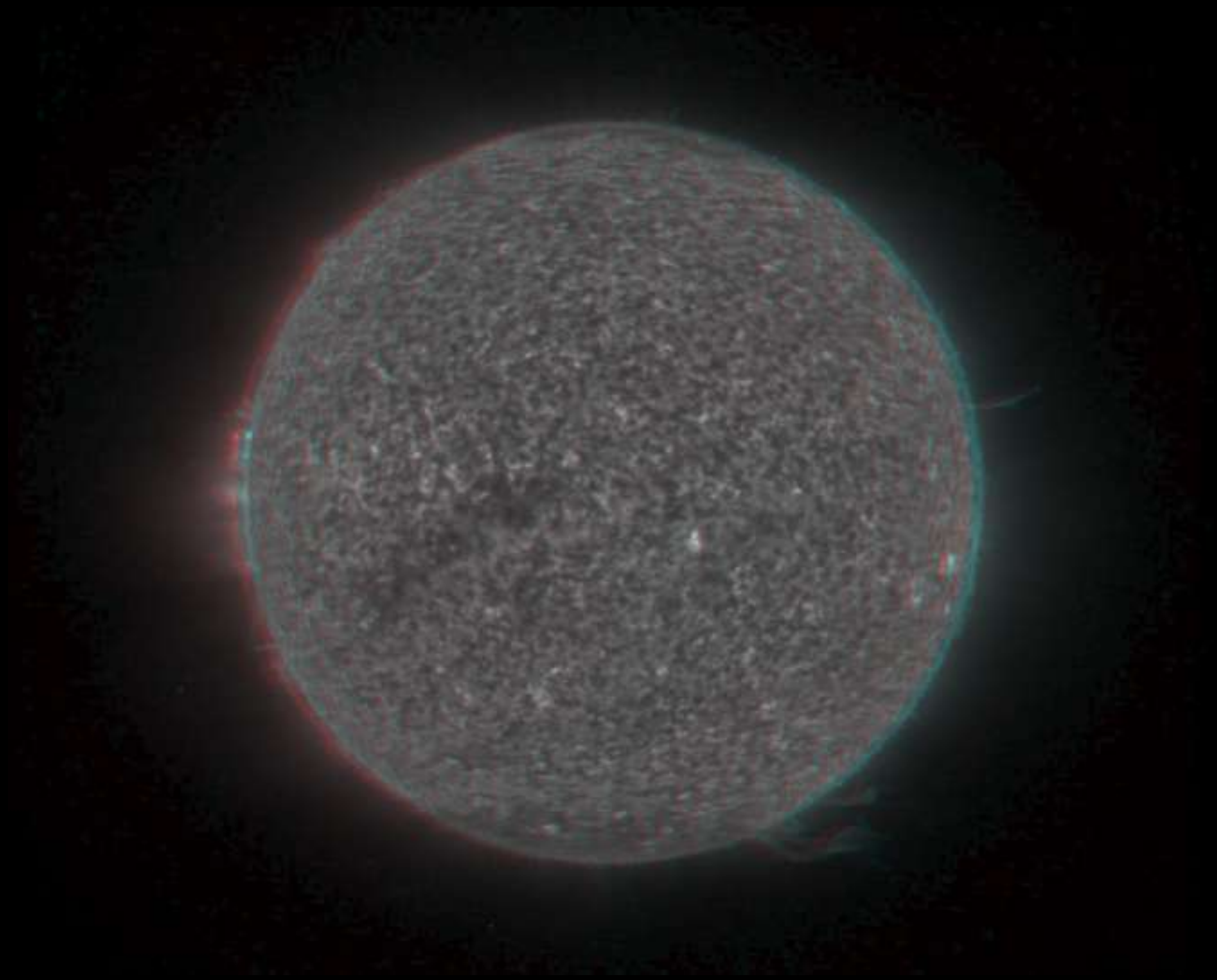
Reactive Fluids

Closing Thoughts

red on left

Sun - NASA’s STEREO
ultraviolet - 1 million degrees



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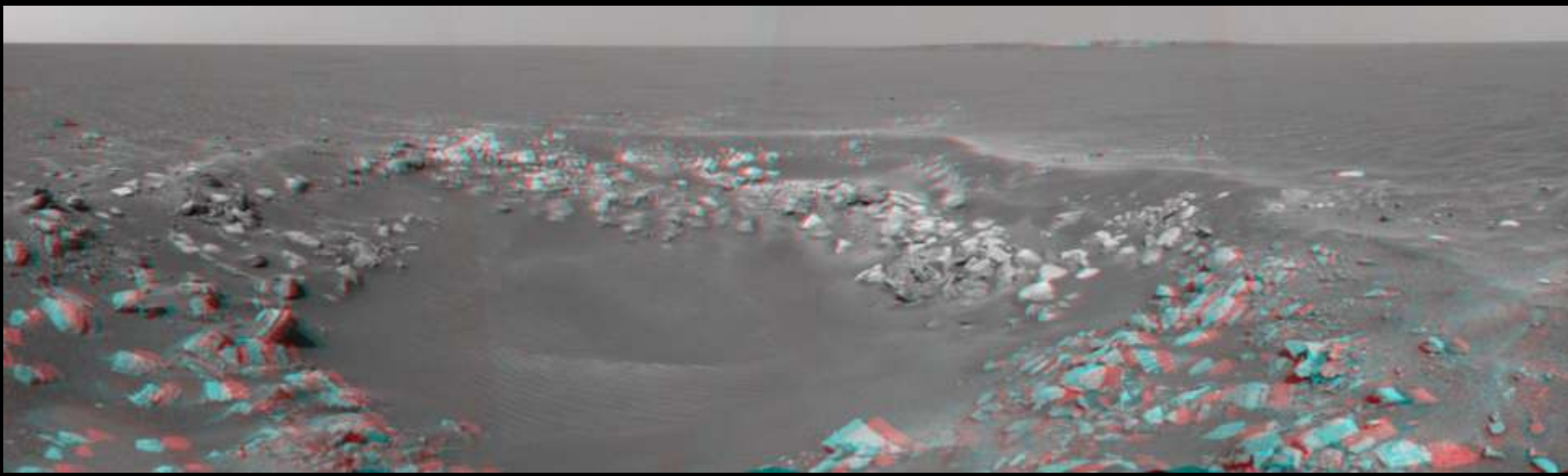
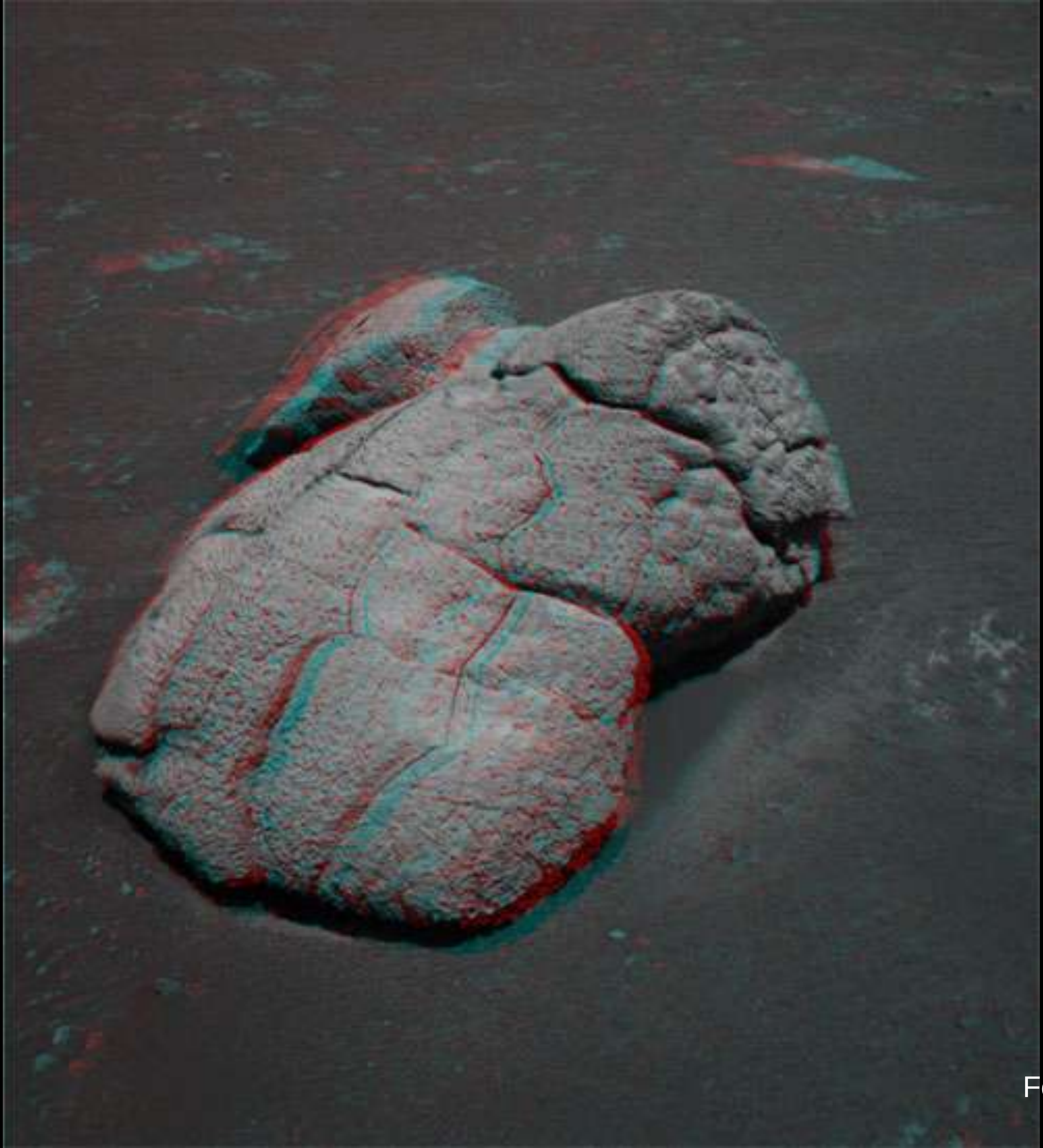


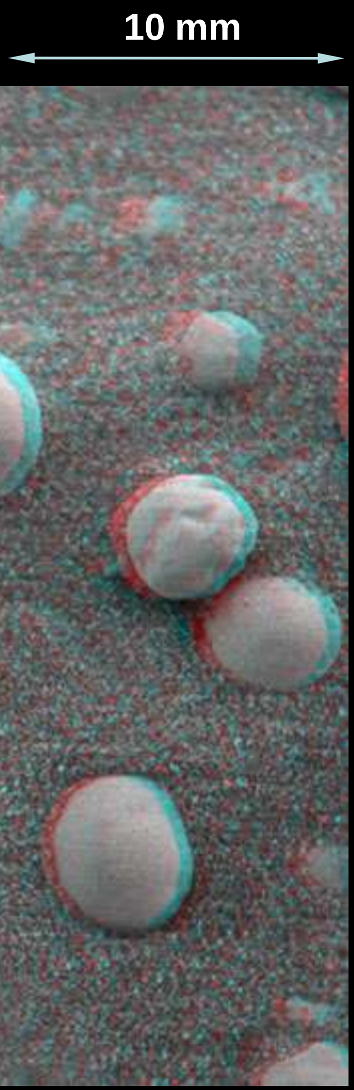
Crater on Mars
NASA

red on left



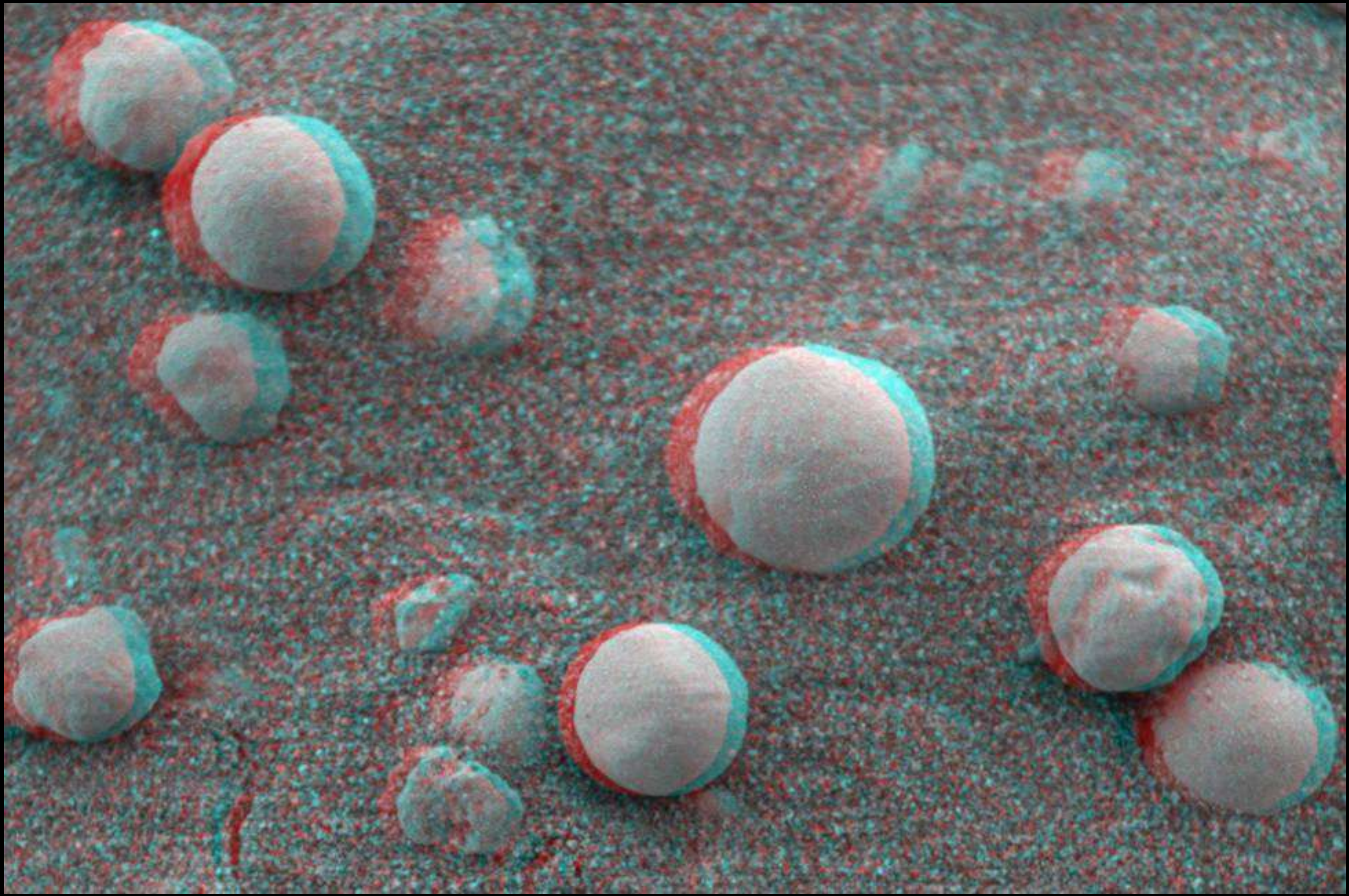
red on left

Formation on Mars
NASA



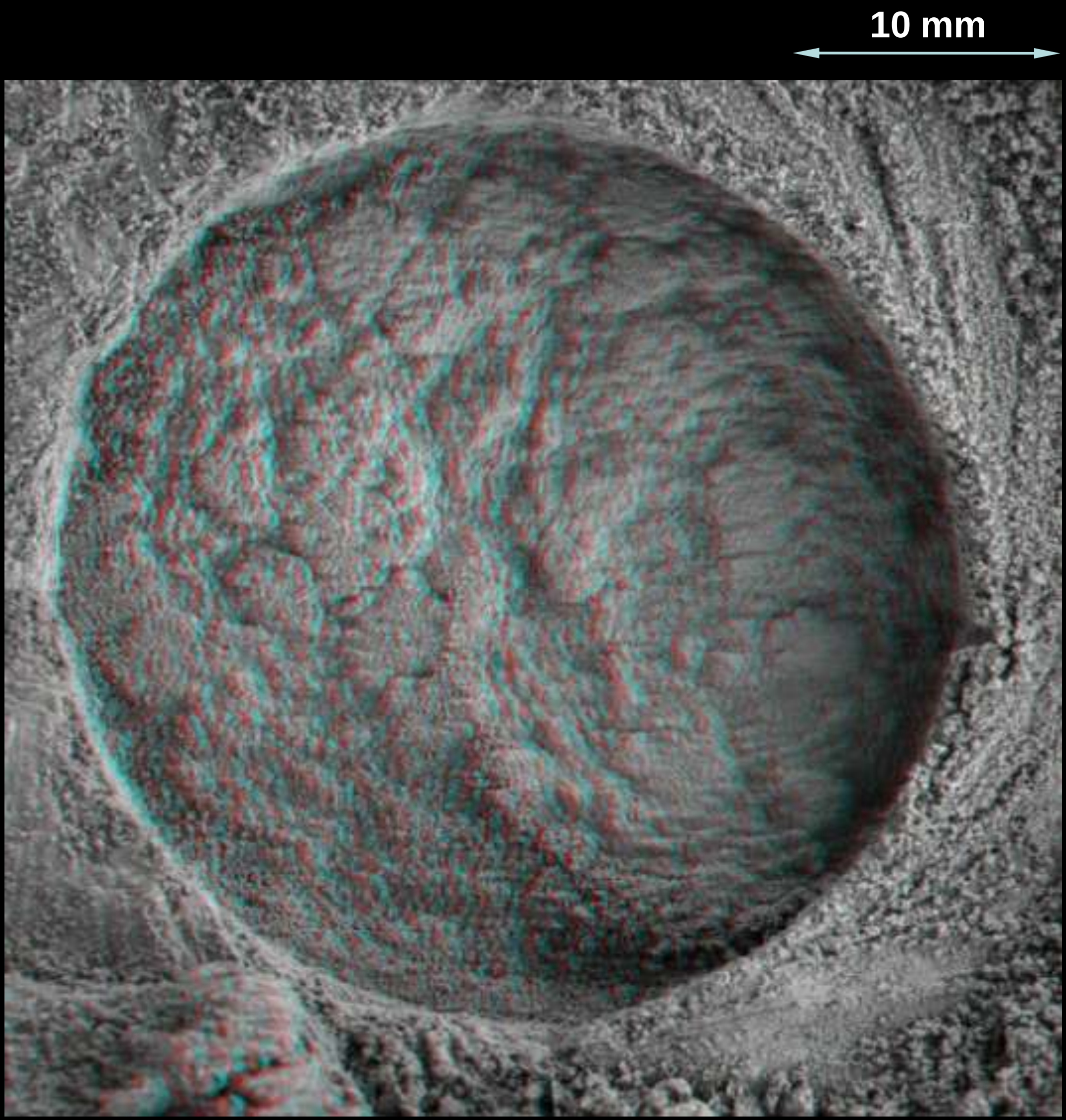
red on left

Berries on Mars
NASA



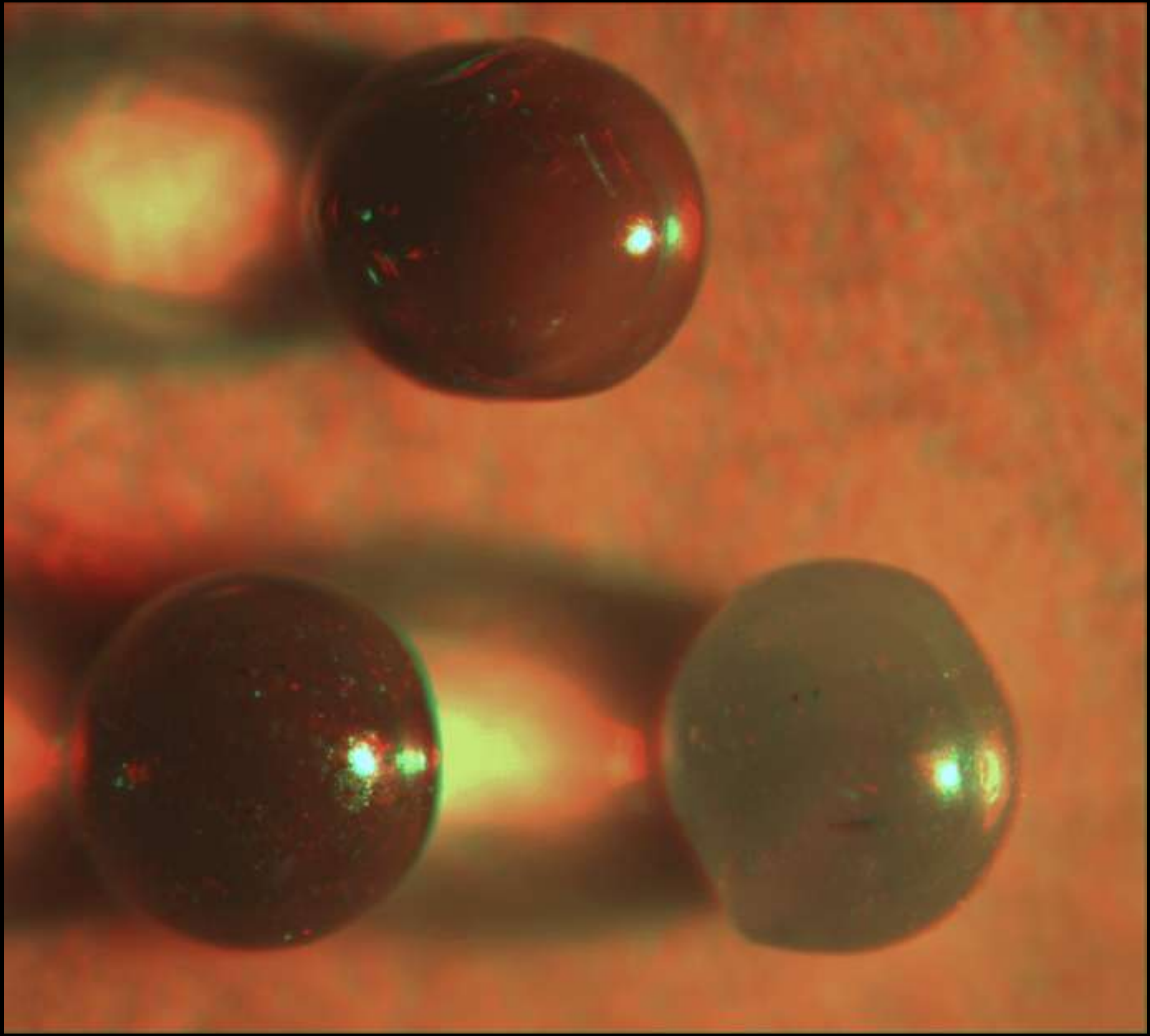
red on left

Berries on Mars
NASA



Diamond coring on Mars
NASA

red on left



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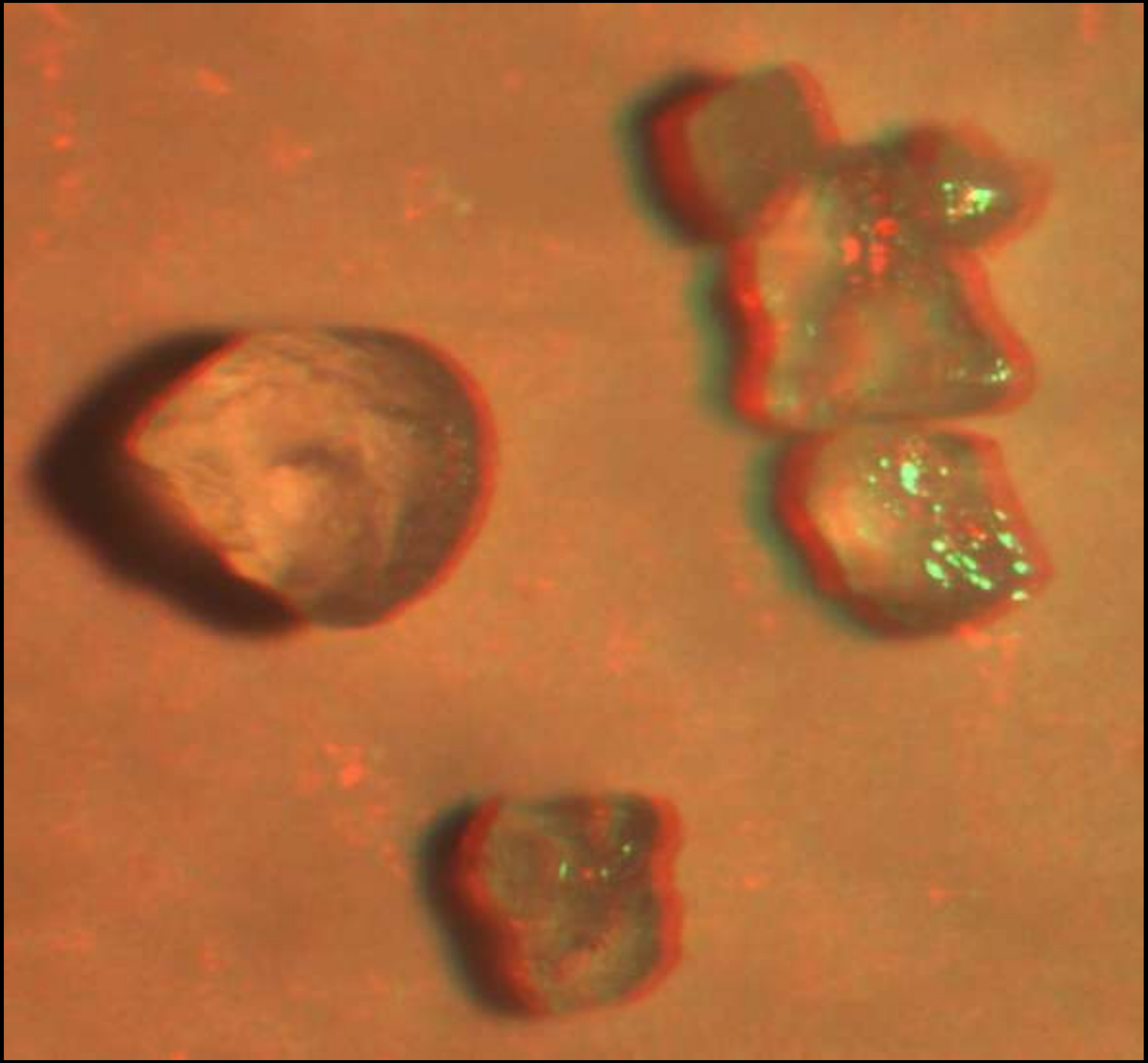
Glass Beads



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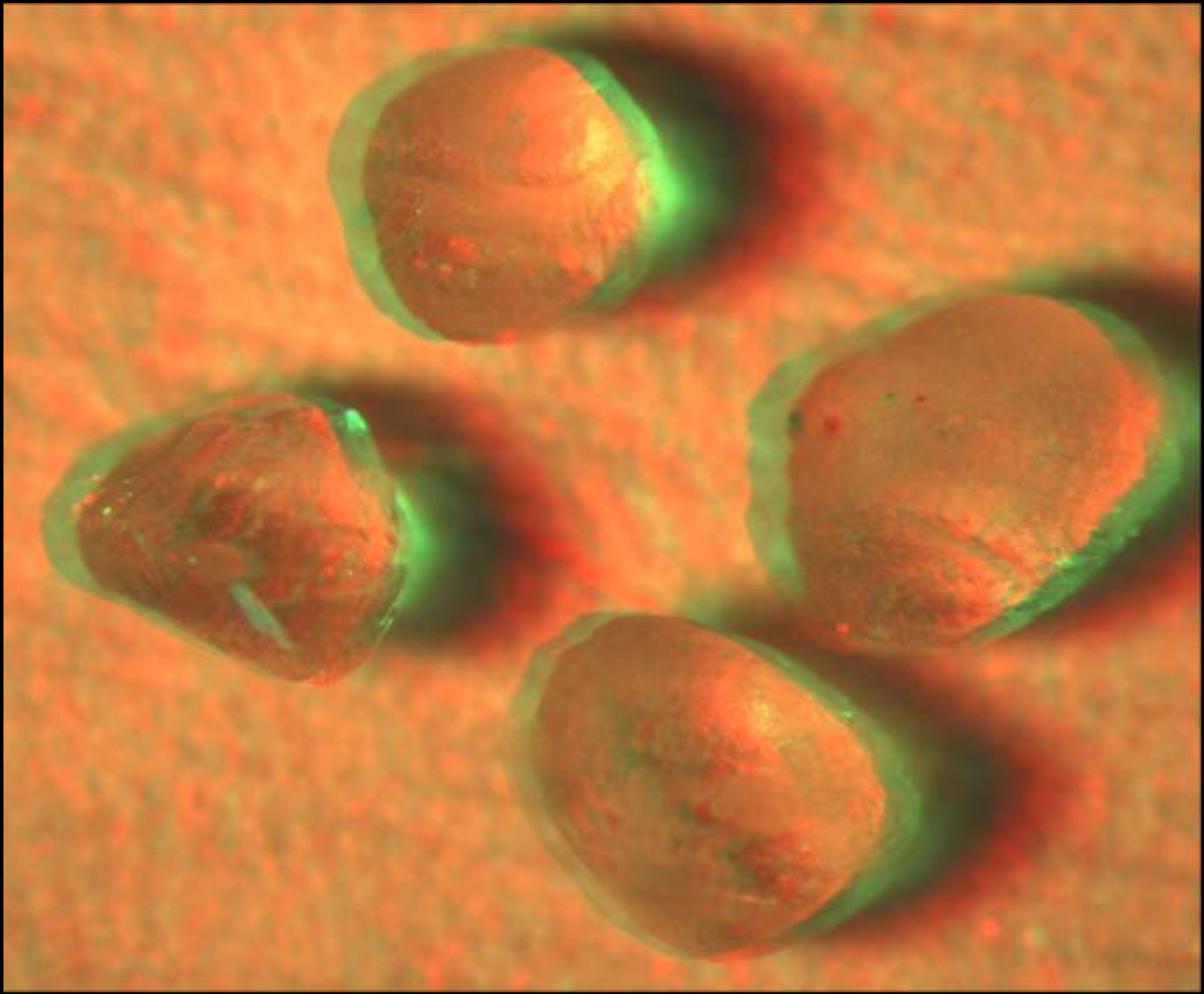
Nevada Sand



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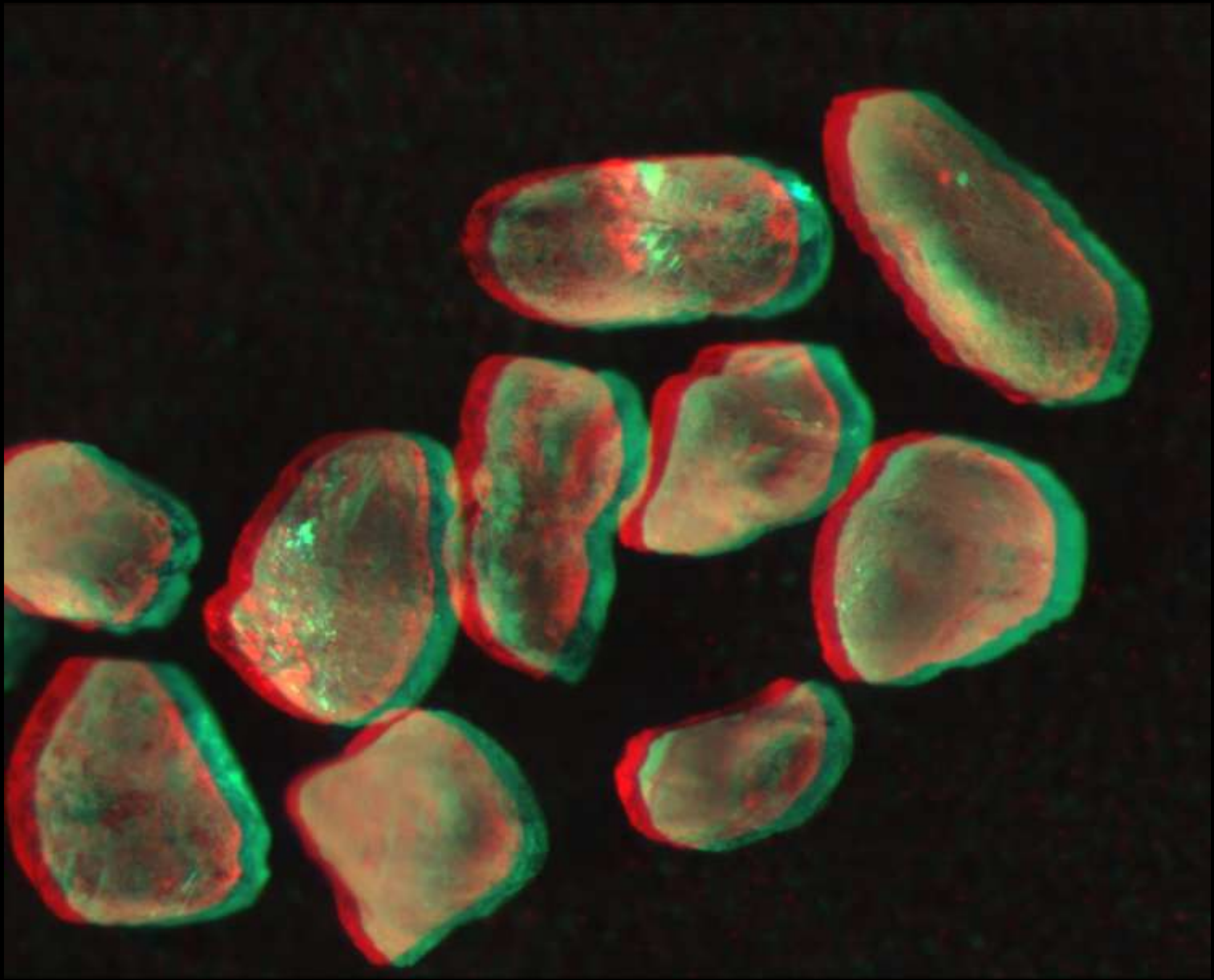

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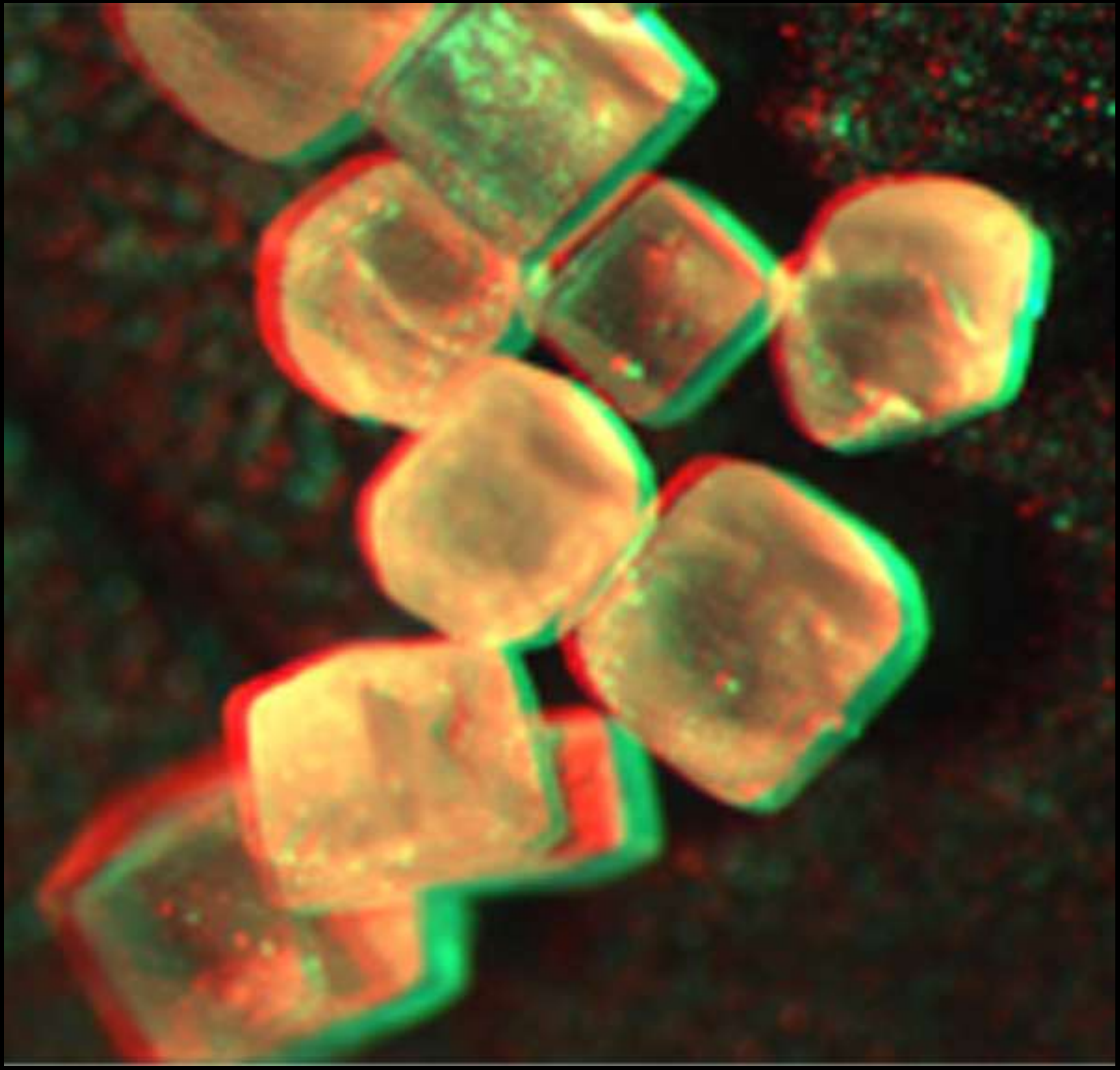
Ottawa 50-70



red on left

Table Salt

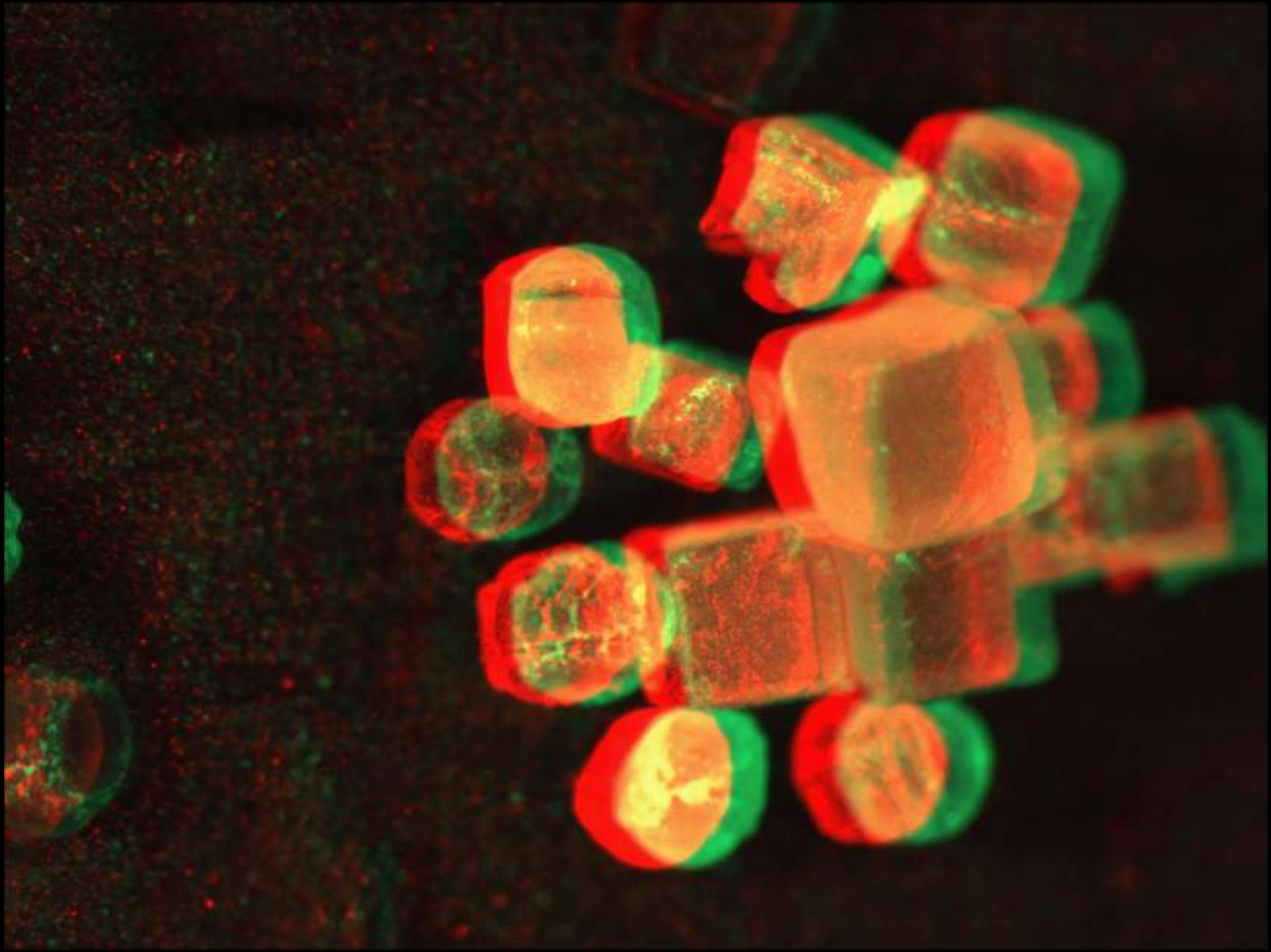
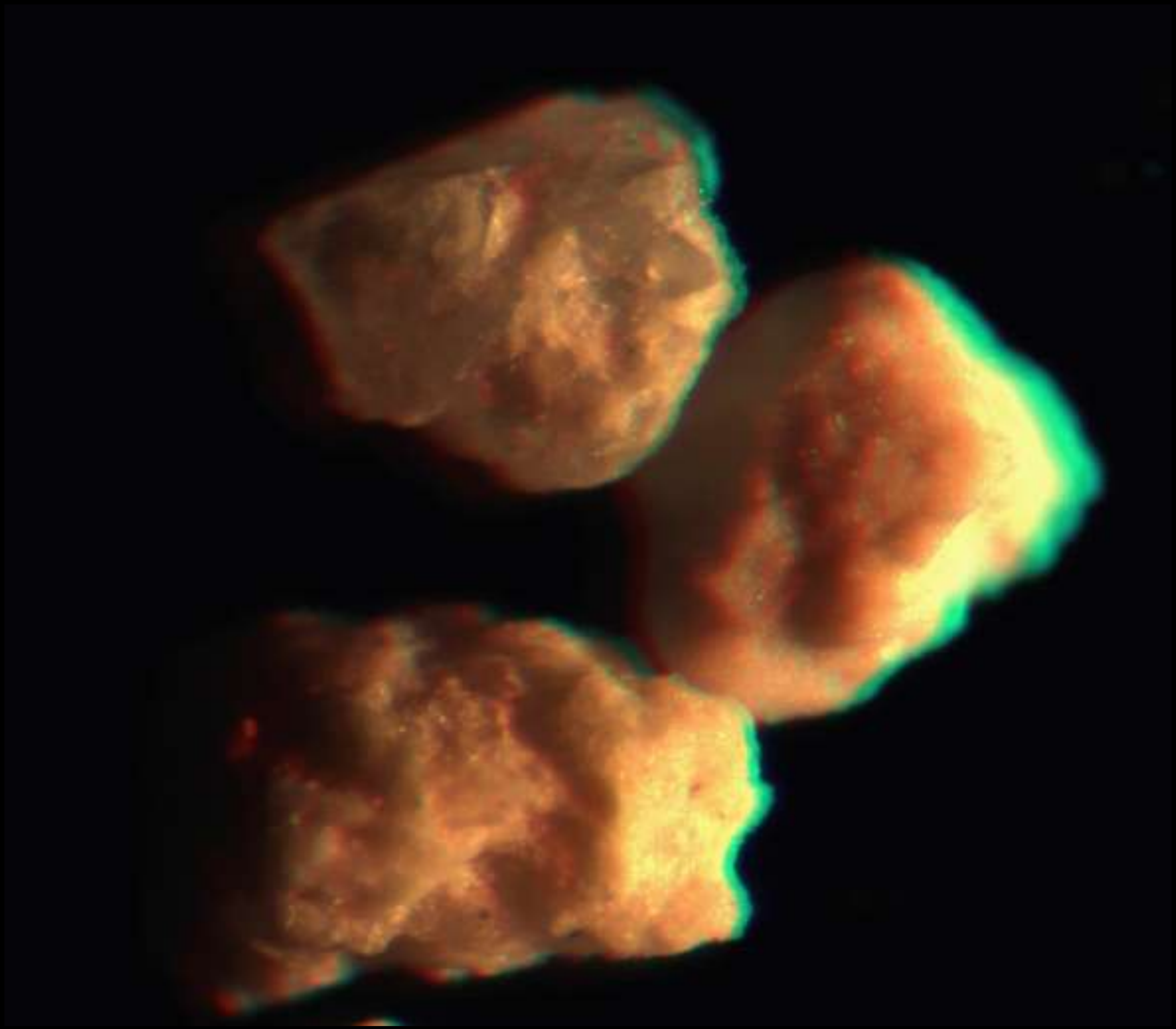


Table Salt



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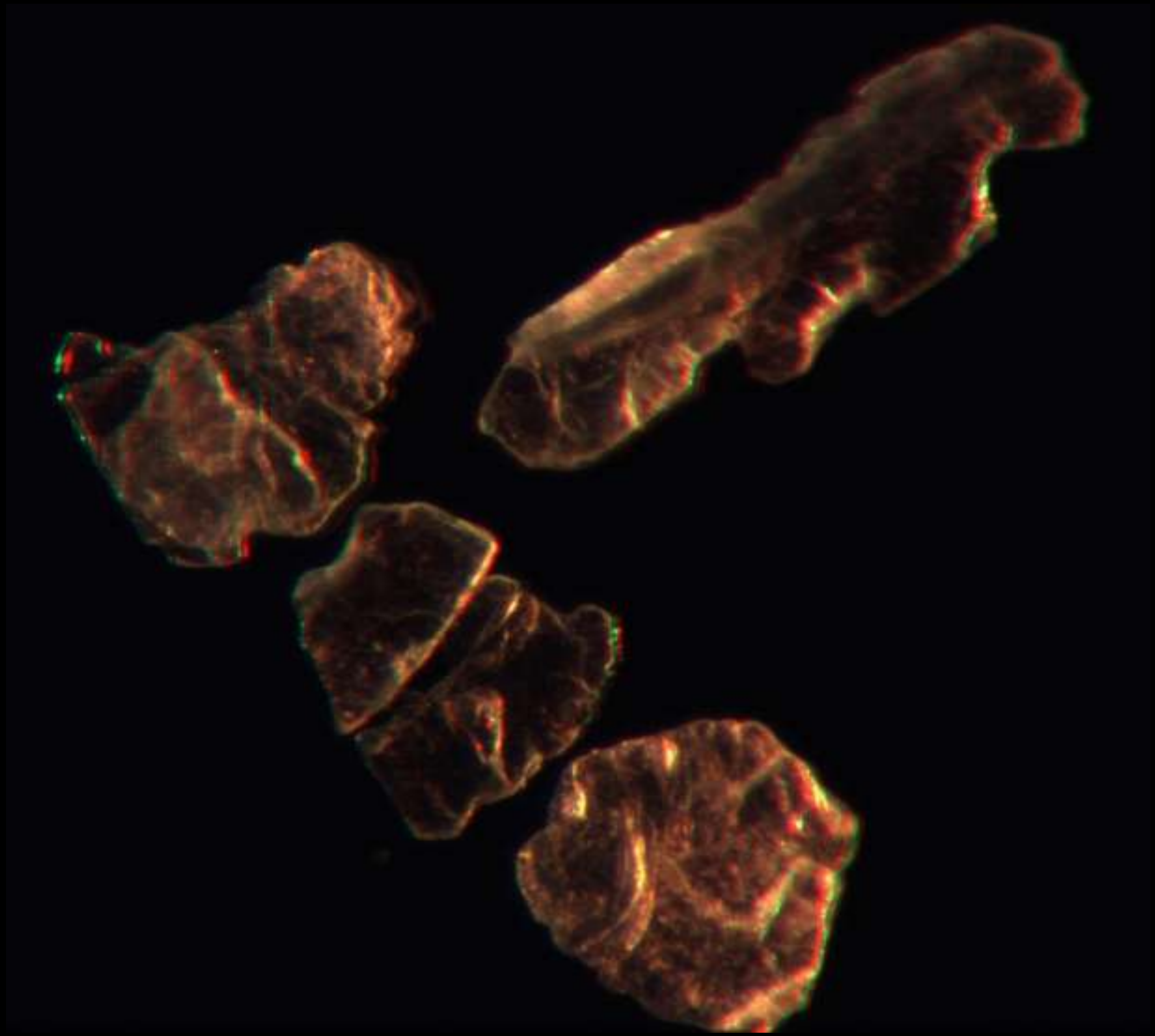
Crushed carbonate



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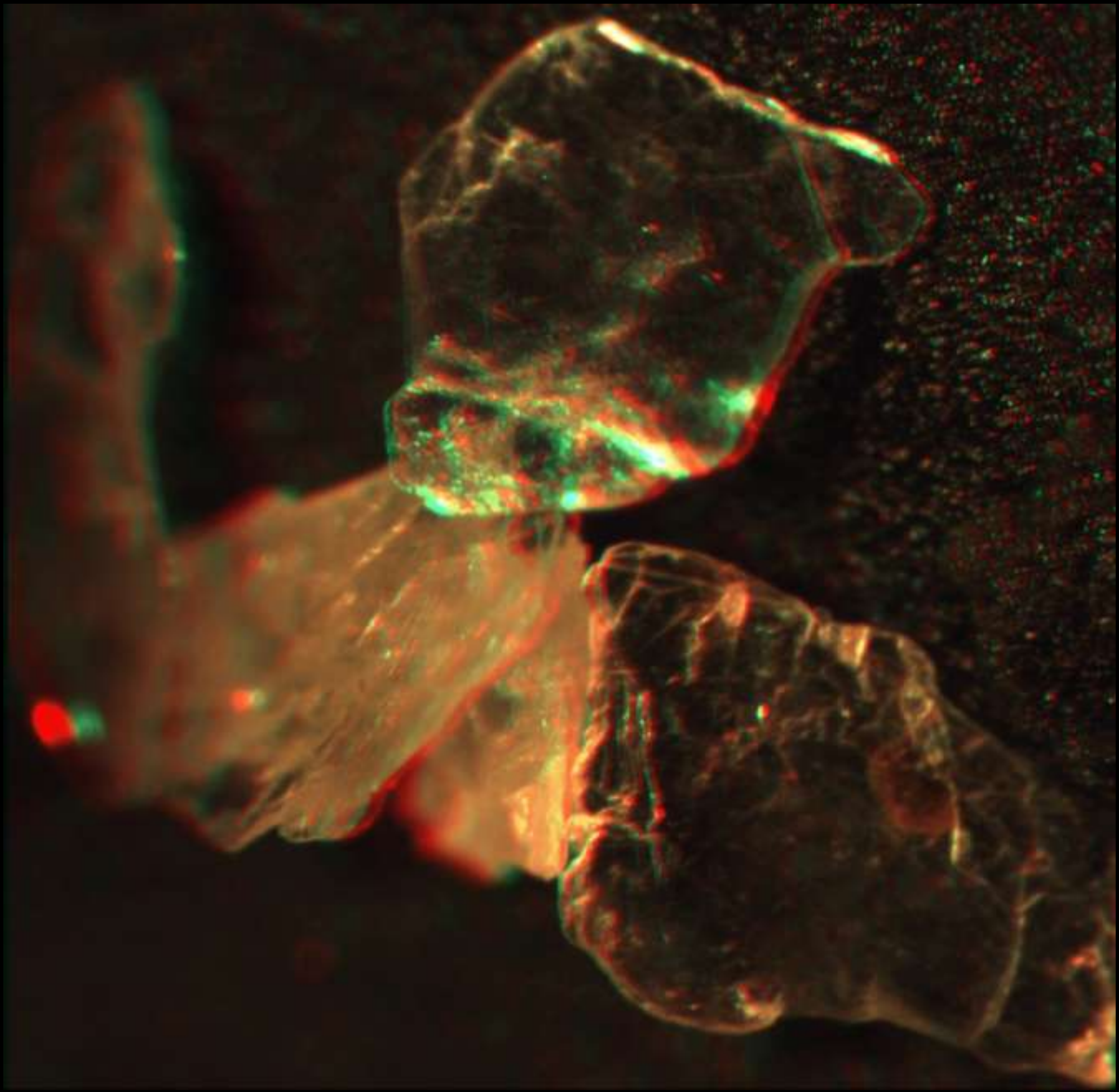
Mica



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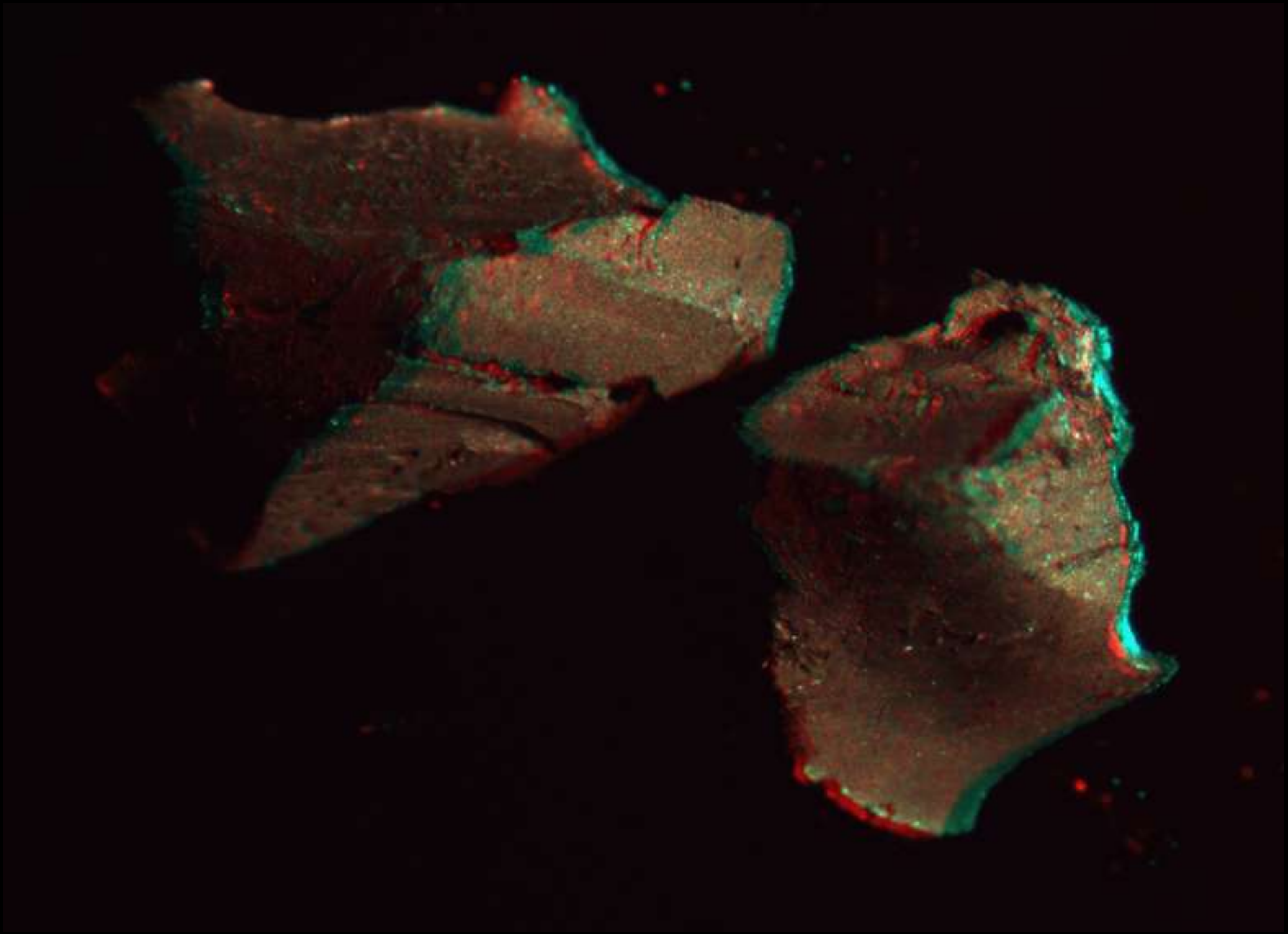
Mica



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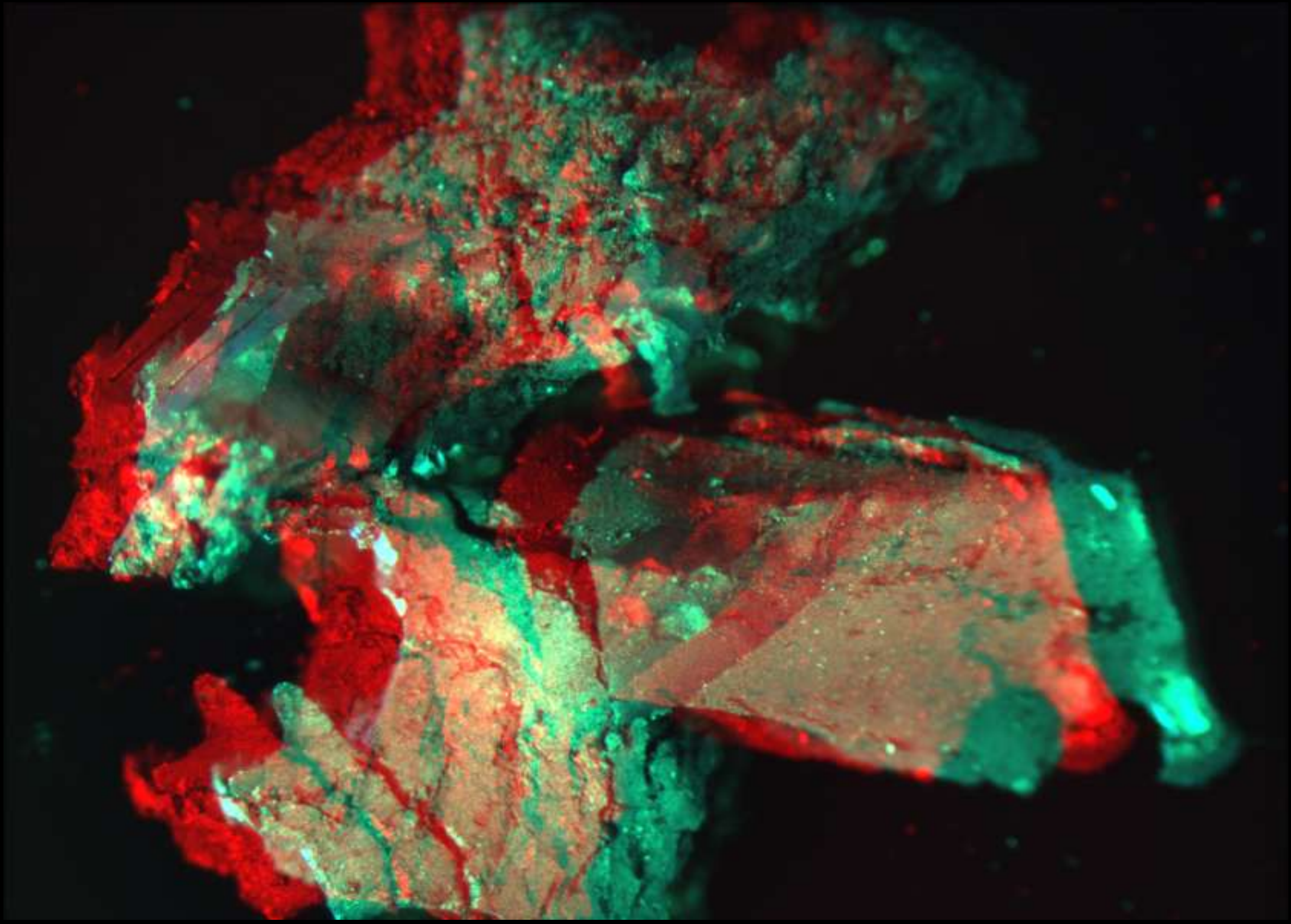
Threaded rubber

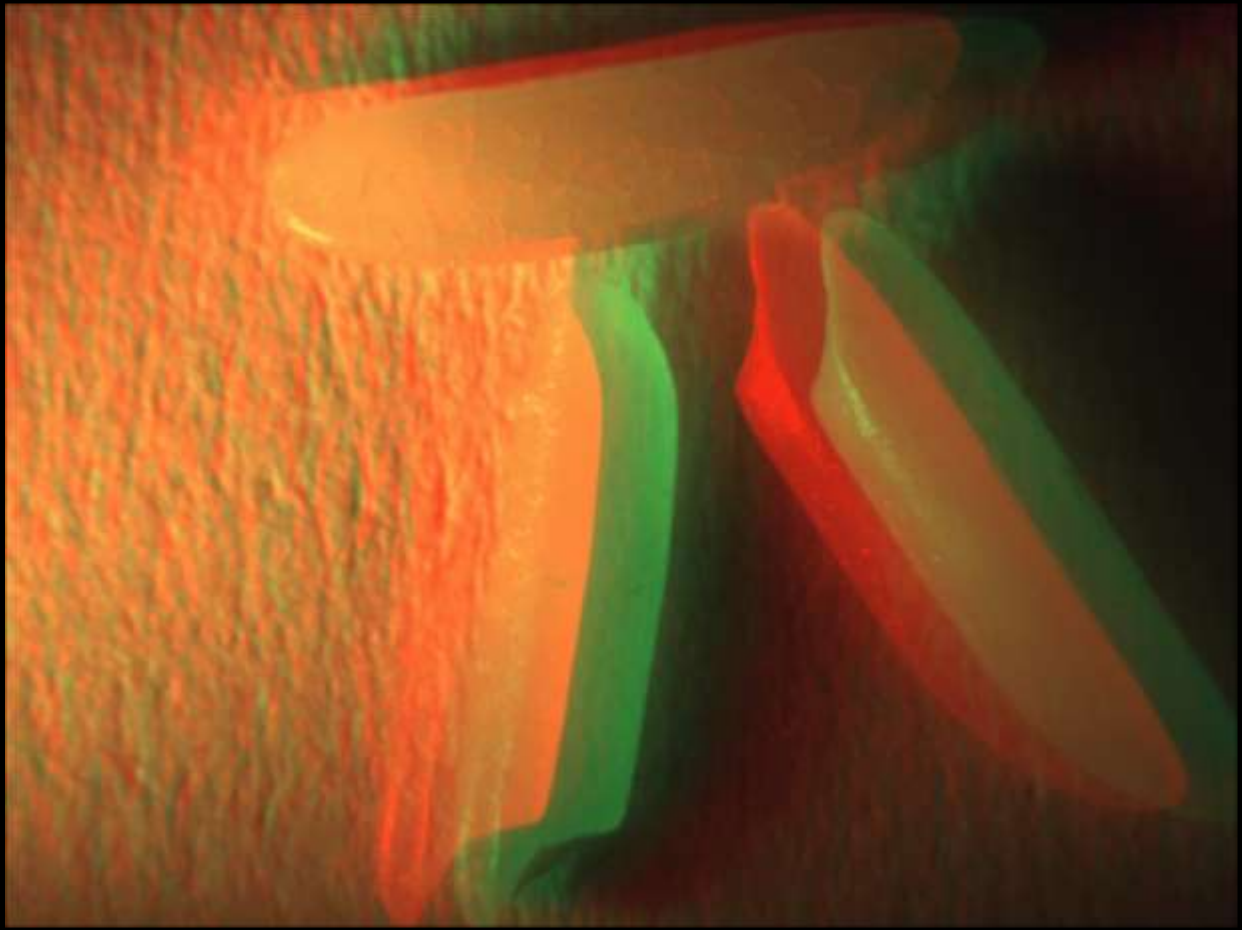


red on left

Threaded rubber

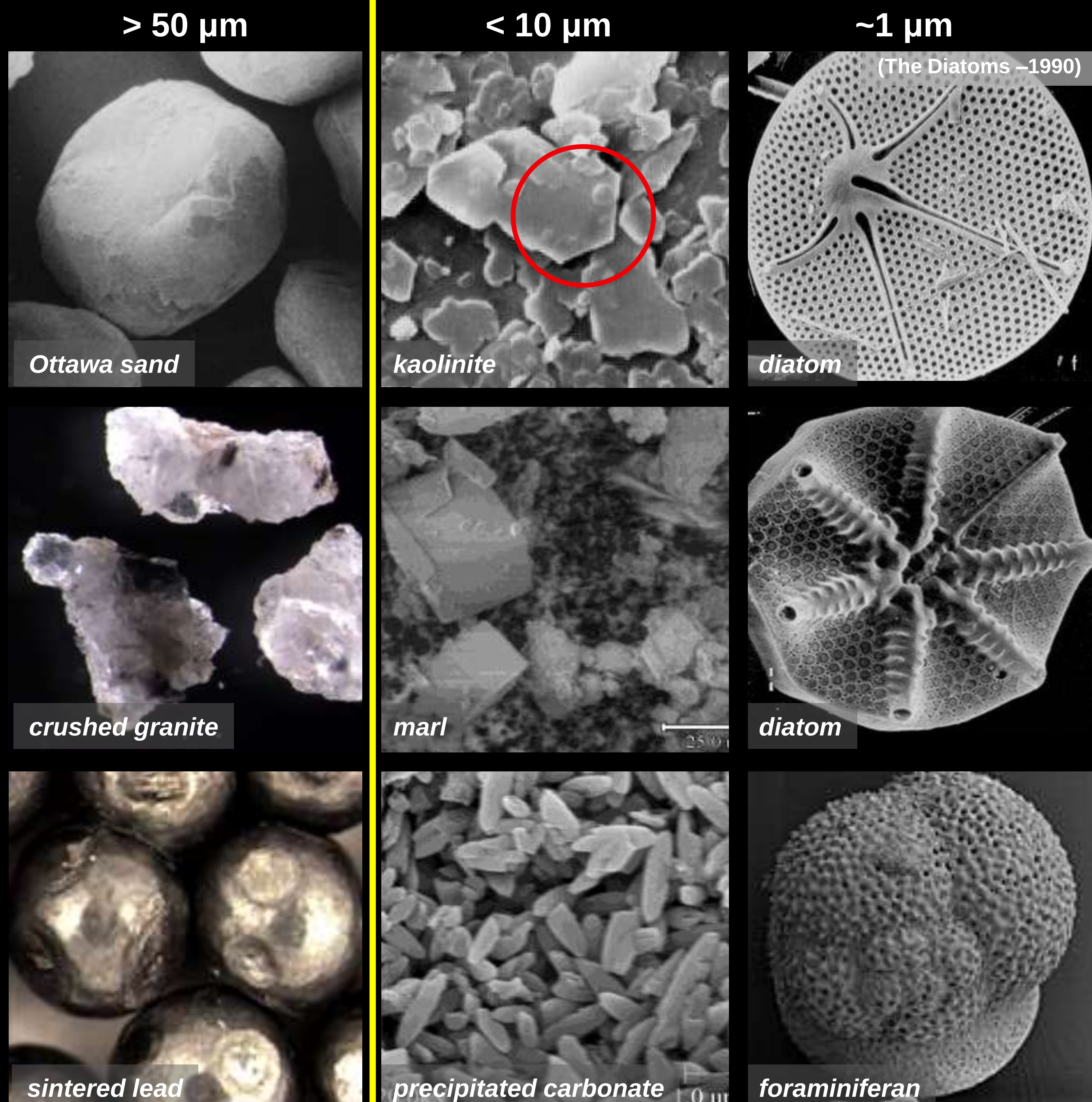
red on left

Rice



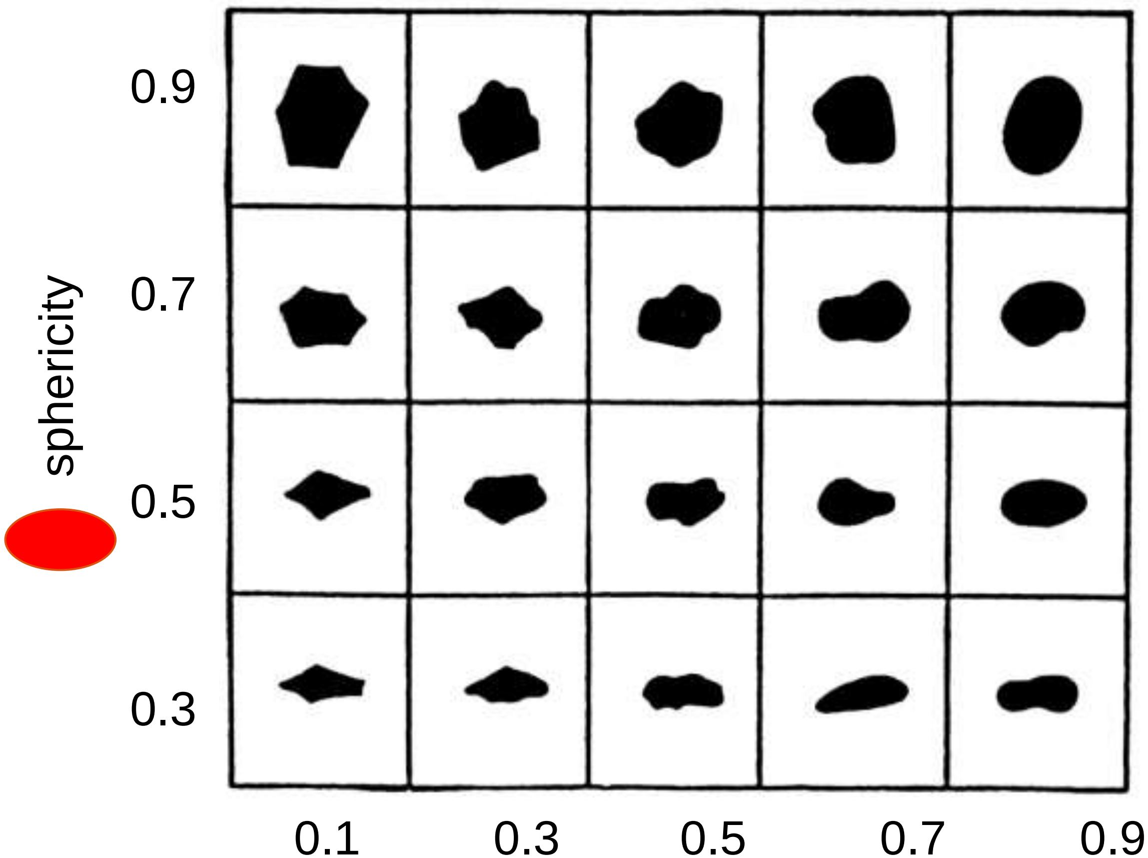
Rice

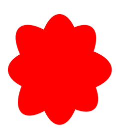

OLD NAVY



> 50 µm
< 10 µm
~1 µm
(The Diatoms –1990)
Ottawa sand
kaolinite
diatom
crushed granite
marl
diatom
sintered lead
precipitated carbonate
foraminiferan

Characterization

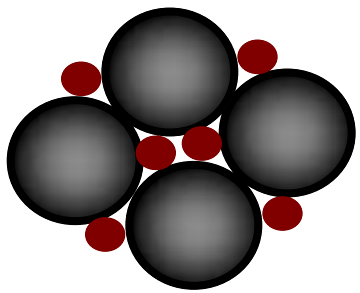




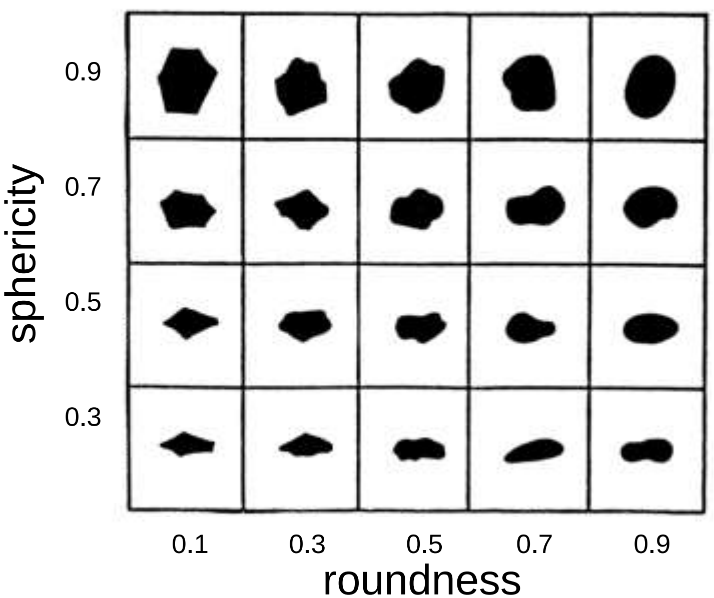
$$\text{roundness} = \frac{\sum r_i / N}{r_{max}}$$

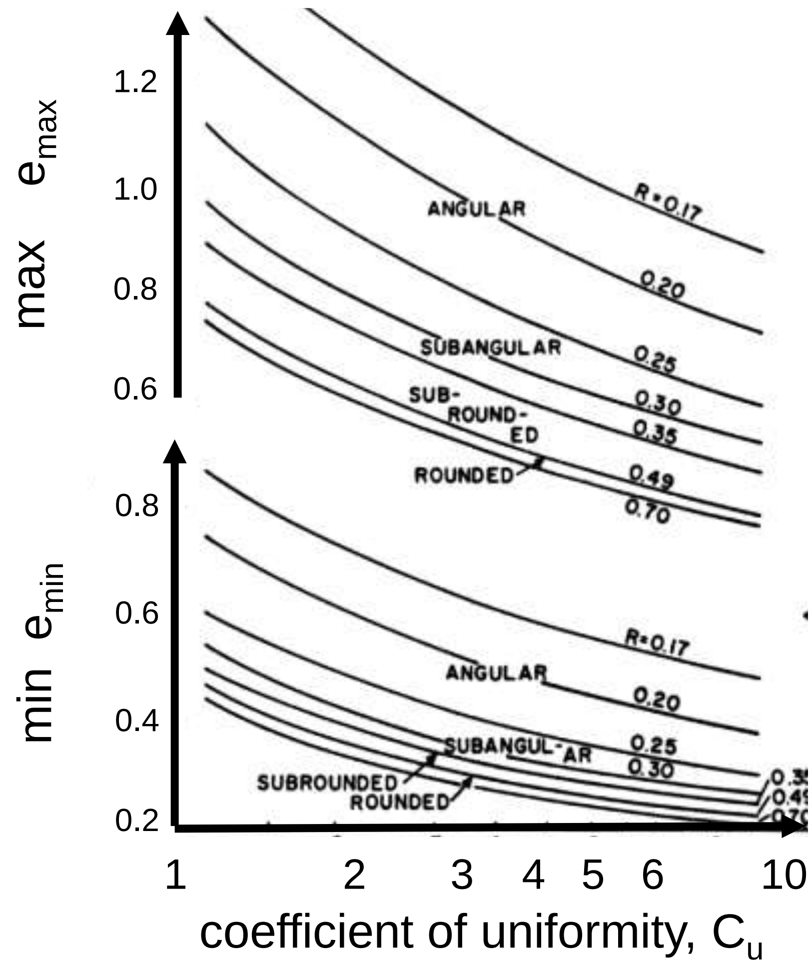
Krumbein and Sloss (1963)

Coarse Grained: Shape + Relative Size



(Krumbein and Sloss, 1963)





(Youd, 1973; see also Maeda, 2001)

Size (F=ma)

Shape

Strength: $\tau=\sigma' \tan\phi$

Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

Pores

Mixed fluids (Unsaturated Soils)

Reactive Fluids

Closing Thoughts

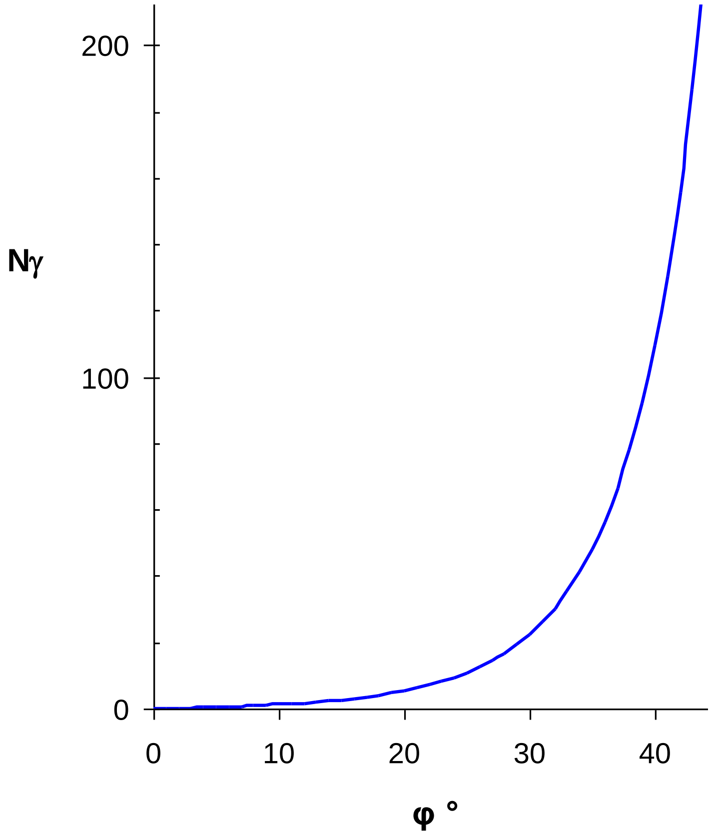

www.dot.ca.gov

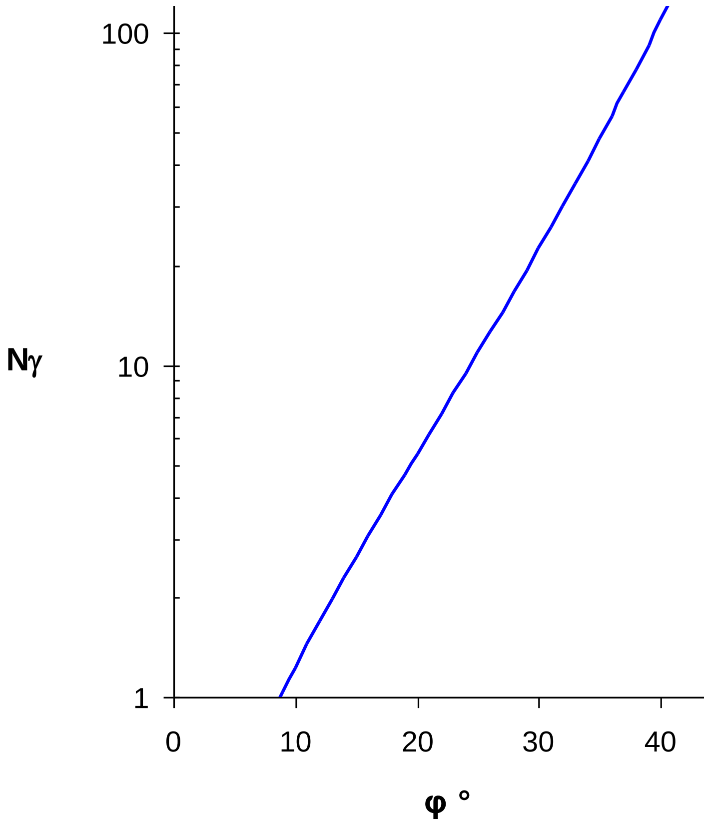

irc.nrc-cnrc.gc.ca


coal mine – Australia – guardian.co.uk


Reinforced Earth

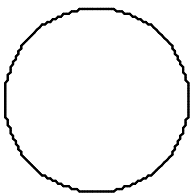
Bearing Capacity – N_g factor

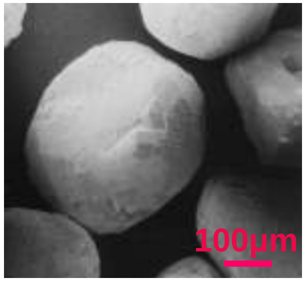




Particle Shape

size d

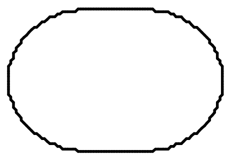


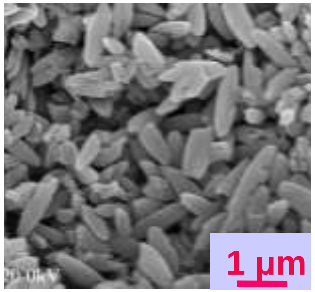


sphericity

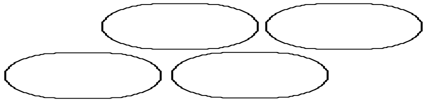
λ=πd/2

ellipticity..platiness





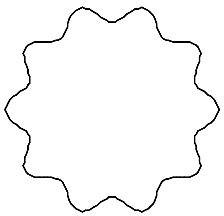
alignment

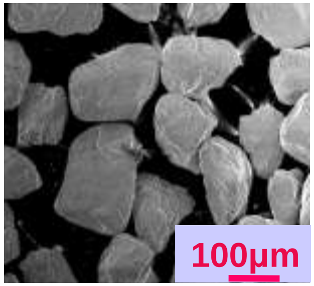


roundness

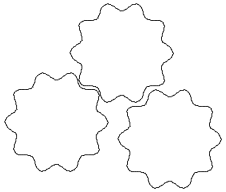
λ=πd/10

angularity





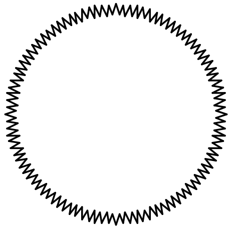
interlocking

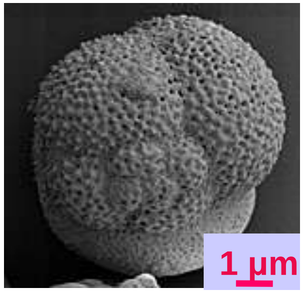


smoothness

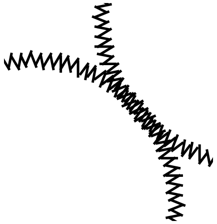
λ=πd/100

roughness

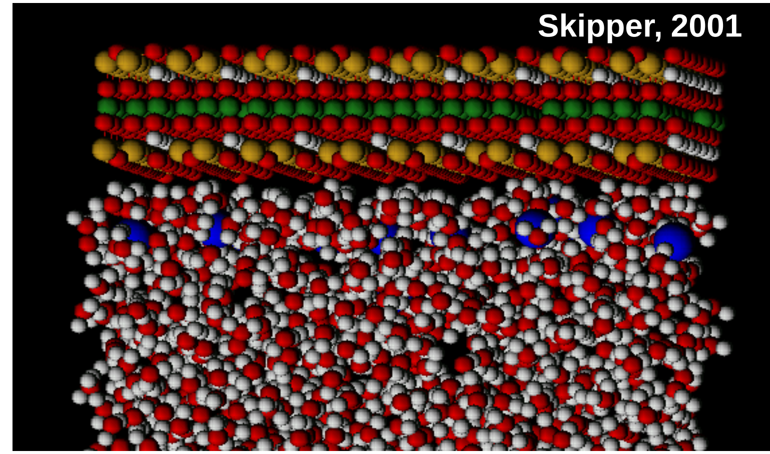




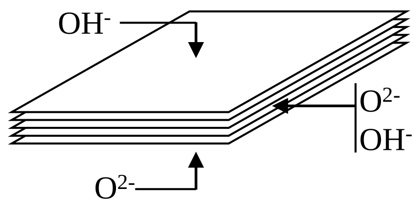
surface μ



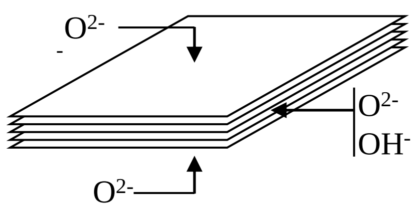
Fine Grained?



Kaolinite

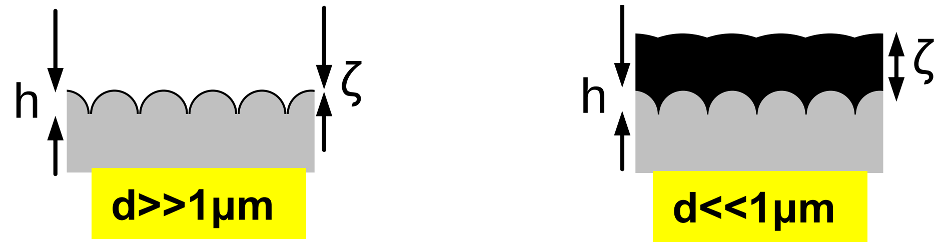


Montmorillonite

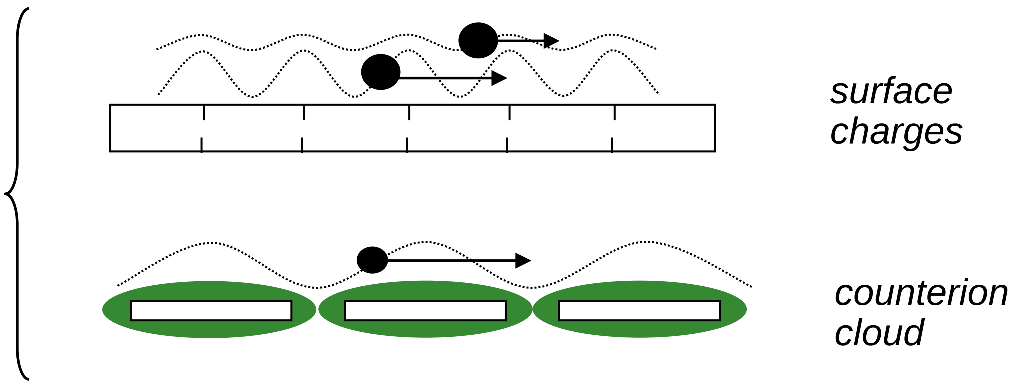


Solid and Electrical Roughness

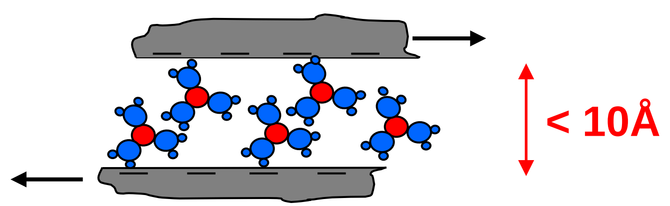
solid roughness h/ζ



electrical roughness

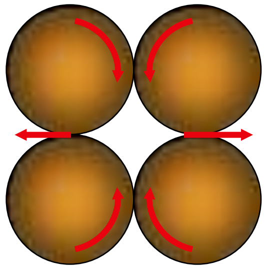


rotational frustration

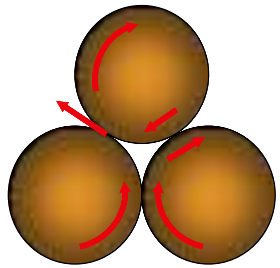


solid-fluid islands

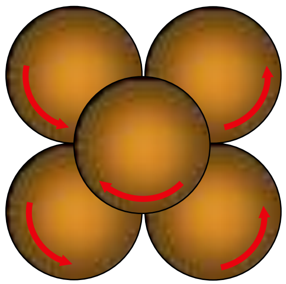
Rotational frustration: coordination ↓



2D Free
(high e)



2D Frustrated
(low e)



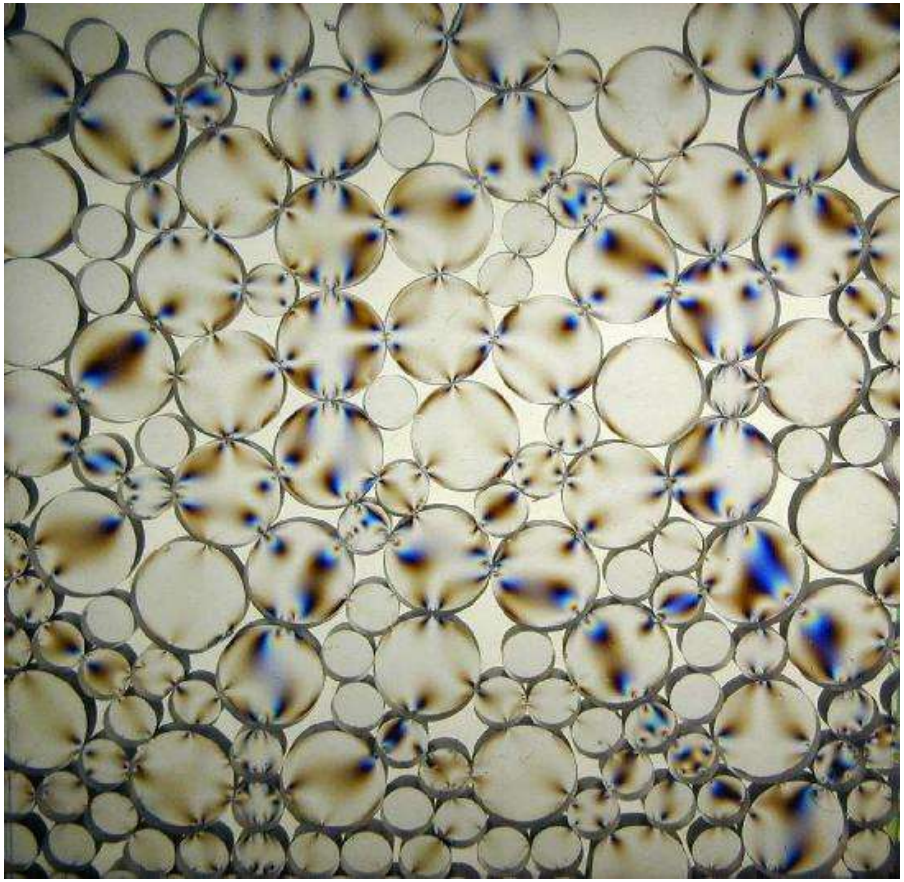
3D Frustrated
(low e)

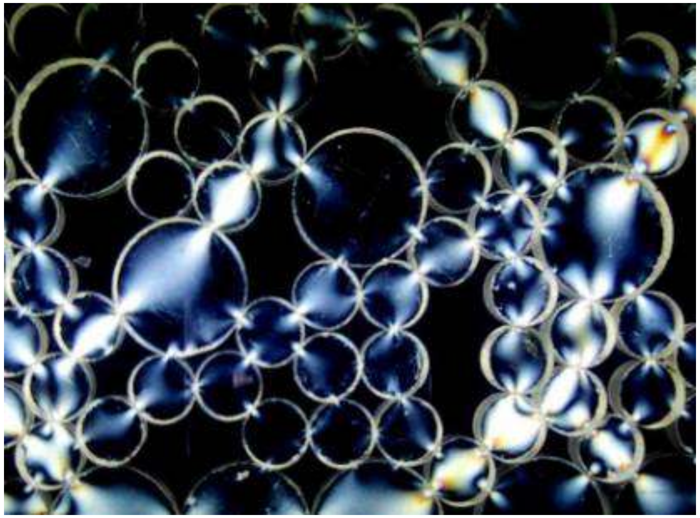
Lower coordination

→ reduce rotational frustration

→ avoid contact slip

Chain Buckling: Coordination↑





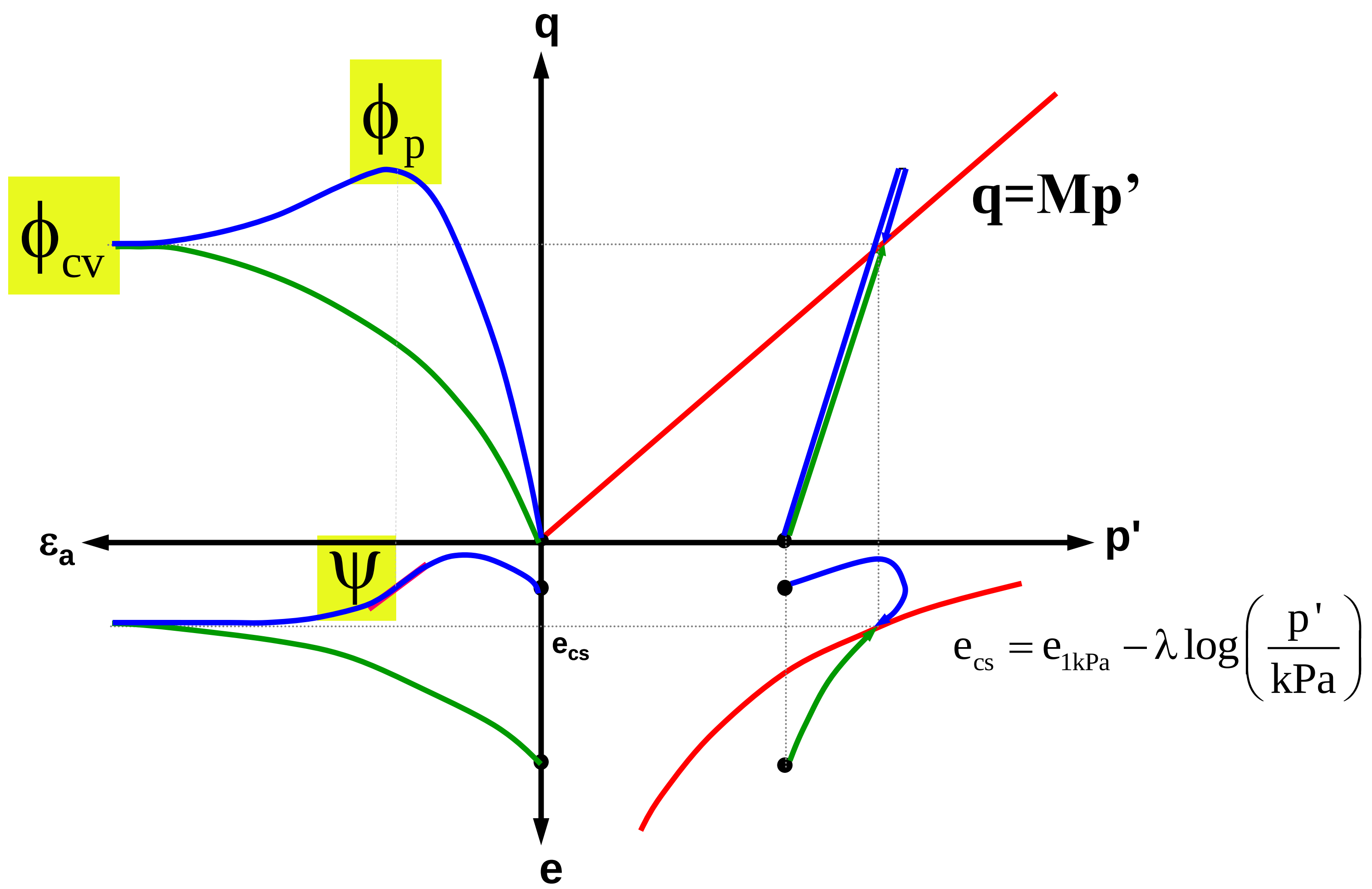
both, coarse and fines (conglomerates)

Evolution of internal micro-scale – 3D

	Isotropic confine.	At peak dev. load	
		AC (b=0)	AE (b=1)
Contact normals			
$\underline{N}(\theta)$			
$\underline{T}(\theta)$ (magnified x5)	.		

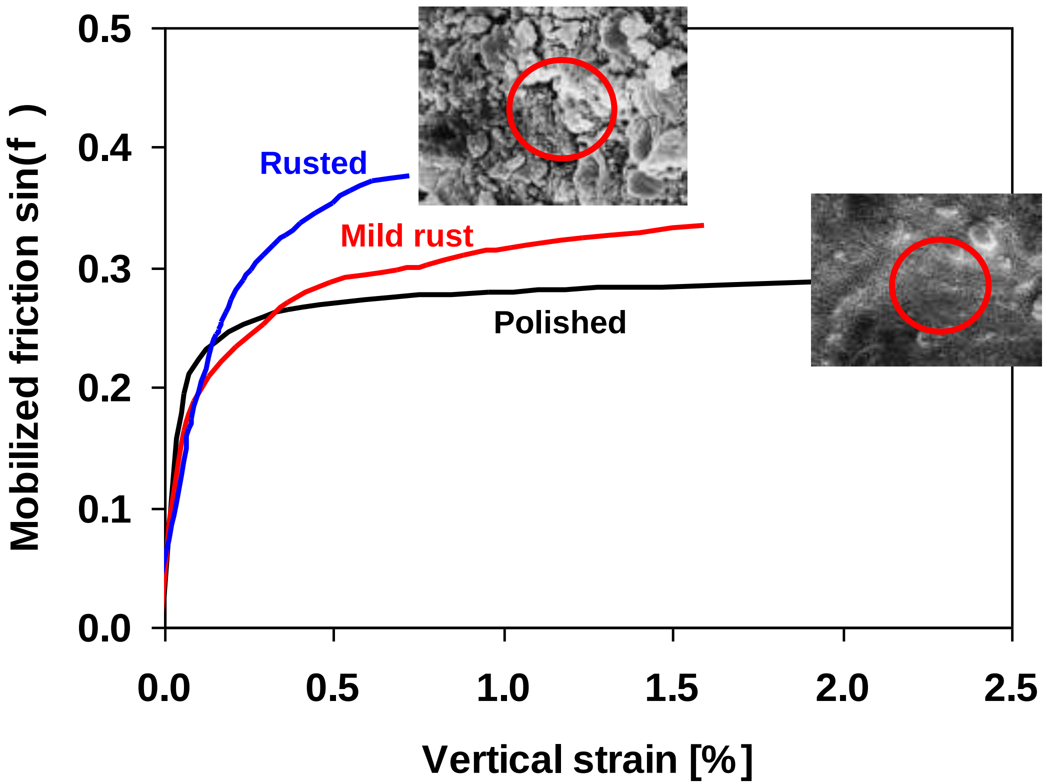
Chantawarangul, 1993

Macroscale Response in q, p', e, ε



Constant Volume Friction - Roughness

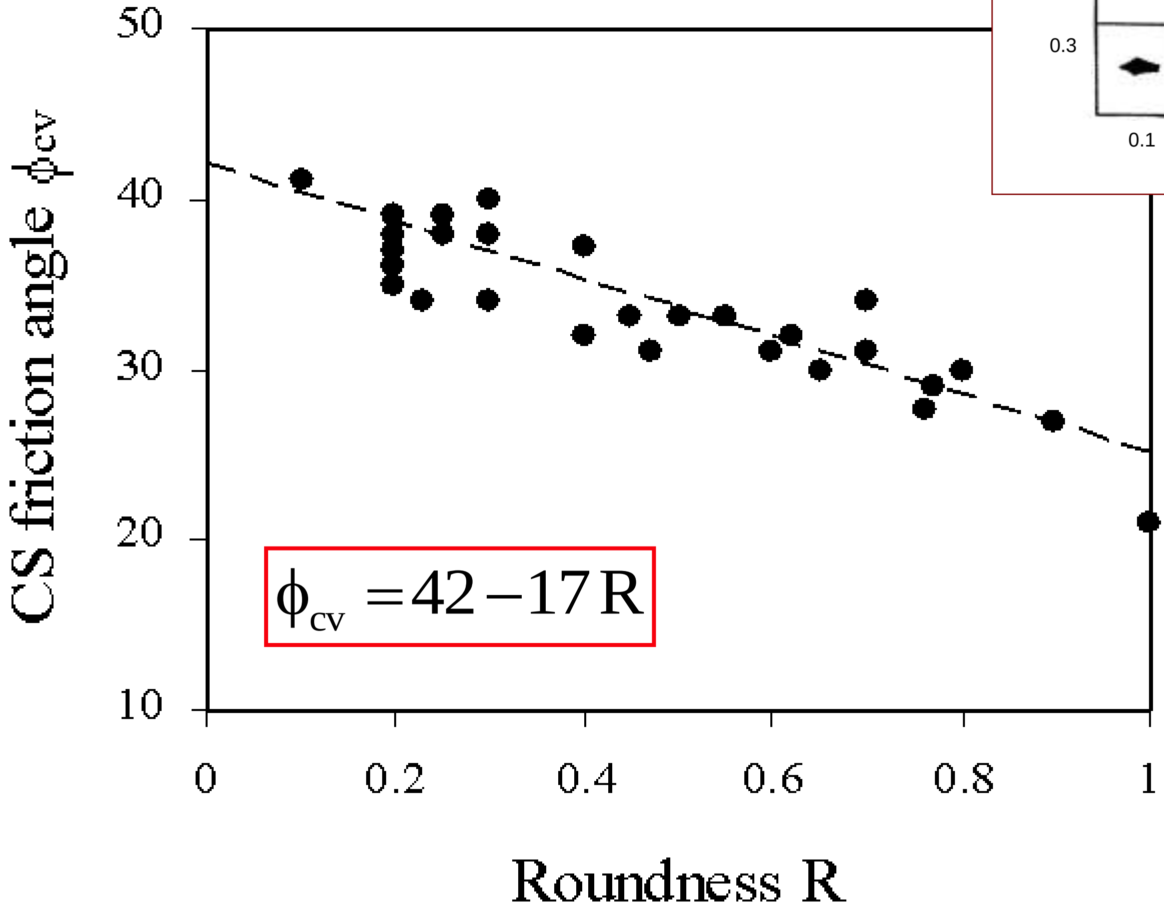
ϕ_{cv}

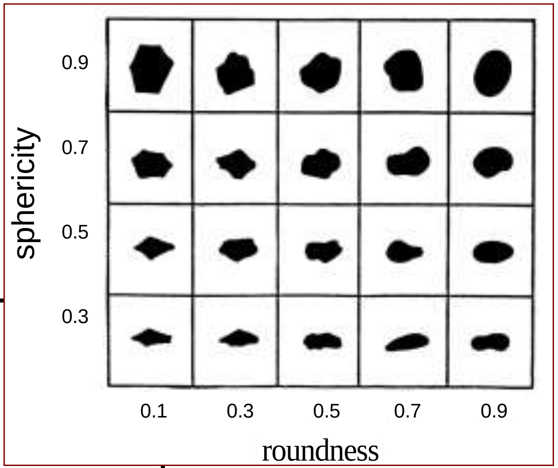


Cascante & JCS 1998

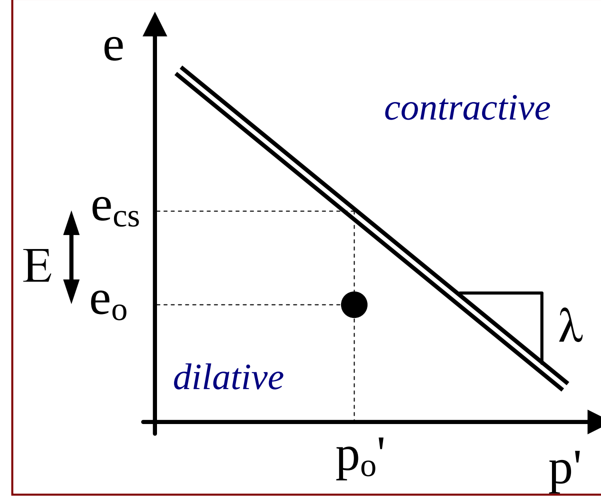
Constant Volume Friction vs. Roundness

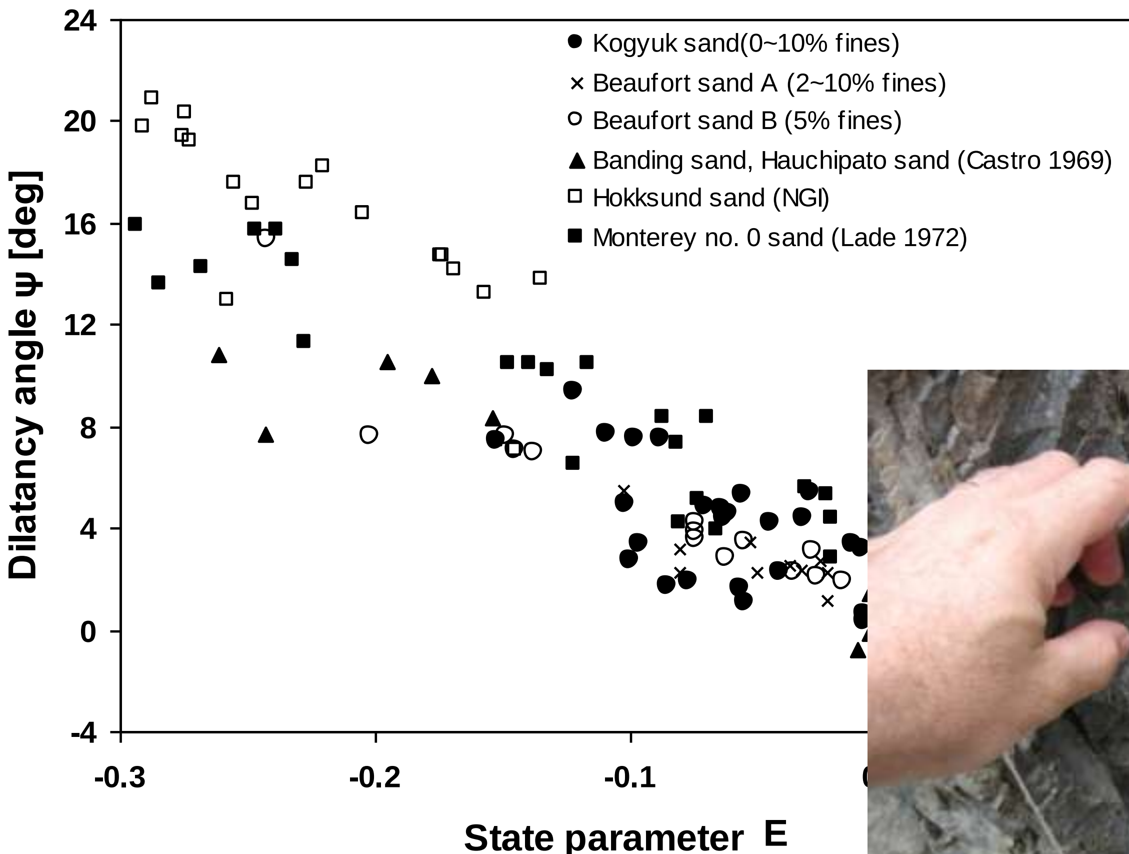
ϕ_{cv}





Dilatency Angle



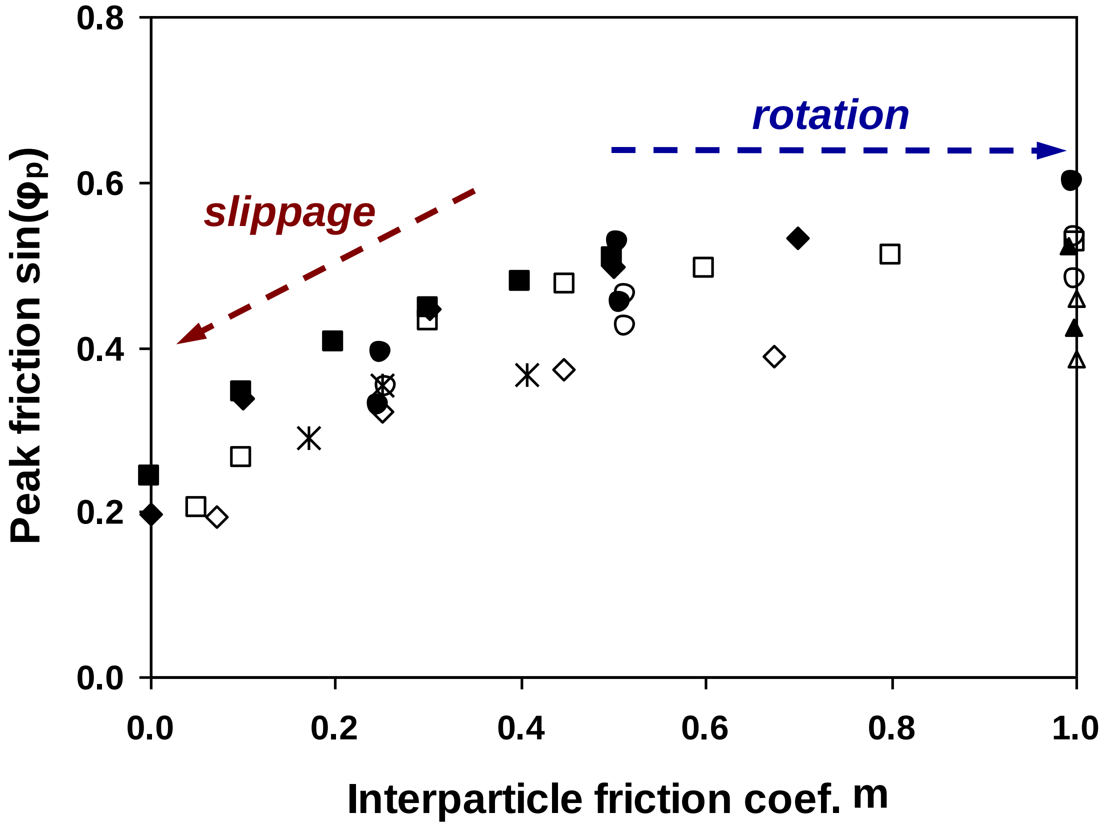


Peak Friction Angle

ϕ_p

Taylor 1948: $$\tan\phi_p = \tan\phi_{cv} + \tan\psi$$

Bolton 1986: $$\phi_p = \phi_{cv} + 0.8\psi$$

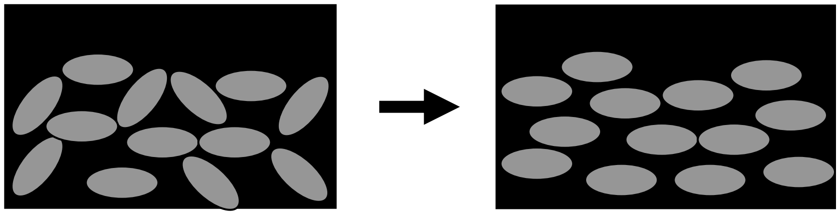


■ (DEM 2D from Kruyt and Rothenberg 2006) Drained TC(r), Undrained TC(p), Drained TE(™), Undrained TE(˜) (DEM-3D from Yimsiri 2001)
□ (DEM 3D from Thornton 2000) ✳ (experiments) ◇ (DEM 3D from Suiker and Fleck 2004) ◆ GT work.

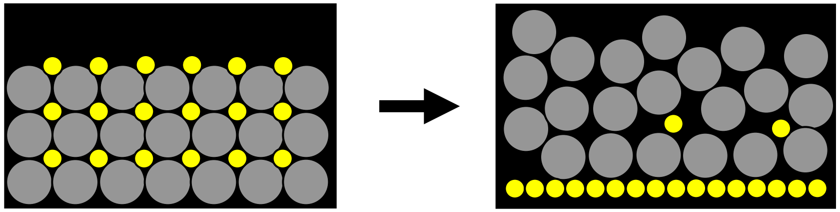
ϕ_r

Residual Friction Angle - very large strains

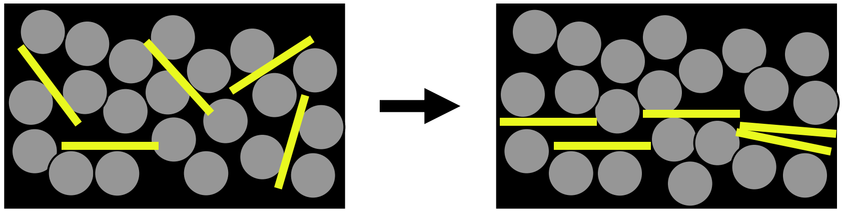
particle alignment



size segregation



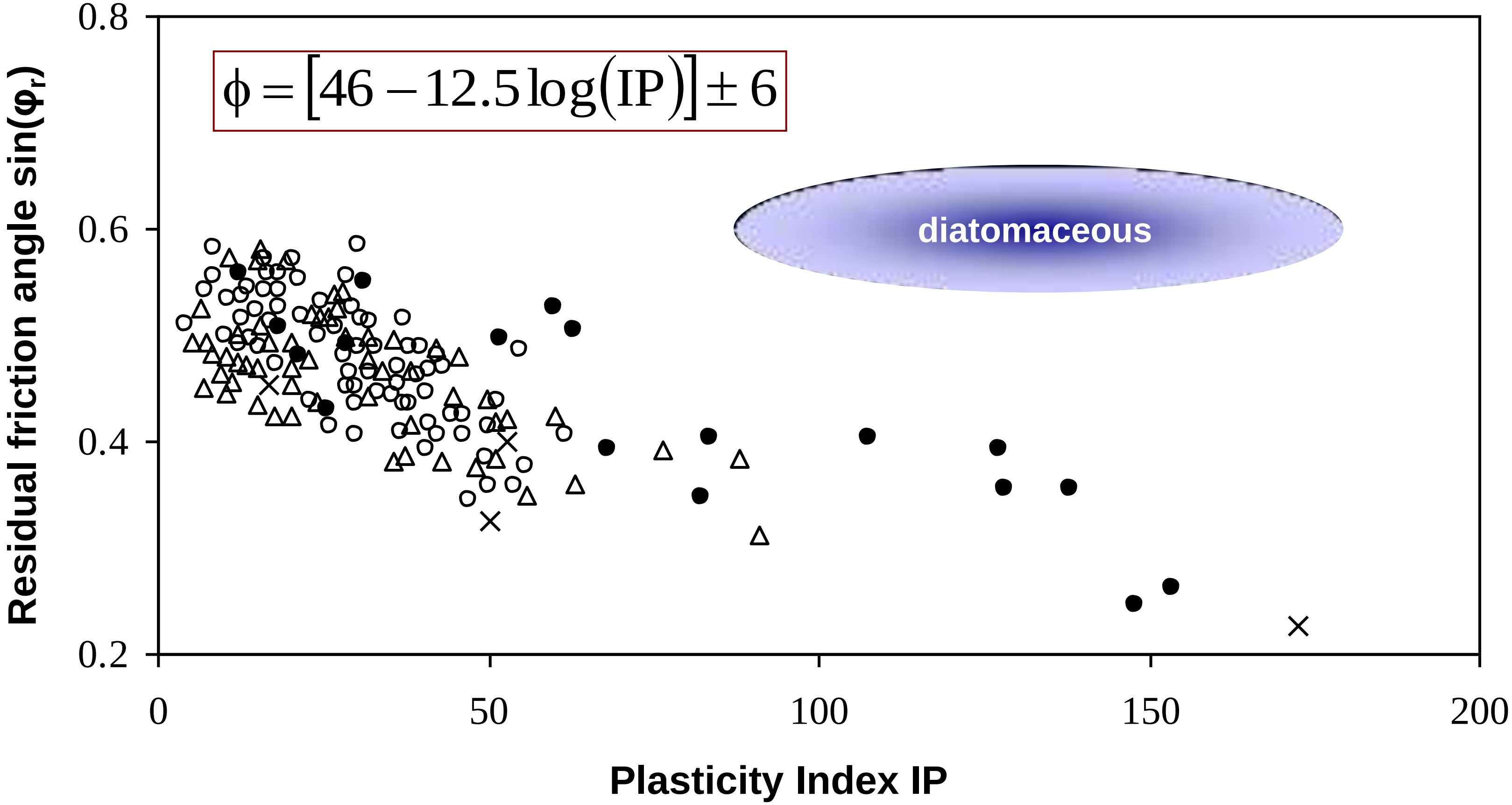
shape segregation



Residual Friction Angle

ϕ_r

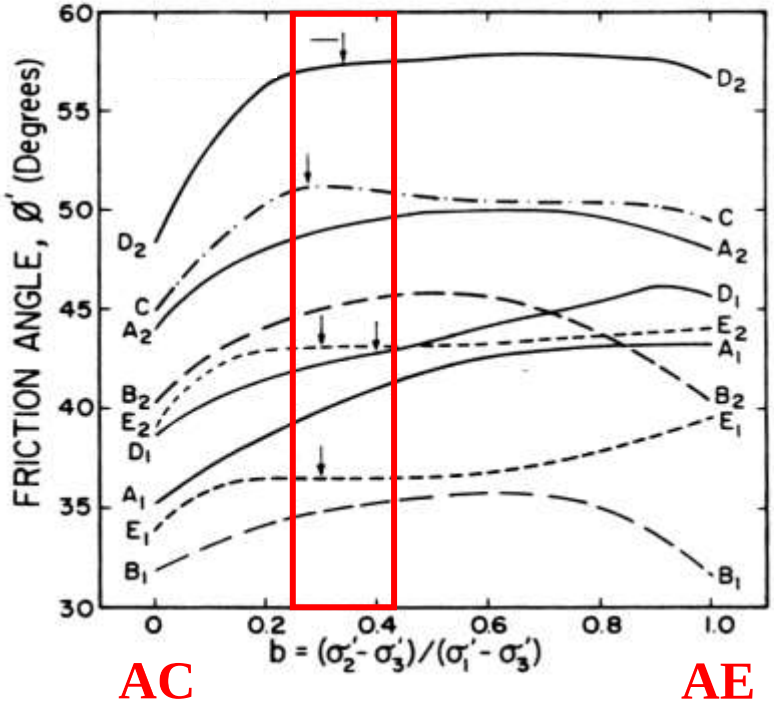
Note: clay fraction must exceed ~20%



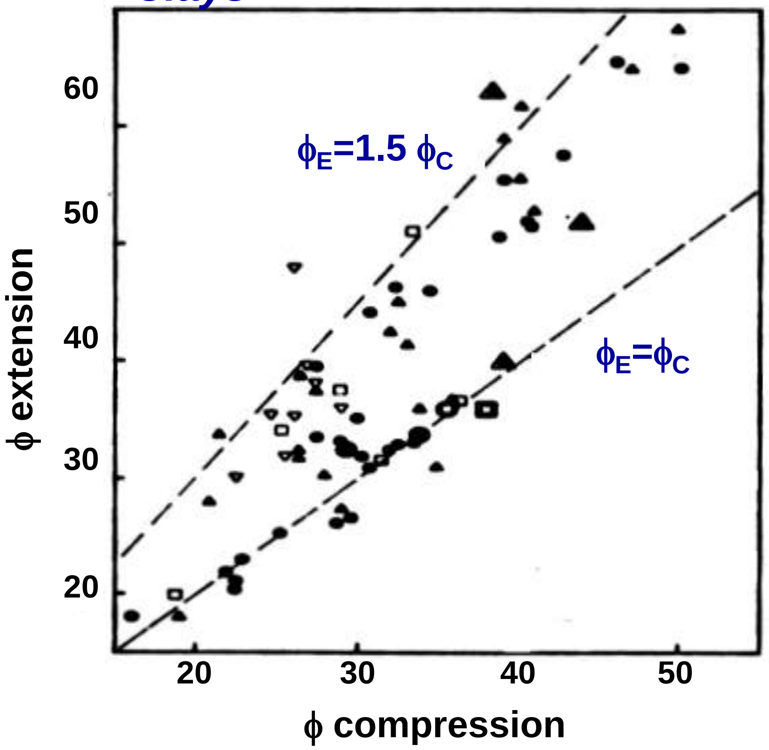
Frictional strength anisotropy

$\phi_E = 1.0 \text{ to } 1.5\ \phi_C$

Sands



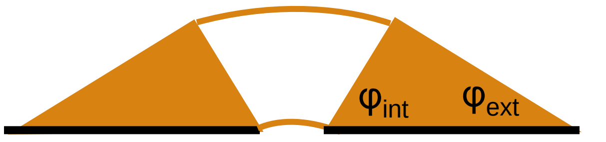
Clays

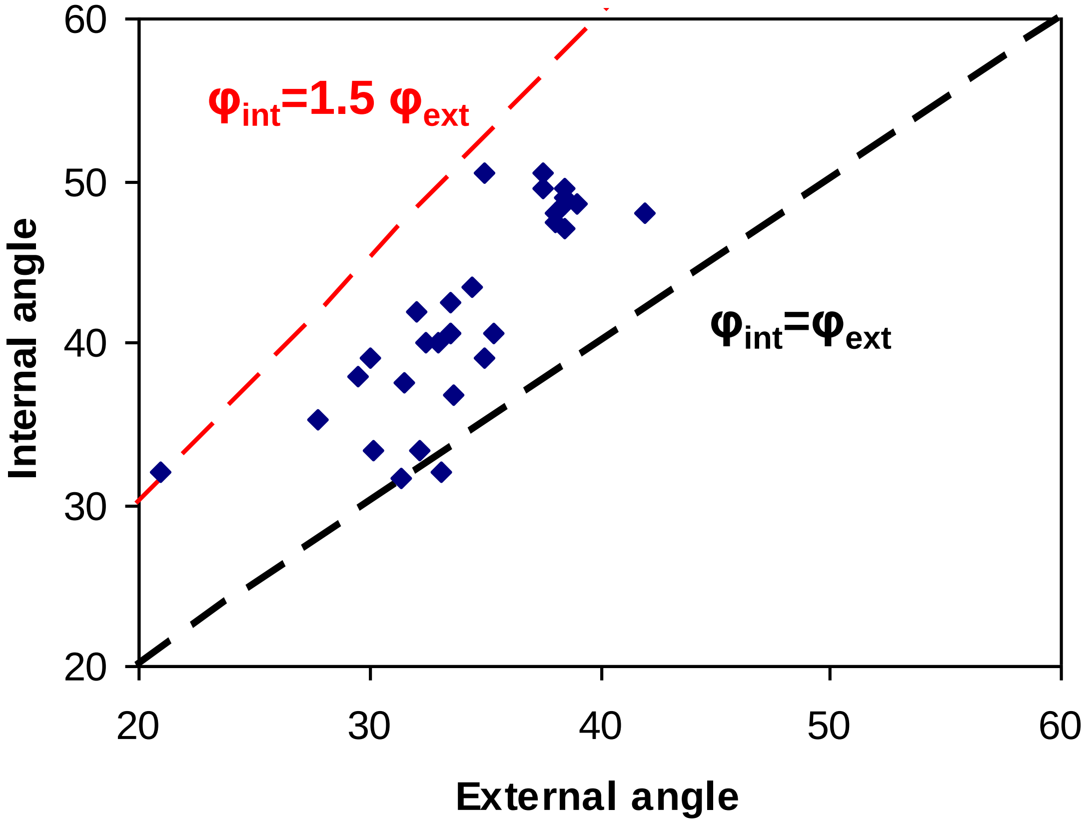


compiled by Ladd et al. 1977

compiled by Mayne and Holtz (1985)

Constant angle of repose?





Narsilio, Dodds, Fugle, Trott, Kim, Yun

Size (F=ma)

Shape

Strength: $\tau=\sigma' \tan\phi$

Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

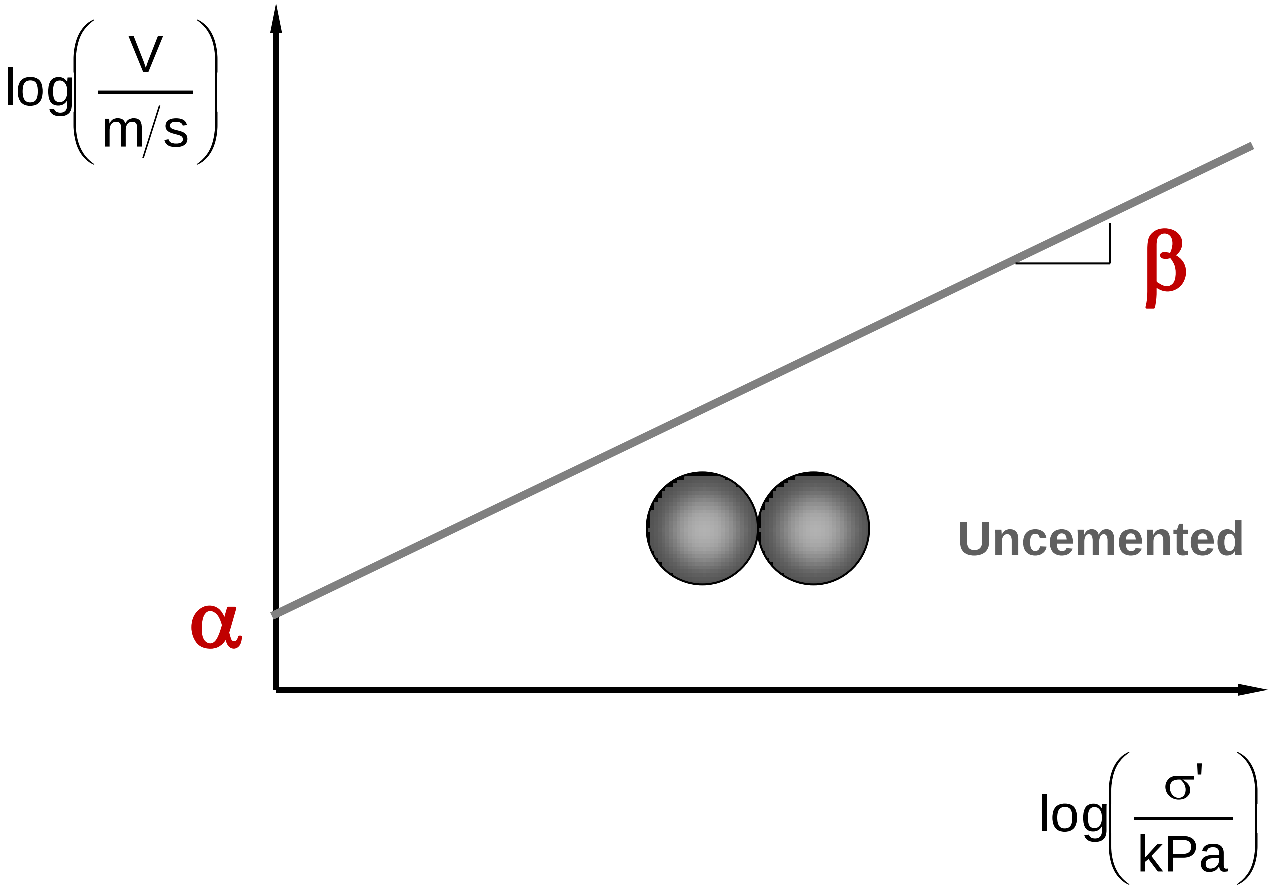
Pores

Mixed fluids (Unsaturated Soils)

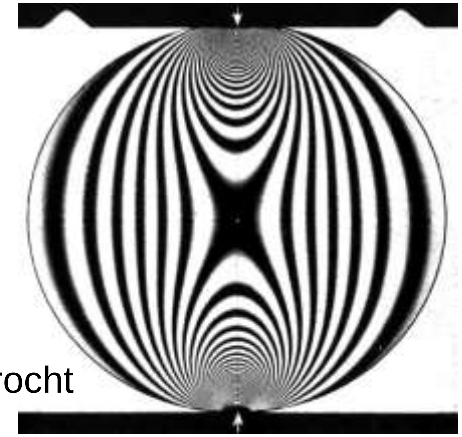
Reactive Fluids

Closing Thoughts

Un-cemented soil

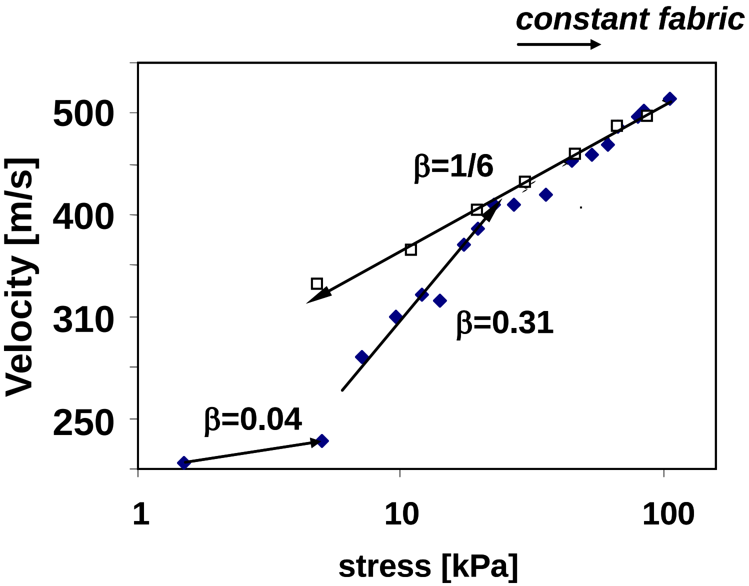


Contact Stiffness + Fabric Change



Frocht

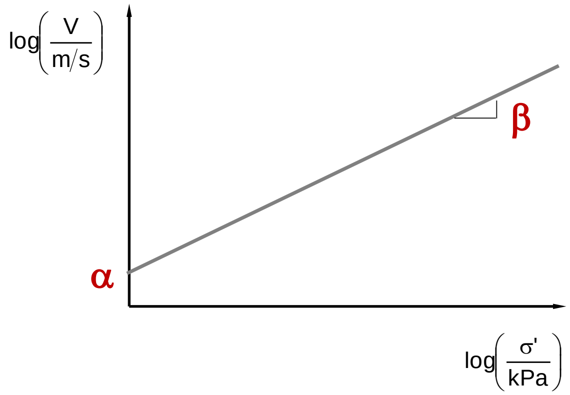
Hertz

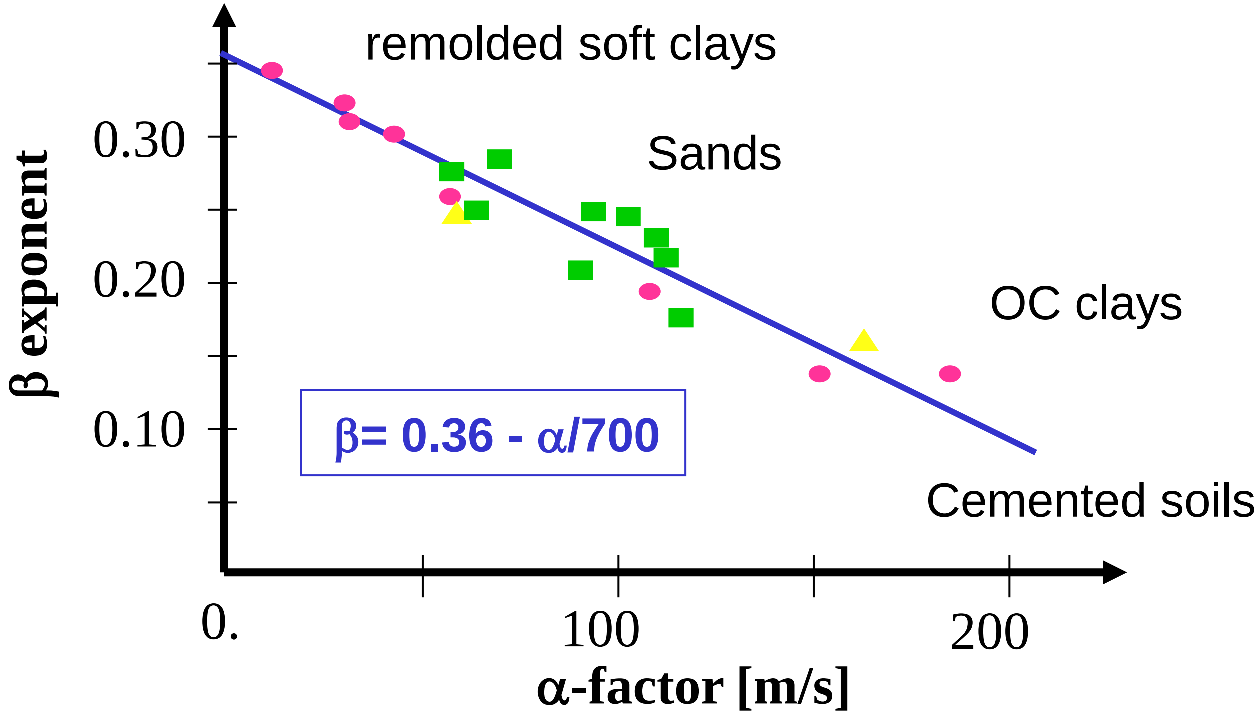


cn ↑ ➔ β ↑

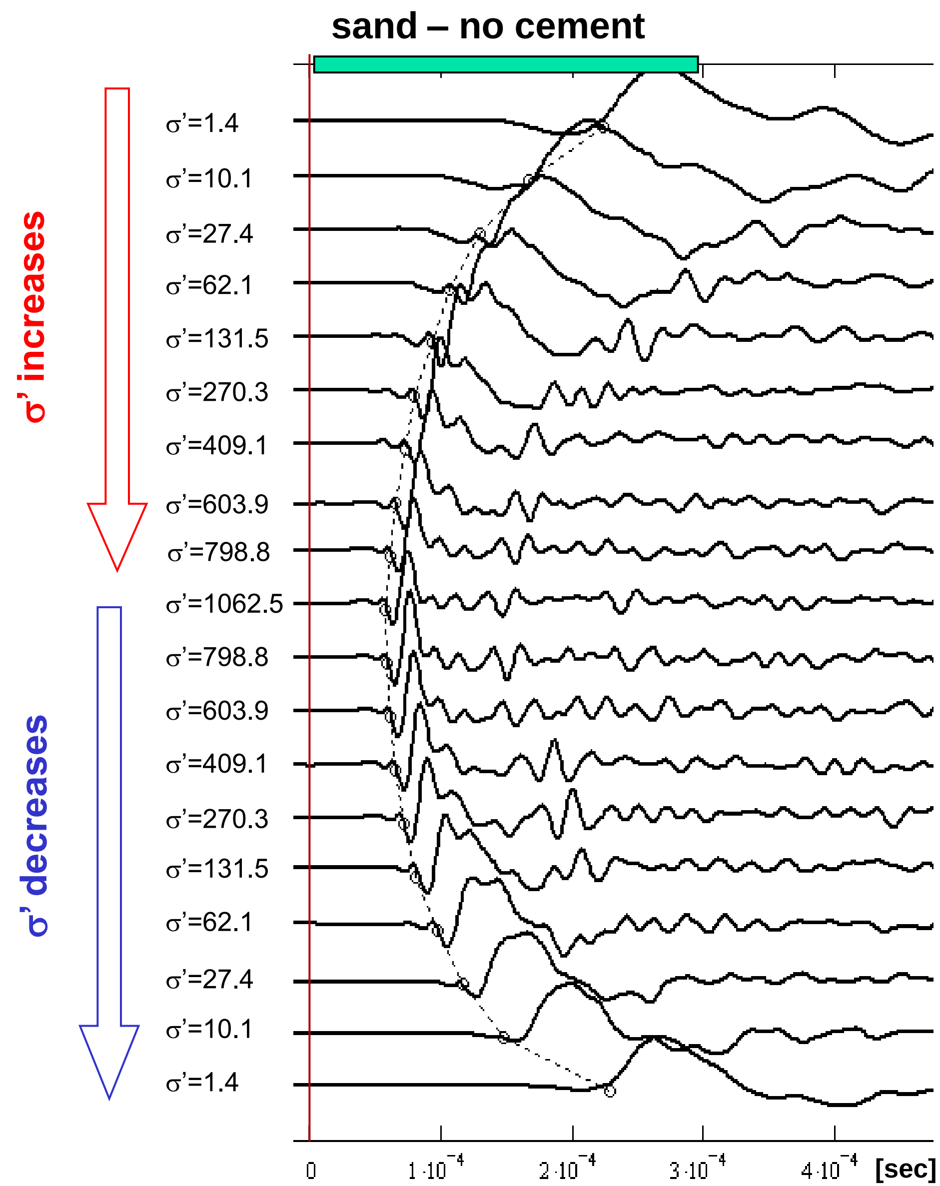
Velocity-Stress: Contact + Fabric

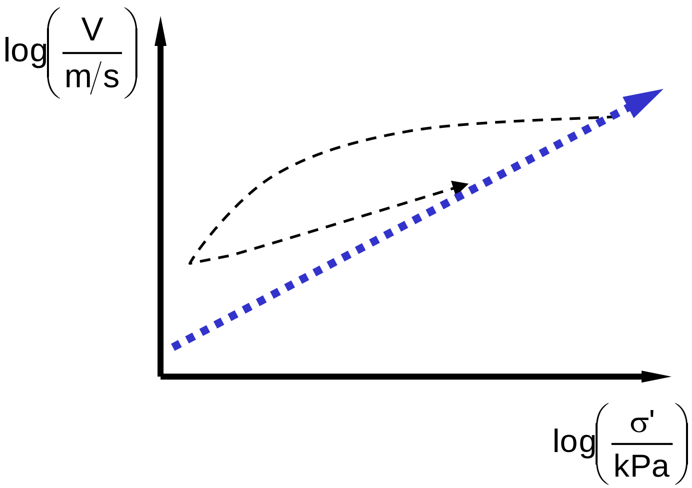
$$V_S = \alpha \left(\frac{\sigma'_x + \sigma'_y}{2 P_a} \right)^{\beta}$$



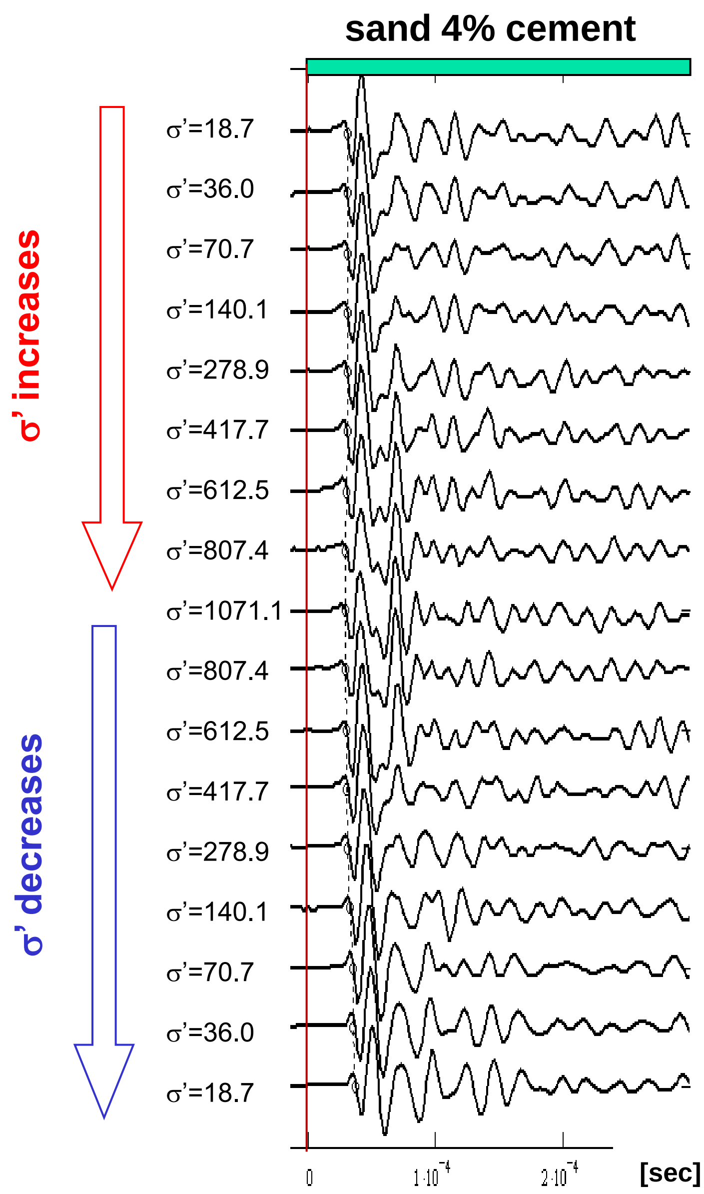


Un-cemented soil – Effective stress



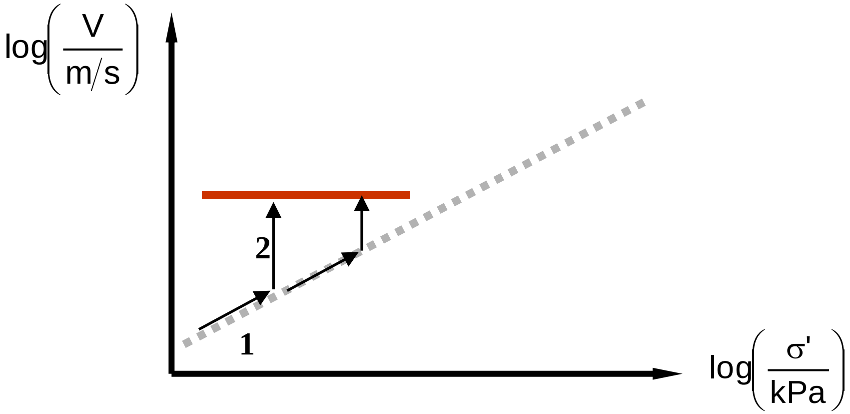


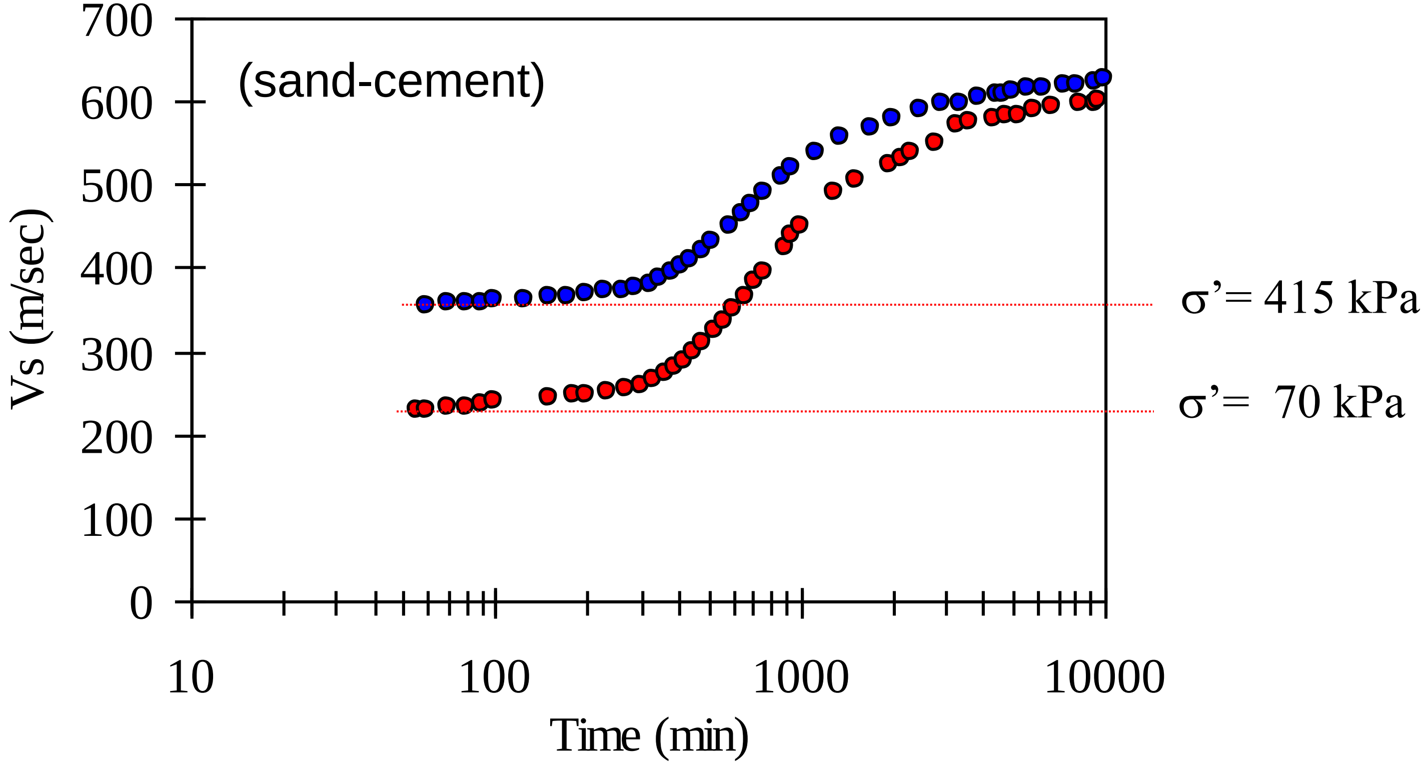
Cementation Controlled Stiffness



1: Confinement

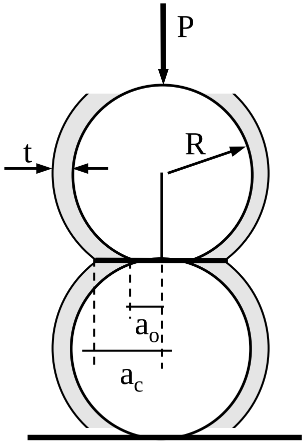
2: Cementation

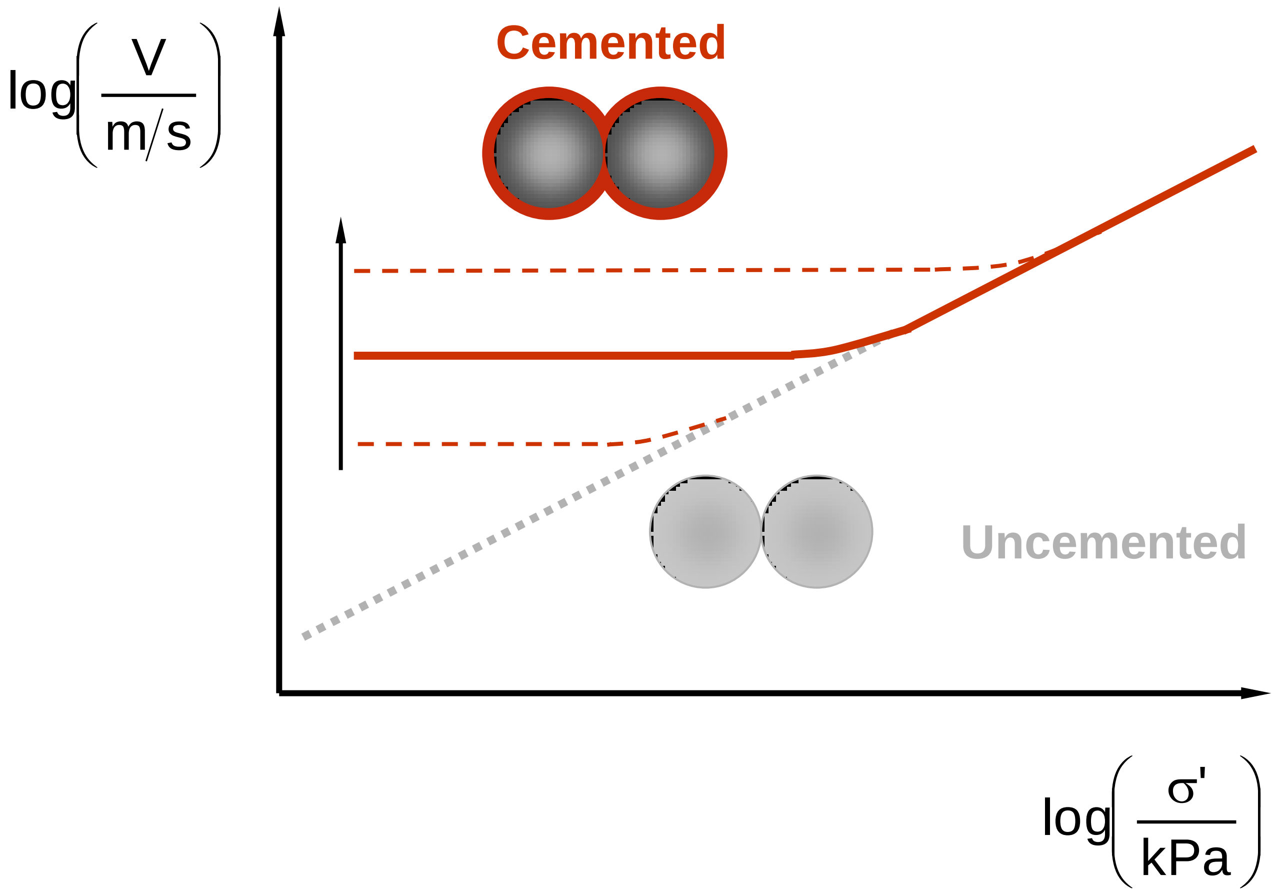




Cemented soil

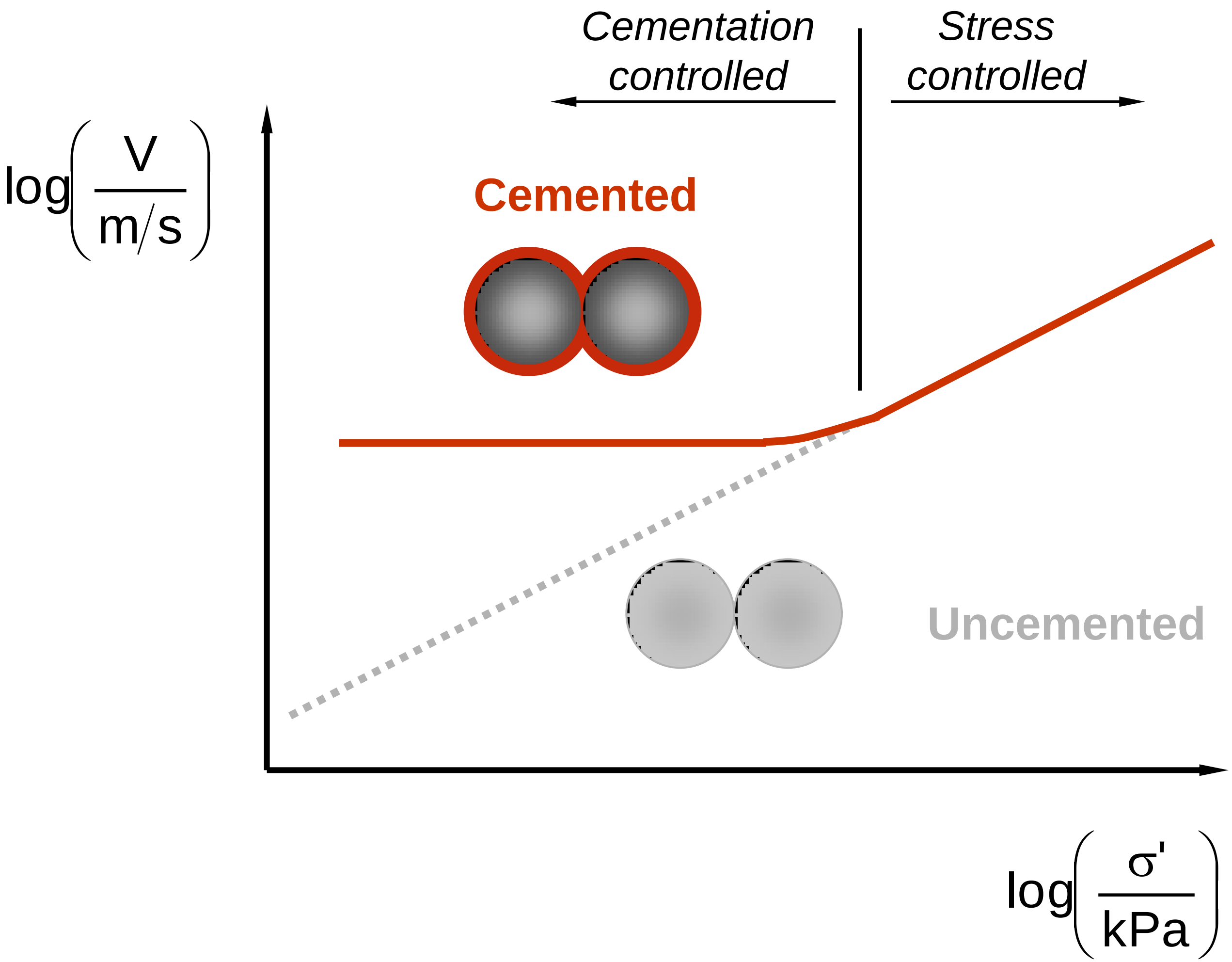
$$\frac{E_T}{G_m} = \frac{1}{(1-\upsilon_m)}\frac{a}{R}$$



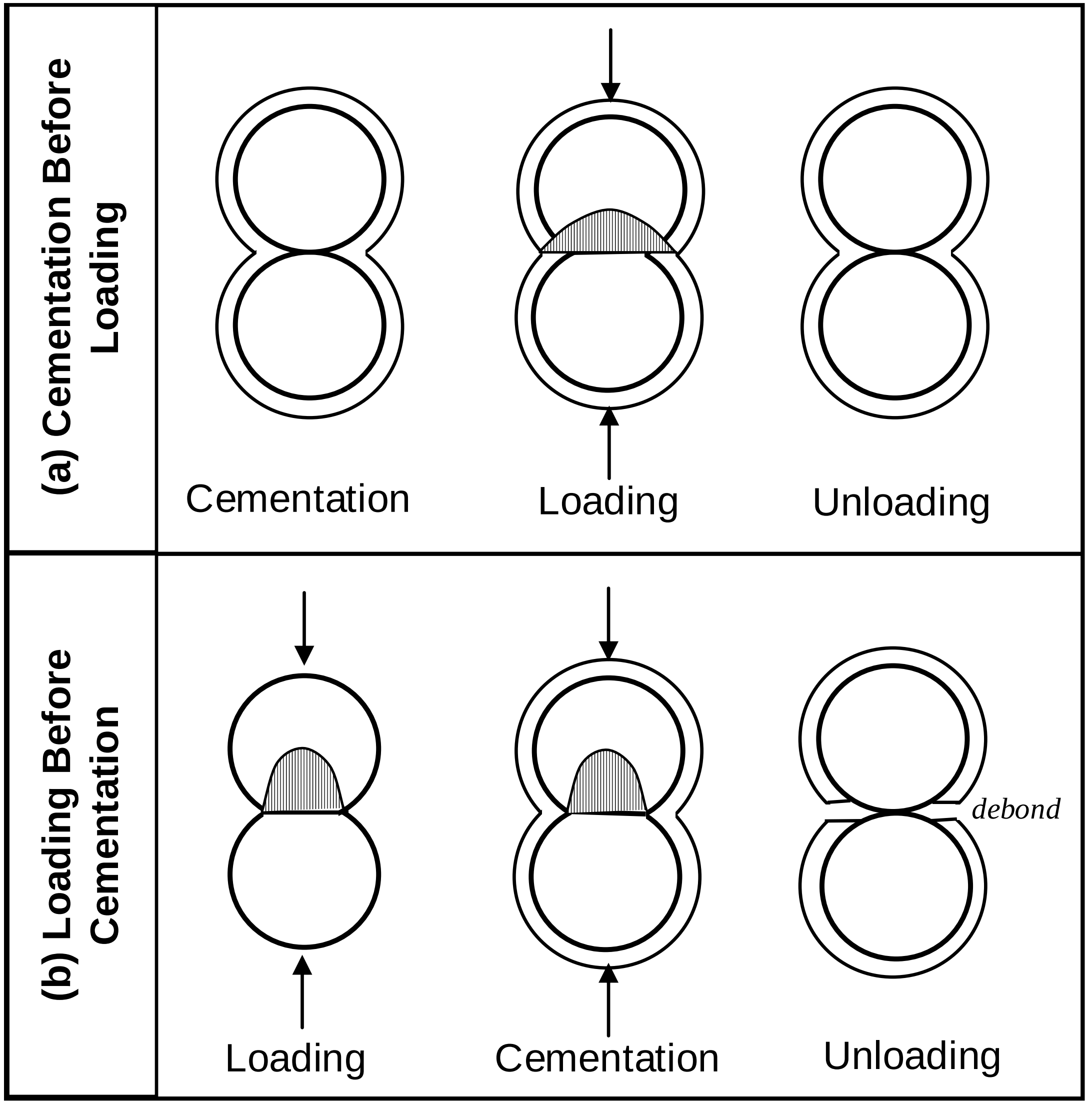


Cemented soil

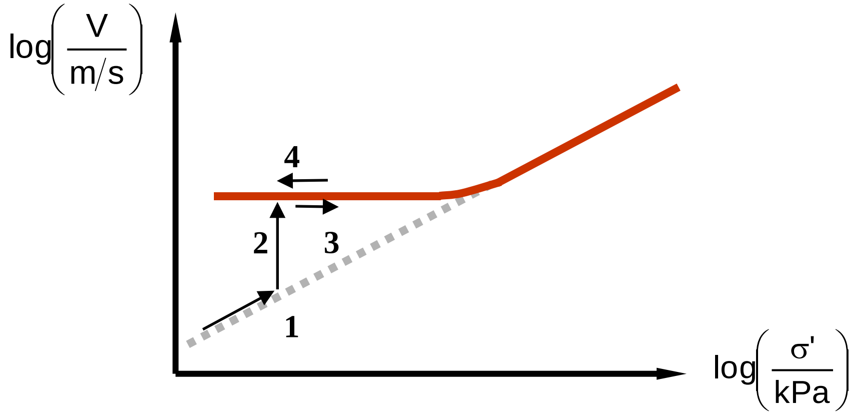
$$\frac{E_T}{G_m} = \frac{1}{(1-\upsilon_m)} \cdot \sqrt{(CC+1)^{2/3} - 1 + \left[\left(\frac{3\cdot(1-\upsilon_m)}{2}\cdot\frac{\sigma}{G_m}\right)^{1/3}\right]^2}$$

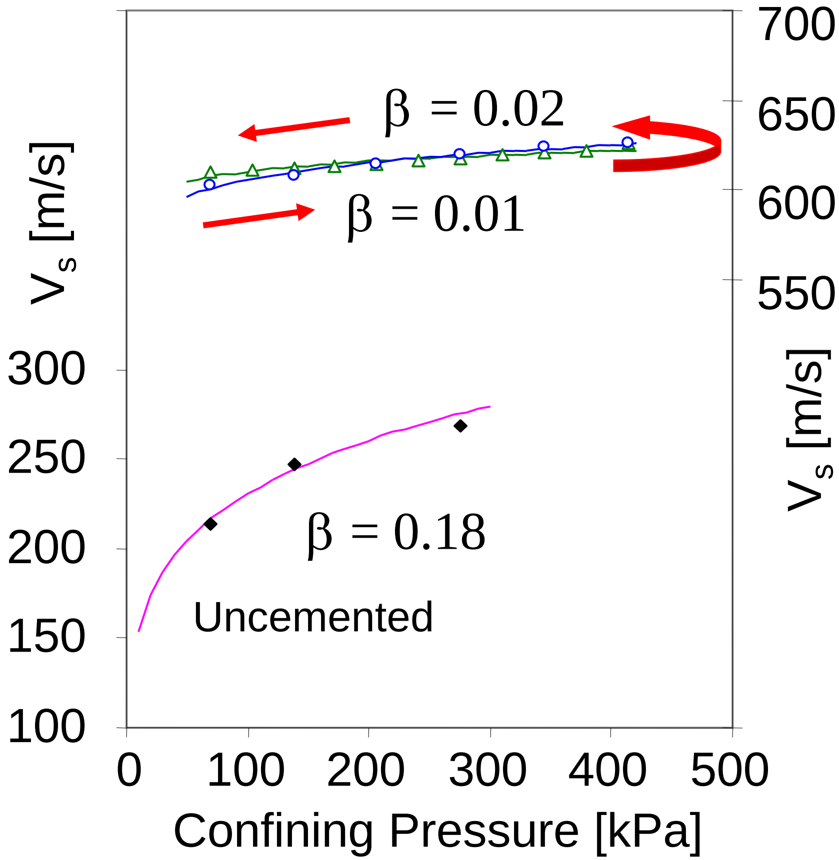


Stress-Cementation History

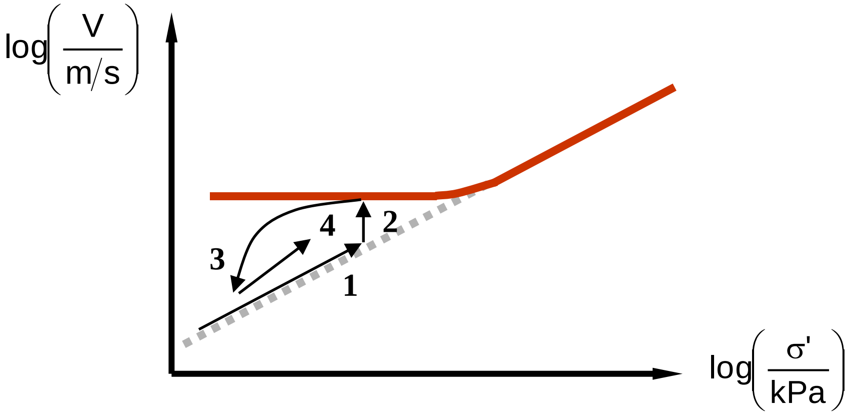


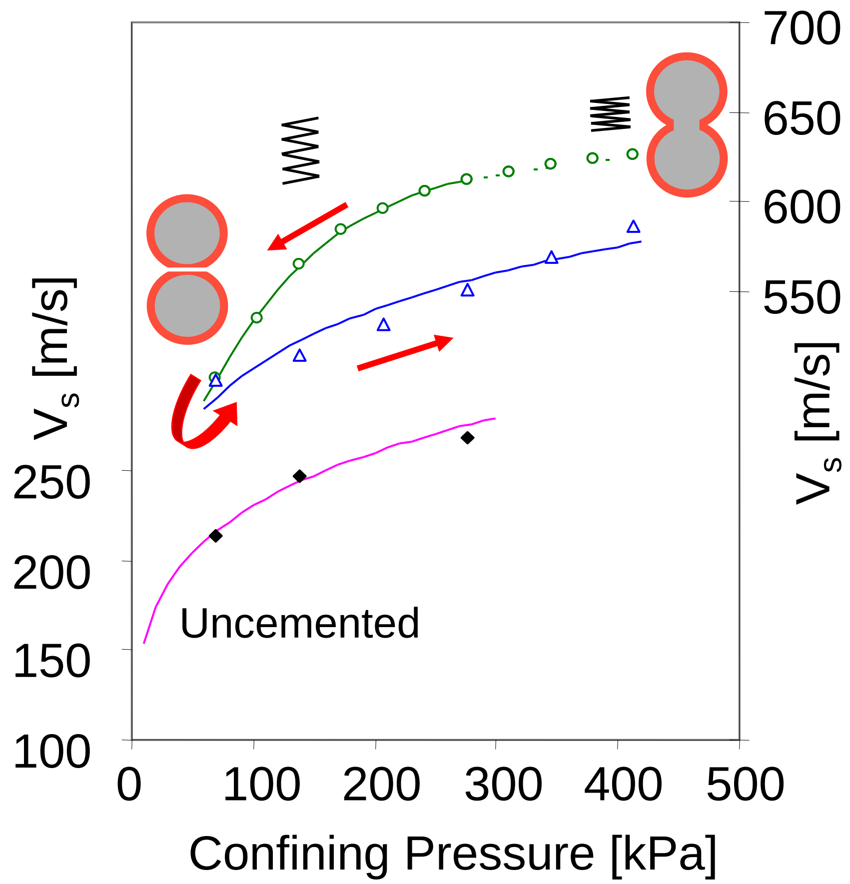
1: Confinement
2: Cementation
3: Load
4: Unload





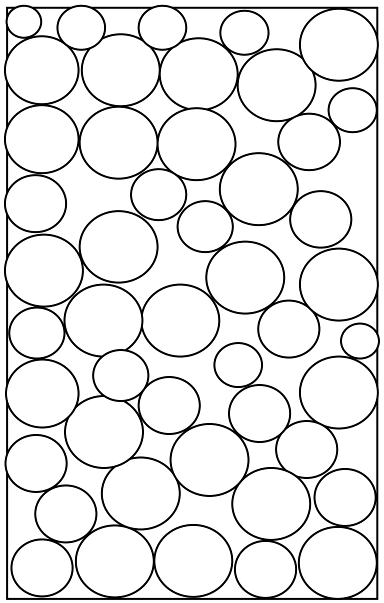
1: Confinement
2: Cementation
3: Unload
4: Re-load



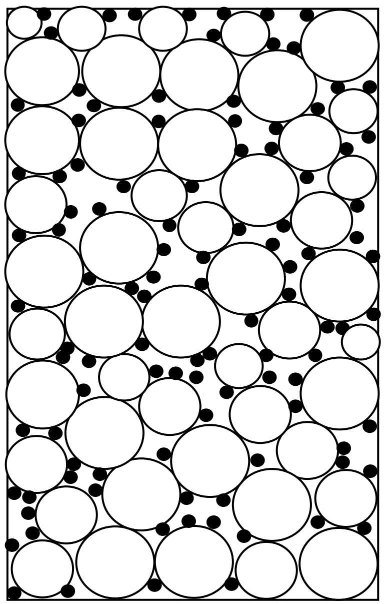


Cementation Pore Habit

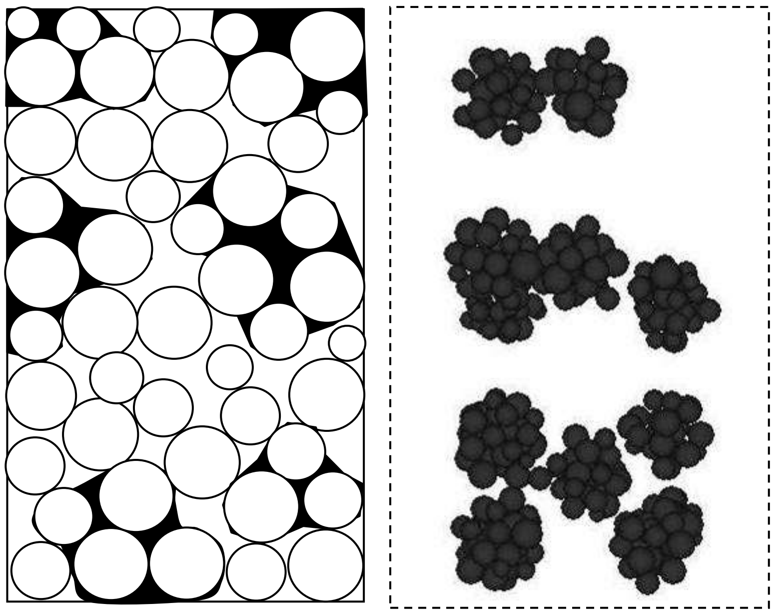
cement-free sediment



distributed cementation



patchy cementation



Mineral

Mineral particle diameter	= 0.62~0.82mm
Mineral particle number	= 3,762~4,806
Initial porosity	= 0.402, 0.532
Effective confining stress	= 0.1-to-1MPa
Normal stiffness	= 1×10^{7} N/m
Shear stiffness	= 1×10^{7} N/m
Friction coefficient	= 0.5

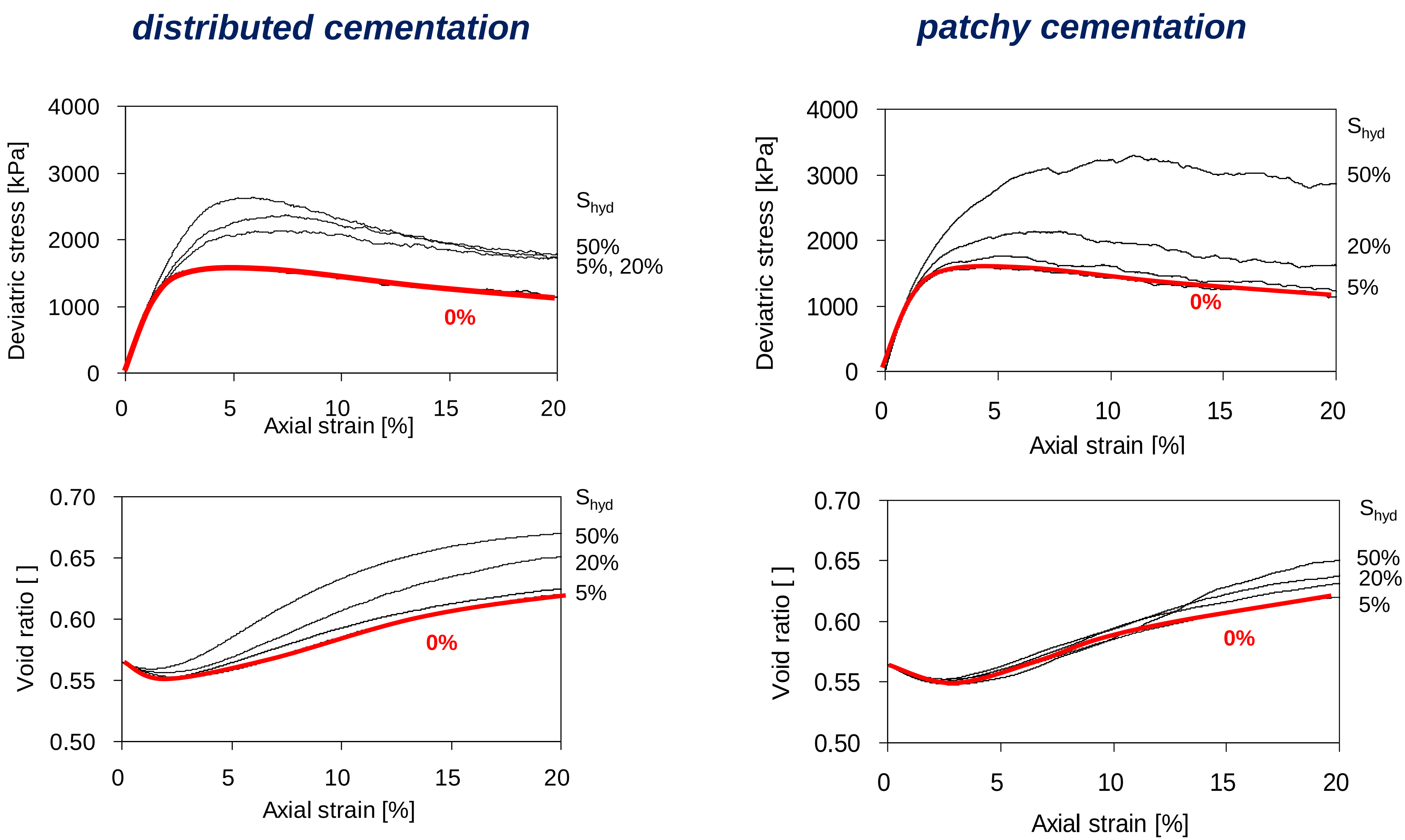
Distributed hydrates

Hydrate saturation	= 0~50%
Hydrate particle diameter	= 0.22mm
Hydrate particle number	= ~74,940
Bonding strength	= 200 kPa

Patchy hydrate saturation

Hydrate saturation	= 0~50%
Cluster number	= 15 groups
Grain numbers in cluster	= 12~160
Parallel Bonding strength	= 5MPa

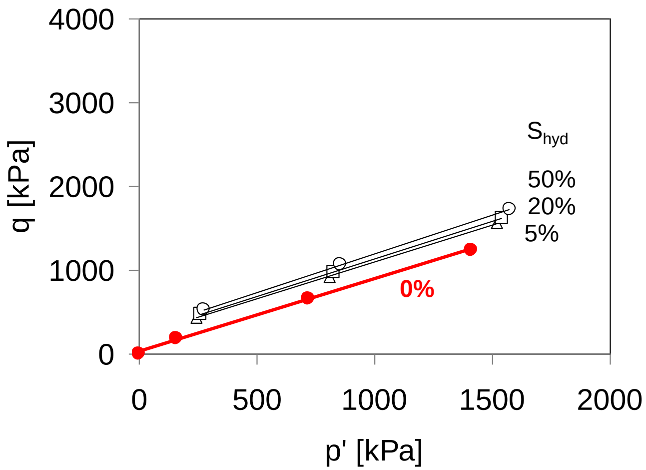
Stress-Strain Response (3D)

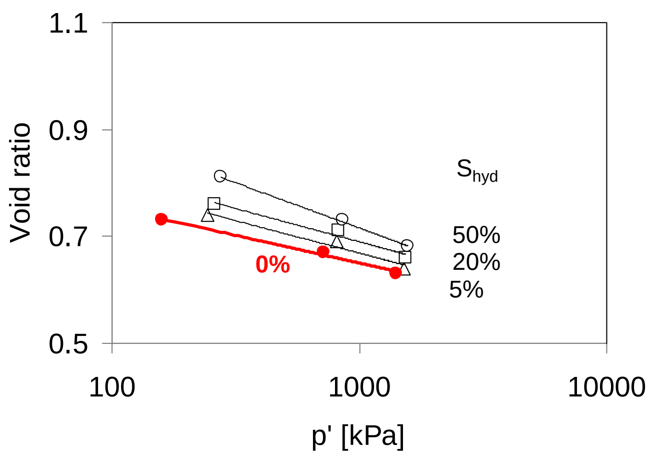


Note: increase in stiffness , strength, dilation with S_{hyd} - pore habit affect dilation

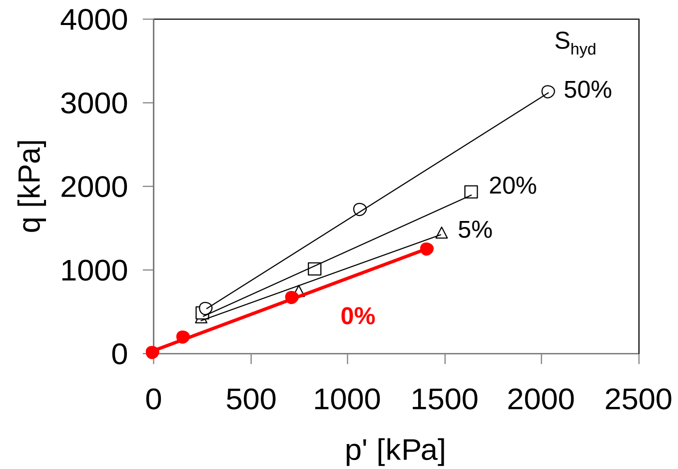
Critical State - *large strain* (3D)

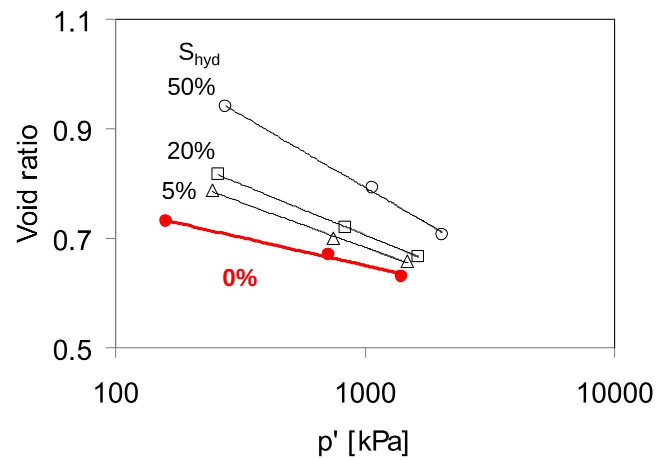
distributed cementation



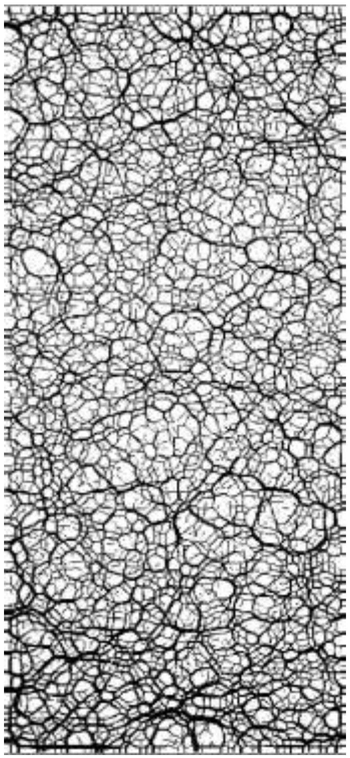
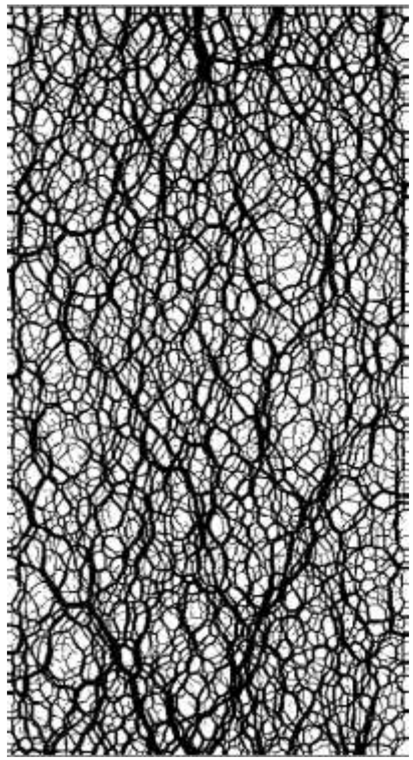
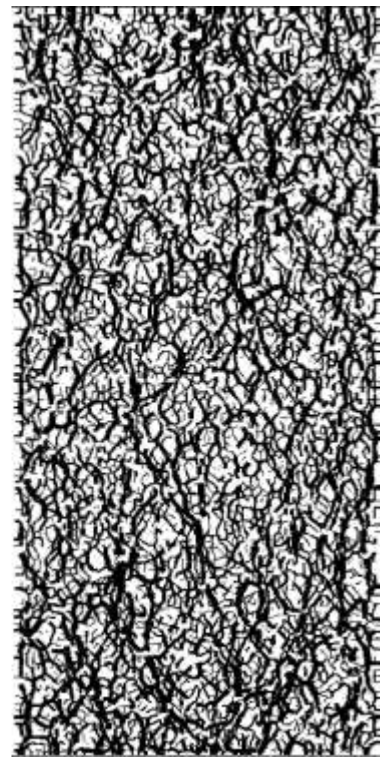
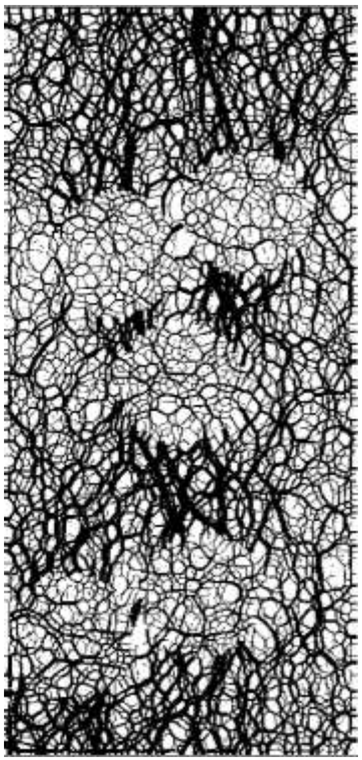


patchy cementation





Contact Force Chains (2D Simulation)

isotropic load (σ_o=1MPa, σ_d=0MPa)	under deviatoric load (σ_o=1MPa, σ_d=1.2MPa)		
	cement-free sediment	distributed cementation (S_h=20%)	patchy cementation (S_h=20%)
			

Size ($F=ma$)

Shape

Strength: $\tau=\sigma' \tan\phi$

Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

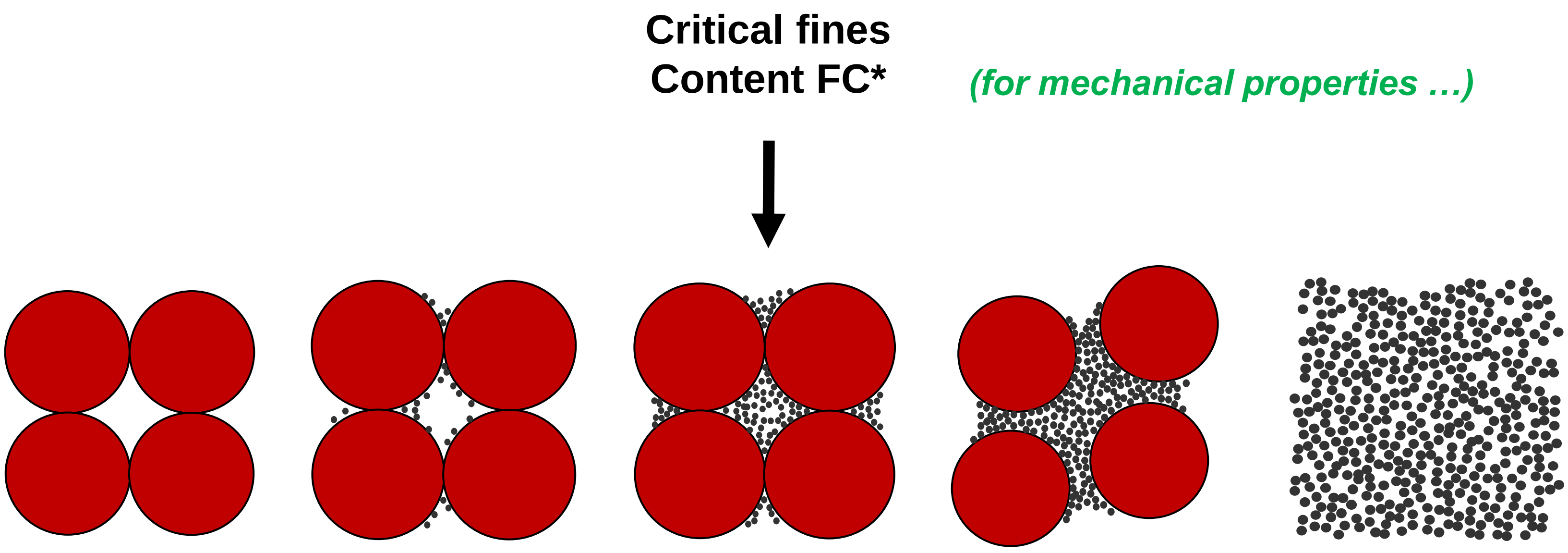
Pores

Mixed fluids (Unsaturated Soils)

Reactive Fluids

Closing Thoughts

Grain Size Distribution: The Role of Fines

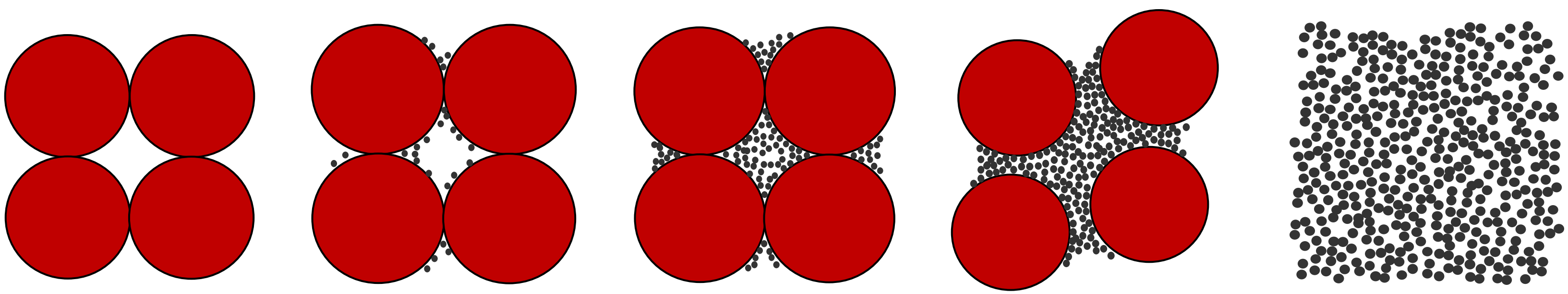


$$FC^{*} = \frac{M_{fine}}{M_{total}} = \frac{e_{coarse}}{1 + e_{coarse} + e_{fine}}$$

Sediment	e_{1kPa}	FC*
Silt	~0.7	~ 25 %
Kaolinite	~1.5	~ 20 %
Illite	~3.7	~ 11 %
Montmorillonite	~5.4	~ 8 %

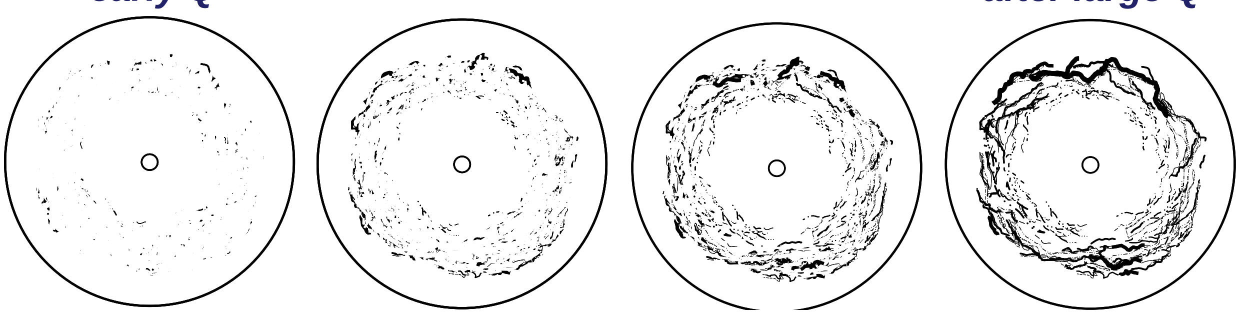
Fines Migration and Clogging

Critical fines
Content FC*



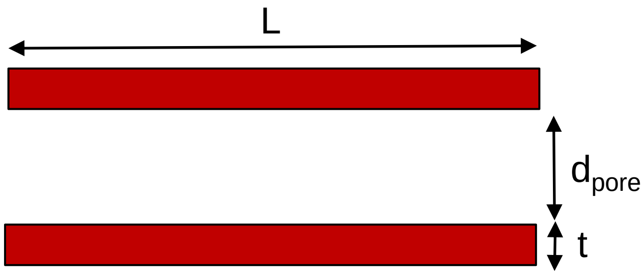
fines migration
& clogging

early Q ***after large Q***



J. Valdez

Grains and Pores: Clays



$$d_{pore} = \frac{2\,e}{S_s\,\rho}$$

MEAN PORE SIZE

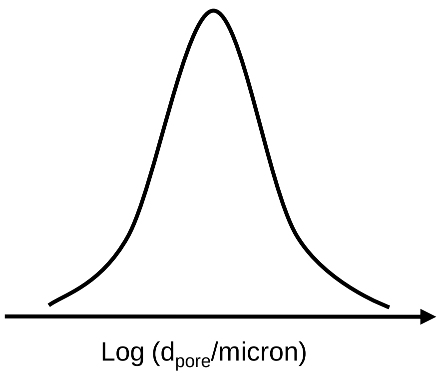
Sediment compaction

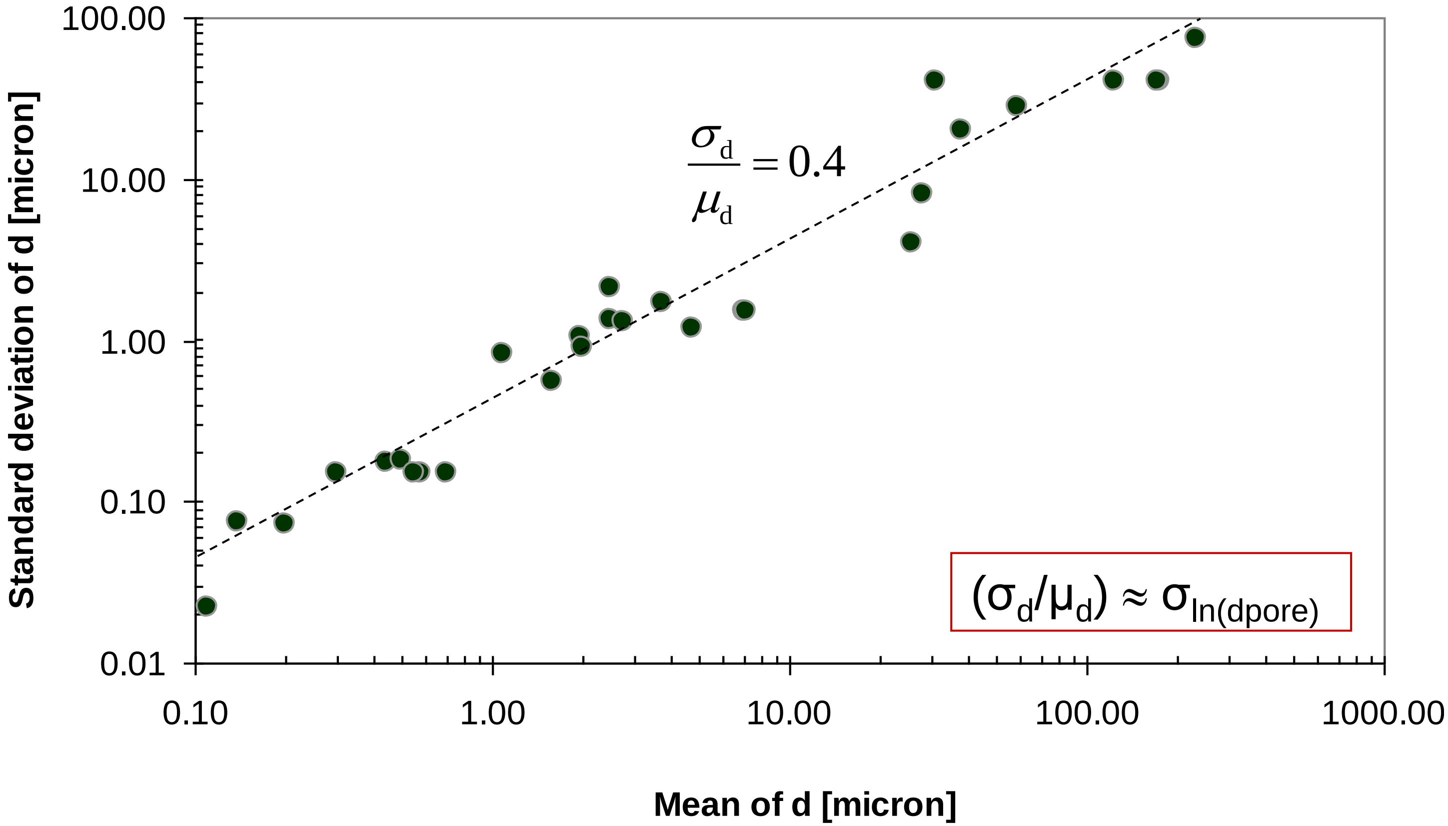
$$e = e_{1kPa} - C_c \log\left(\frac{\sigma'}{1kPa}\right)$$

Sediment	e_{1kPa}	C_C	S [m^2/g]	mean d_{pore}	ΔP [Mpa]
Silt	~0.7	0.02-0.09	0.045-1	5 μm	0.05
Kaolinite	~1.5	0.19-0.3	10-20	0.5 μm	0.5
Illite	~3.7	0.5-1.1	65-100	0.05 μm	5
Montmorillonite	~5.4	1-2.6	300-780	0.005 μm	50

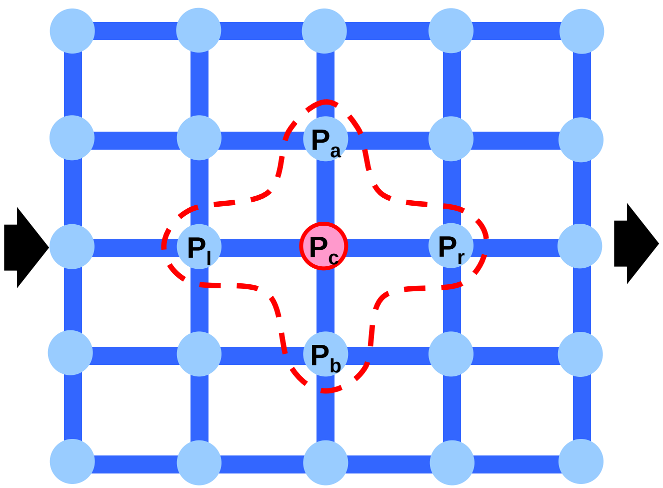
@ σ'=100 kPa σ_{LV}=70 mN/m

Pore Size Distribution





Network Models – Upscaling



Poiseuille's Eq.

$$q = \frac{\pi R^4}{8\eta\,\Delta L}\Delta P \quad \left(\alpha = \frac{\pi R^4}{8\eta\,\Delta L}\right)$$

Mass Balance at Nodes

$$0 = \sum q_c$$

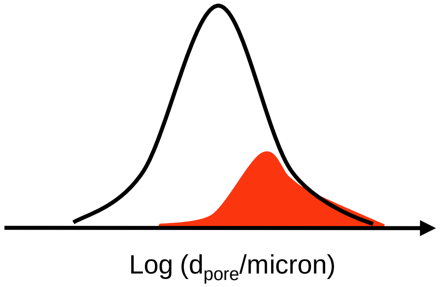
$$0 = \alpha_a\left(P_a - P_c\right) + \alpha_b\left(P_b - P_c\right) + \alpha_r\left(P_r - P_c\right) + \alpha_l\left(P_l - P_c\right)$$

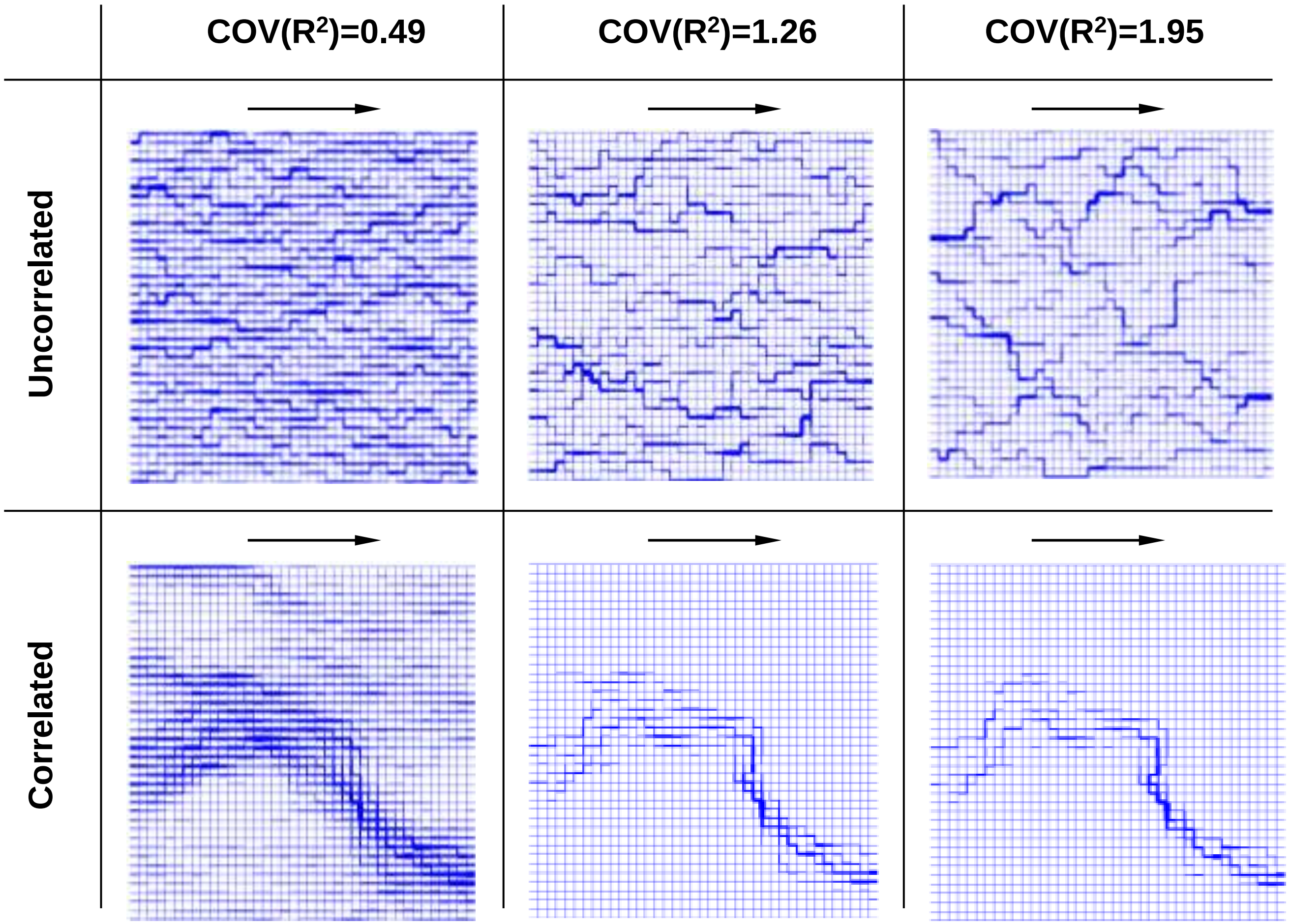
$$P_c = \frac{\alpha_a P_a + \alpha_b P_b + \alpha_r P_r + \alpha_l P_l}{\left(\alpha_a + \alpha_b + \alpha_r + \alpha_l\right)}$$

System of Equations

$$\underline{B} = \underline{\underline{A}}\,\underline{P} \qquad \text{then} \qquad \underline{P} = \underline{\underline{A}}^{-1}\underline{B}$$

Spatially Correlated Porosity





Size (F=ma)

Shape

Strength: $\tau=\sigma' \tan\phi$

Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

Pores

Mixed fluids (Unsaturated Soils)

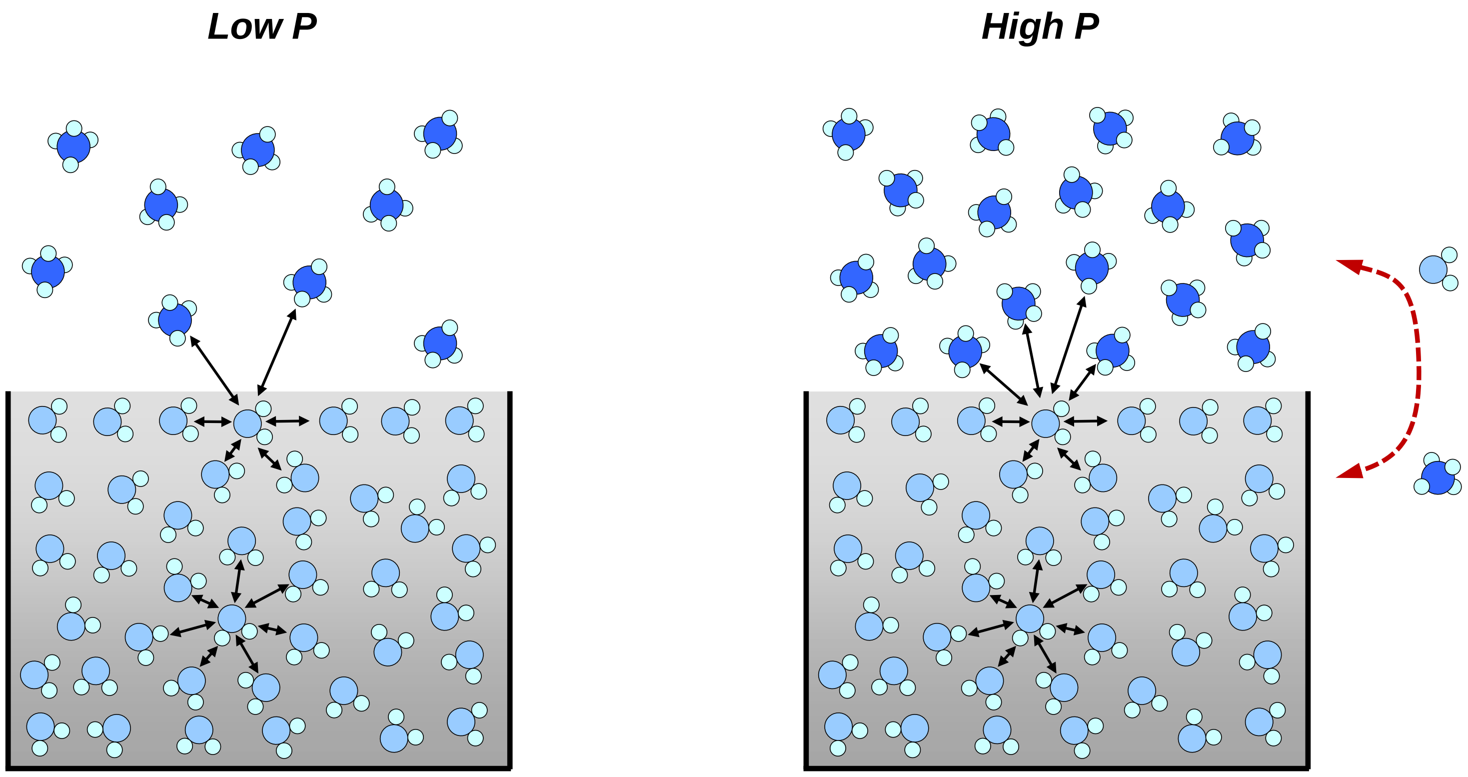
Reactive Fluids

Closing Thoughts

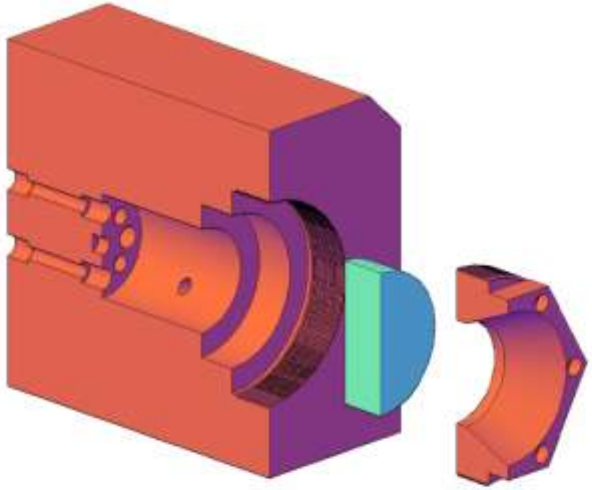
Surface Tension

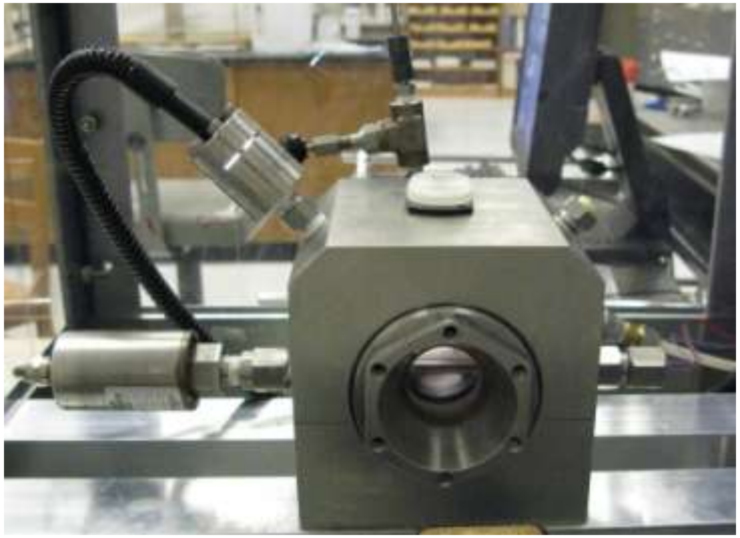
BBC News In pictures Visions of Science.jpg

CO_2-H_2O: Interfacial Interaction



Surface Tension and Contact Angle

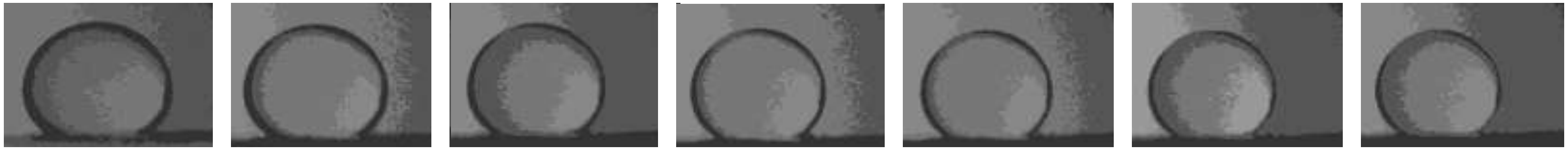




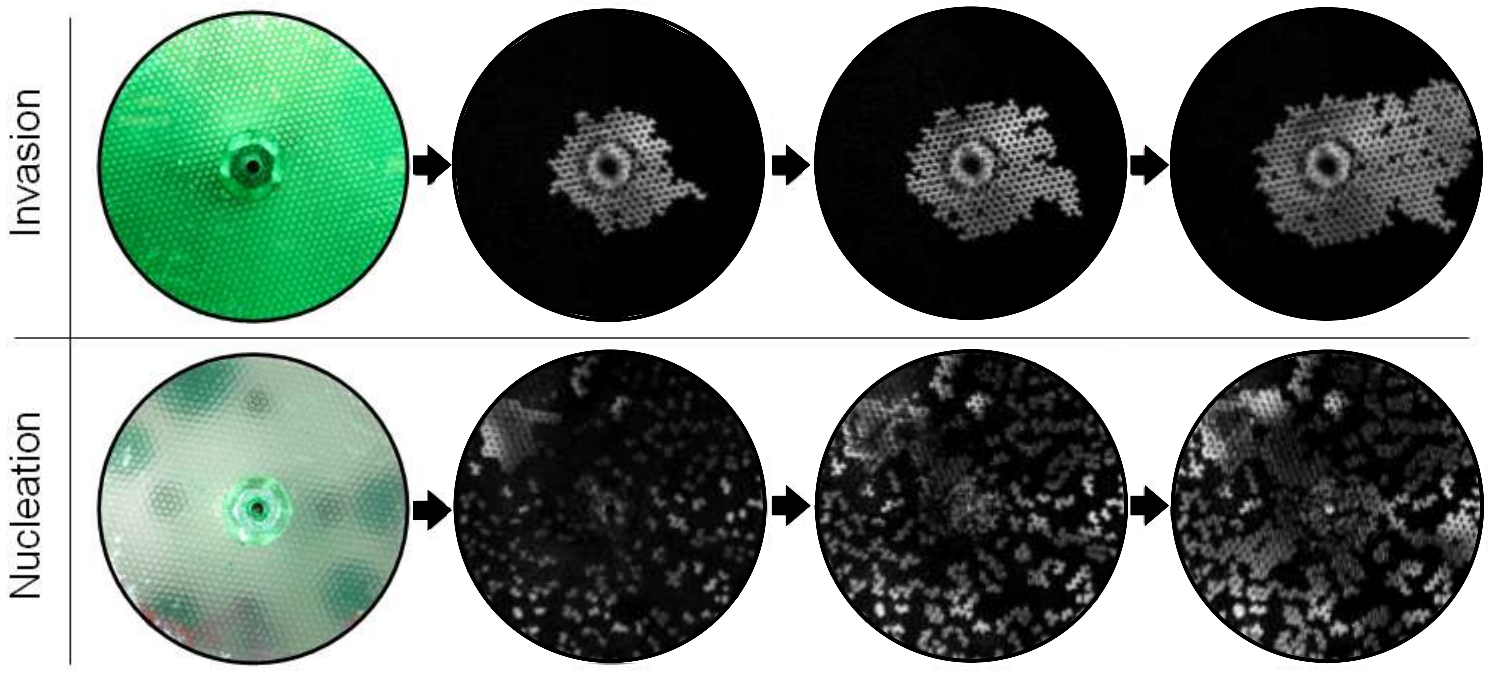
Water droplet in

CO_2 gas

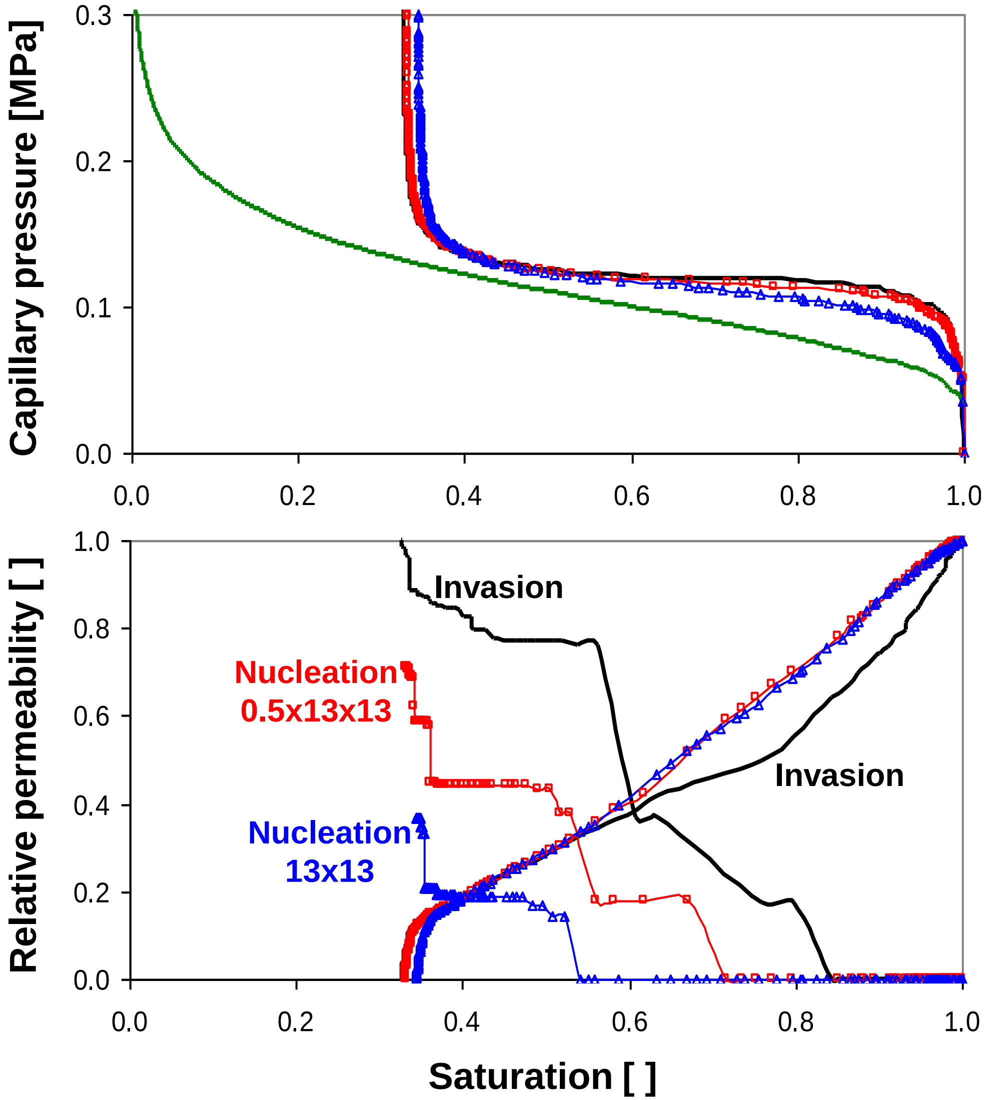
CO_2 liquid



Invasion vs. Nucleation



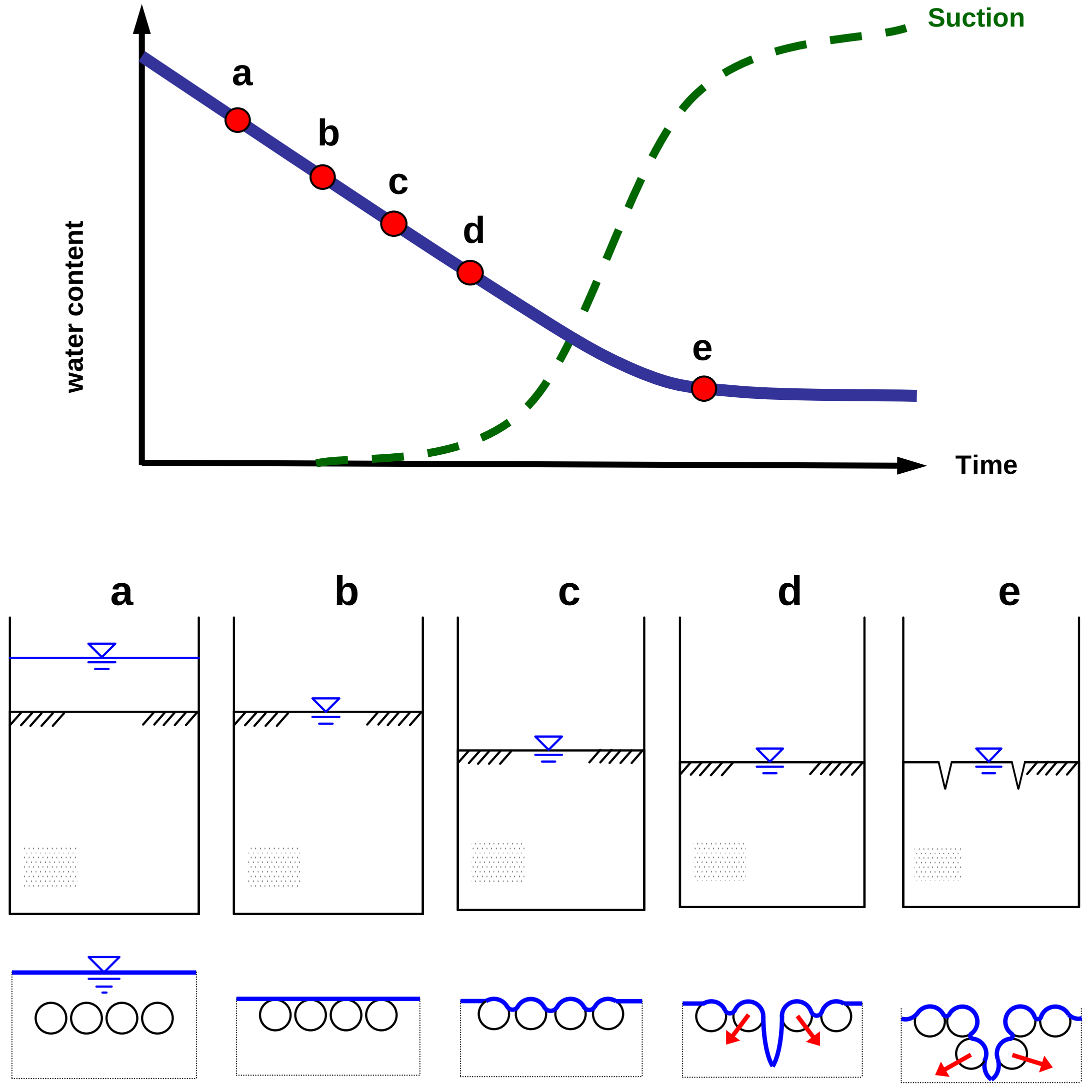
Characteristic Curve & k_r



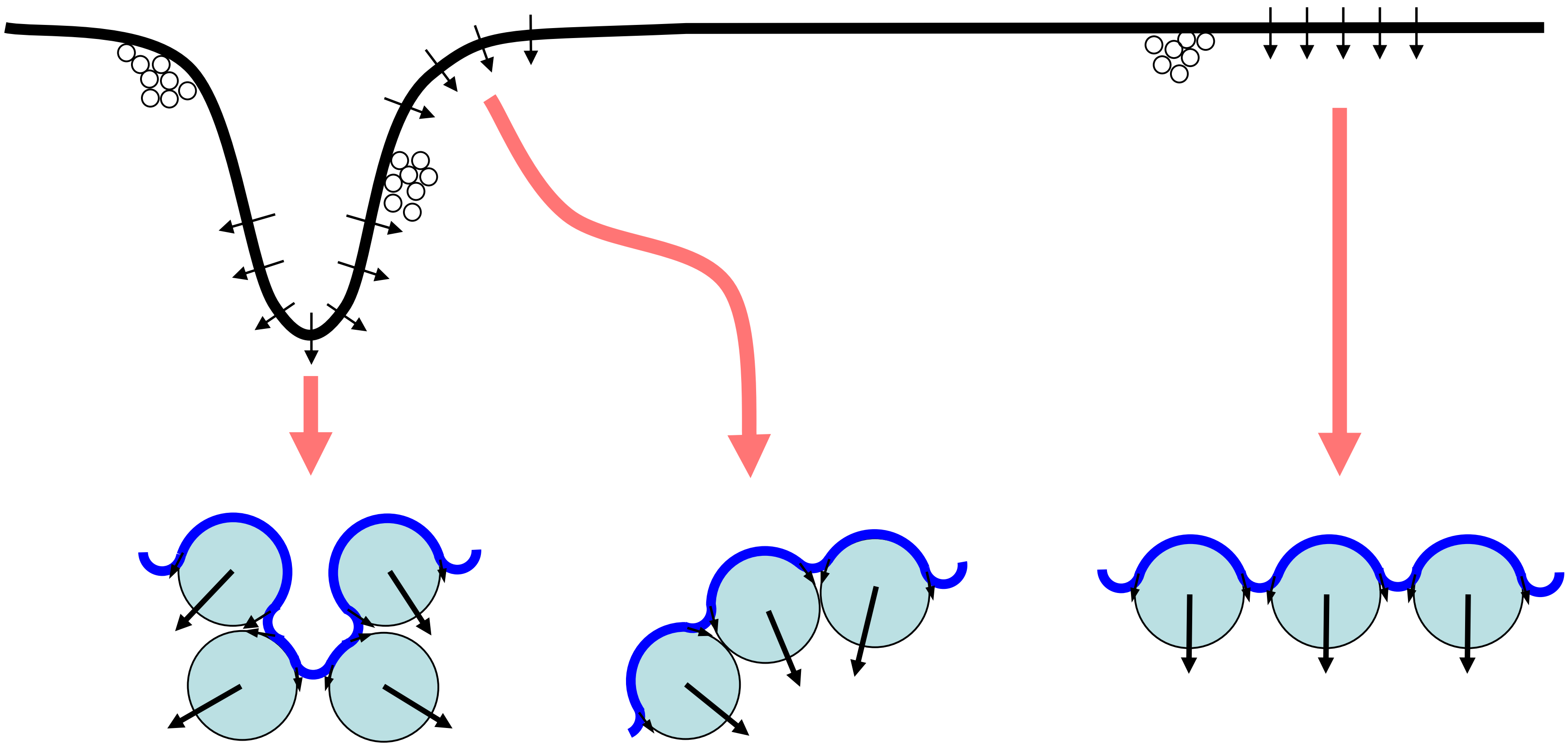
Log-normal distribution of R^2, $\sigma(\ln(R/[\mu m]))=0.4$, Network size: 3D 13x13x13, $cn=6$, $P_c=2T_s\cos\theta/R$, $T_s=72$mN/m, $\cos\theta=1$

Forcing Gas Into Sediment

Evolution



Gas-Driven Fracture

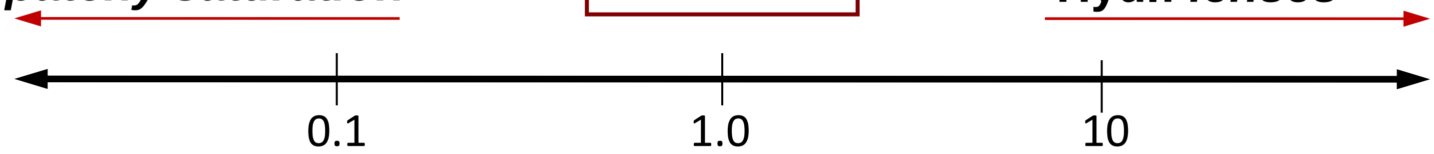


Invasion vs. Localization

INVASION
Fluid invasion
Crystal growth in pores
Hyd.: *patchy saturation*

$$\frac{F_c}{N} = \frac{2\pi\sigma_{LV}}{\sigma' d}$$

LOCALIZATION
Lenses
Fractures
Hyd.: *lenses*



coarse grained soils
high effective stress

fine grained soils
low effective stress

Size (F=ma)

Shape

Strength: $\tau = \sigma' \tan\phi$

Stiffness: $G = \alpha(\sigma'/kPa)^{\beta}$ … Cementation

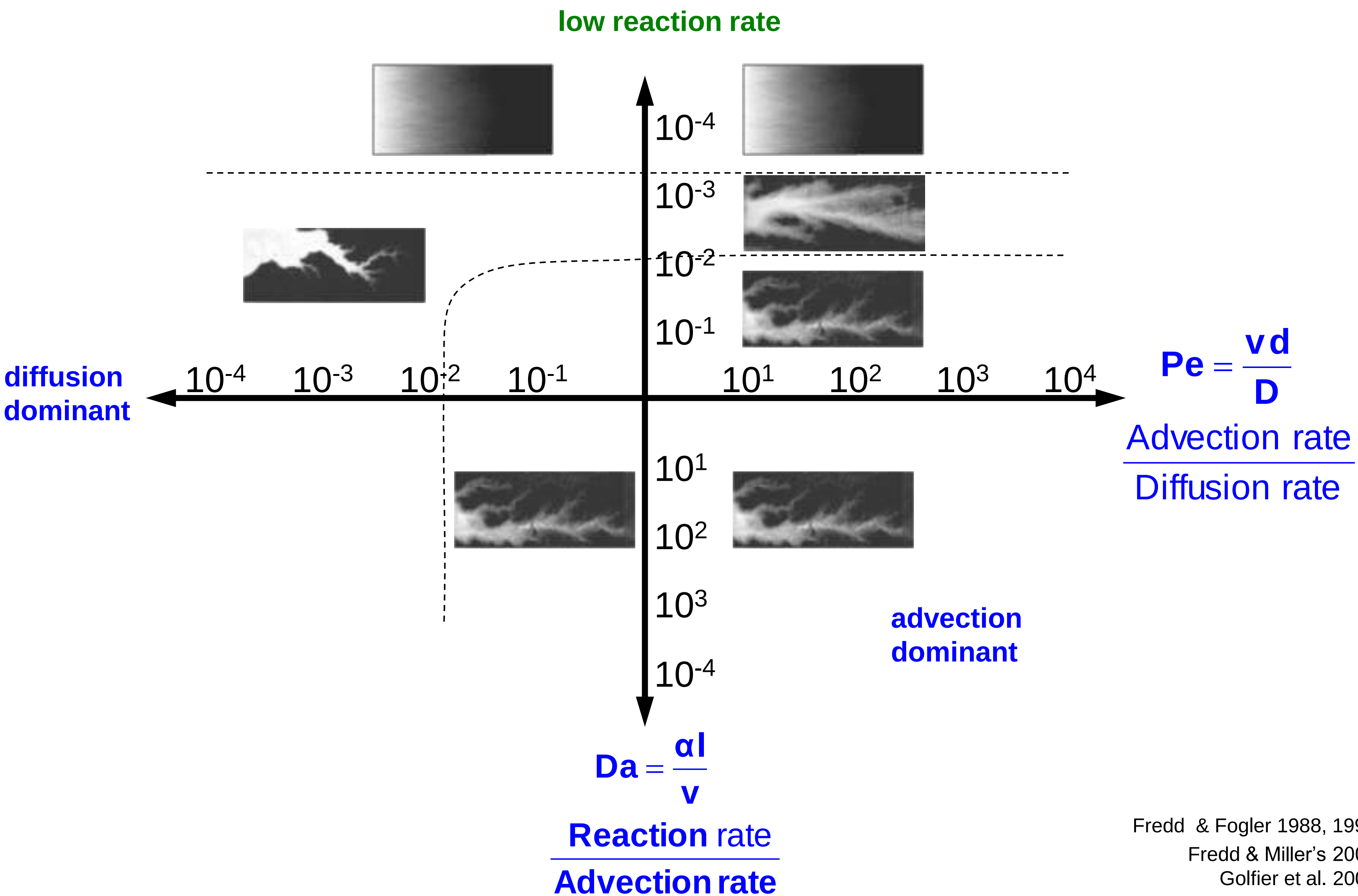
Pores

Mixed fluids (Unsaturated Soils)

Reactive Fluids

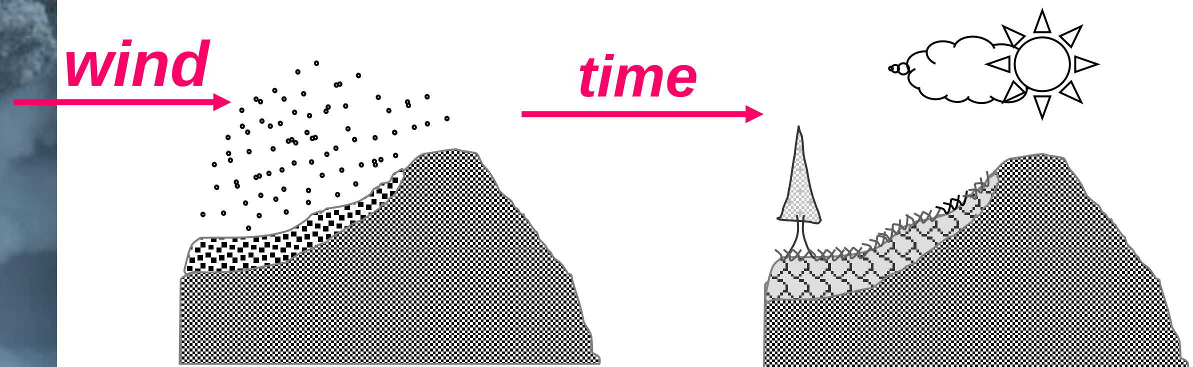
Closing Thoughts

Reactive Fluid Transport



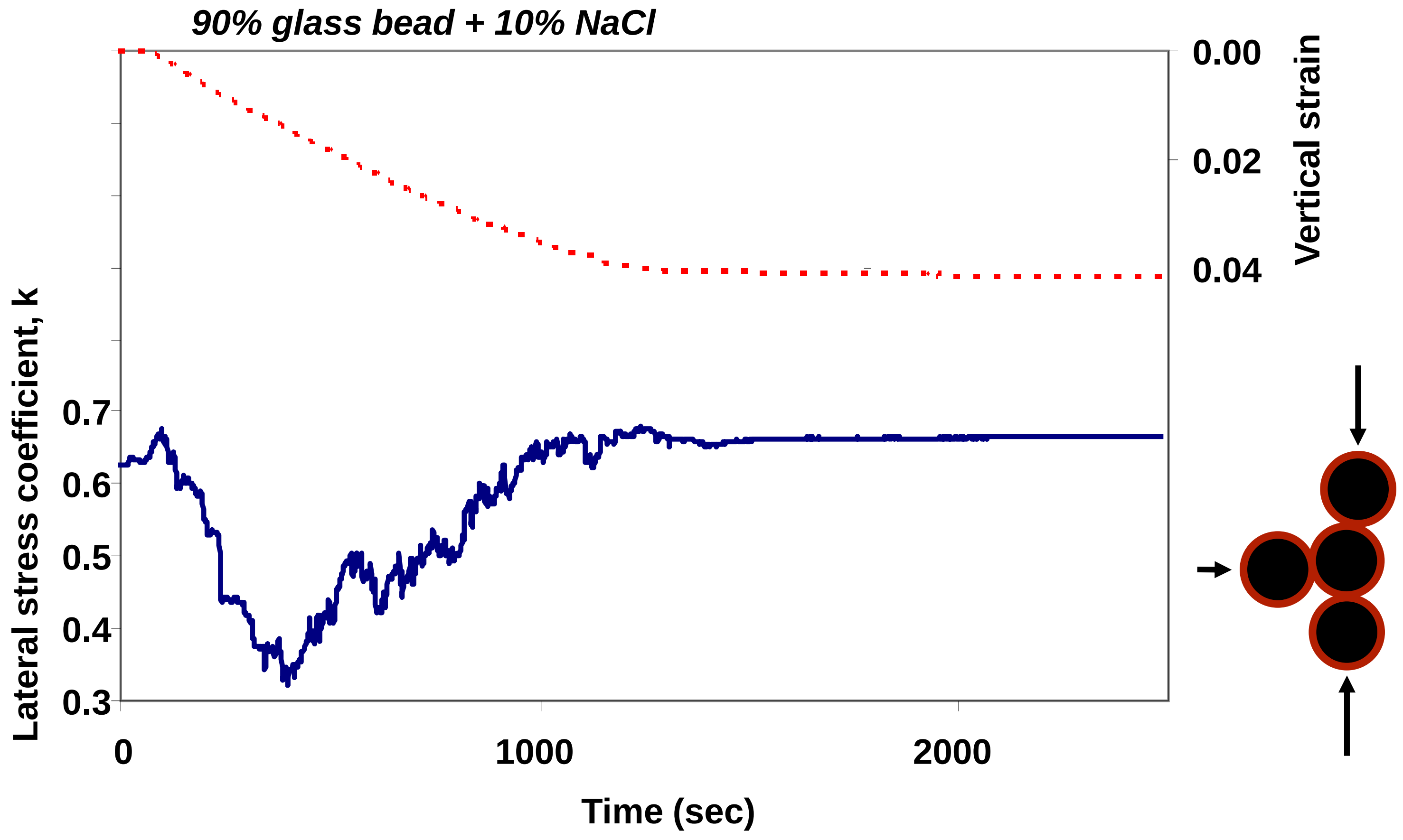
Fredd & Fogler 1988, 1998
Fredd & Miller's 2000
Golfier et al. 2002

Volcanic Ash Soils: Formation



$e = 0.8\text{-}1.5$	**$e = 2.0\text{-}7.0$**
$S_s \sim 0.1\text{-}1\ m^2/g$	**$S_s = 50\text{-to-}200\ m^2/g$**
volcanic glass	**hallosite** **imogolite** **alophane**
$k_o = 1 - \sin\varphi$	**$k_o = ??$**

Experimental Results



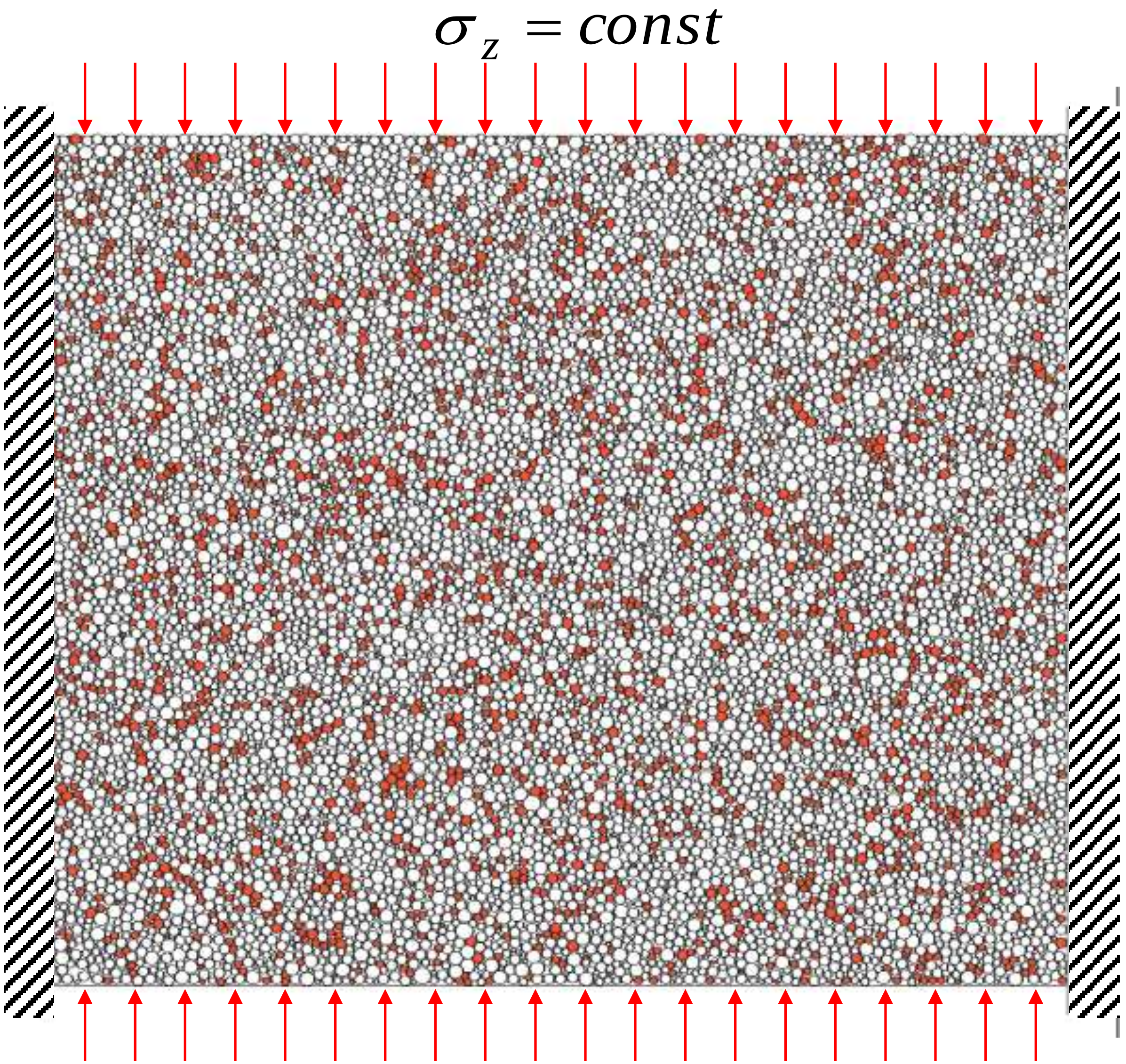
DEM Simulation

N= 9999 (in 2D) - 8000 (in 3D)

cov particle diameter: 0.25

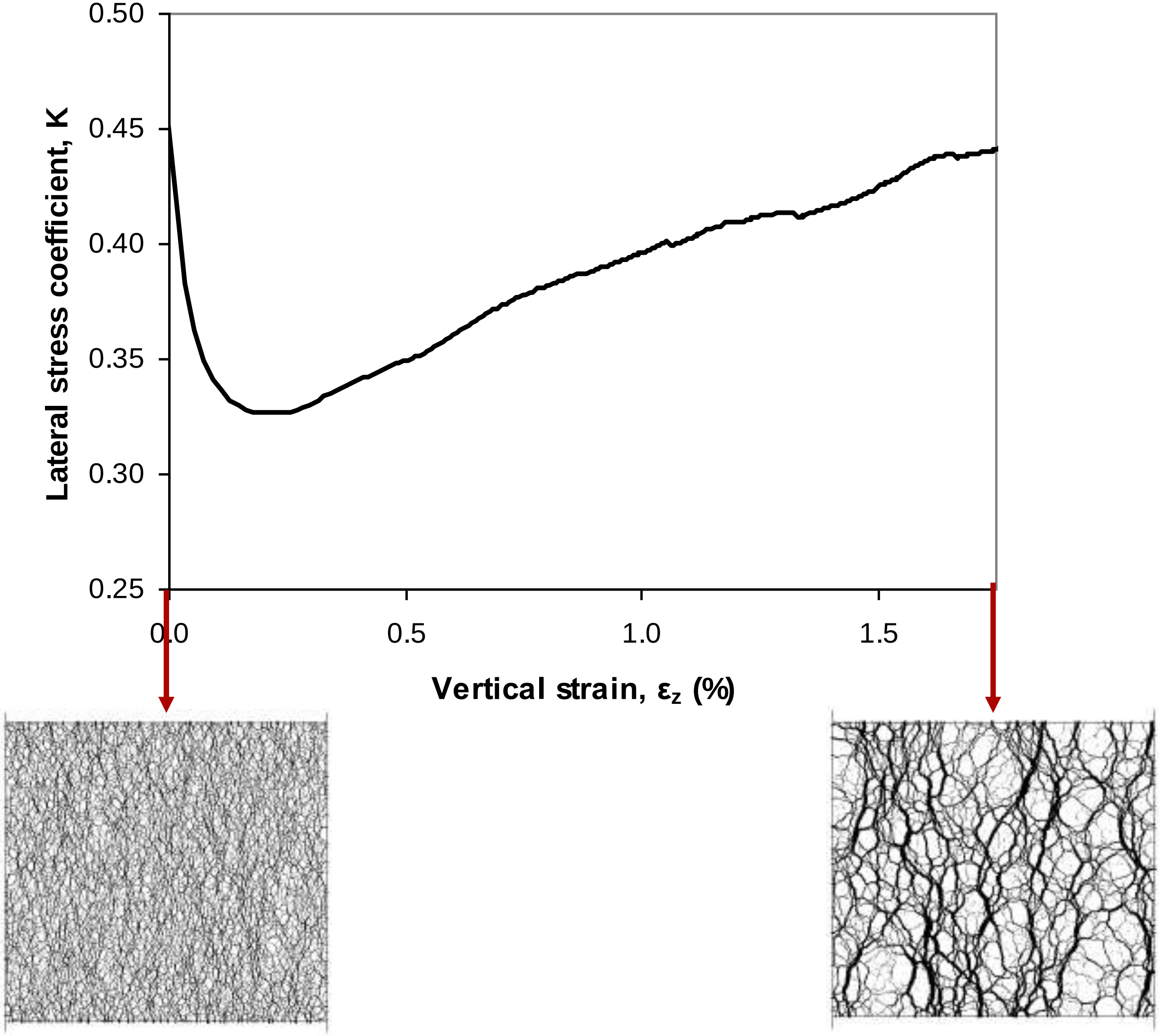
Interparticle friction: 0.5

Simulation: reduce D or G

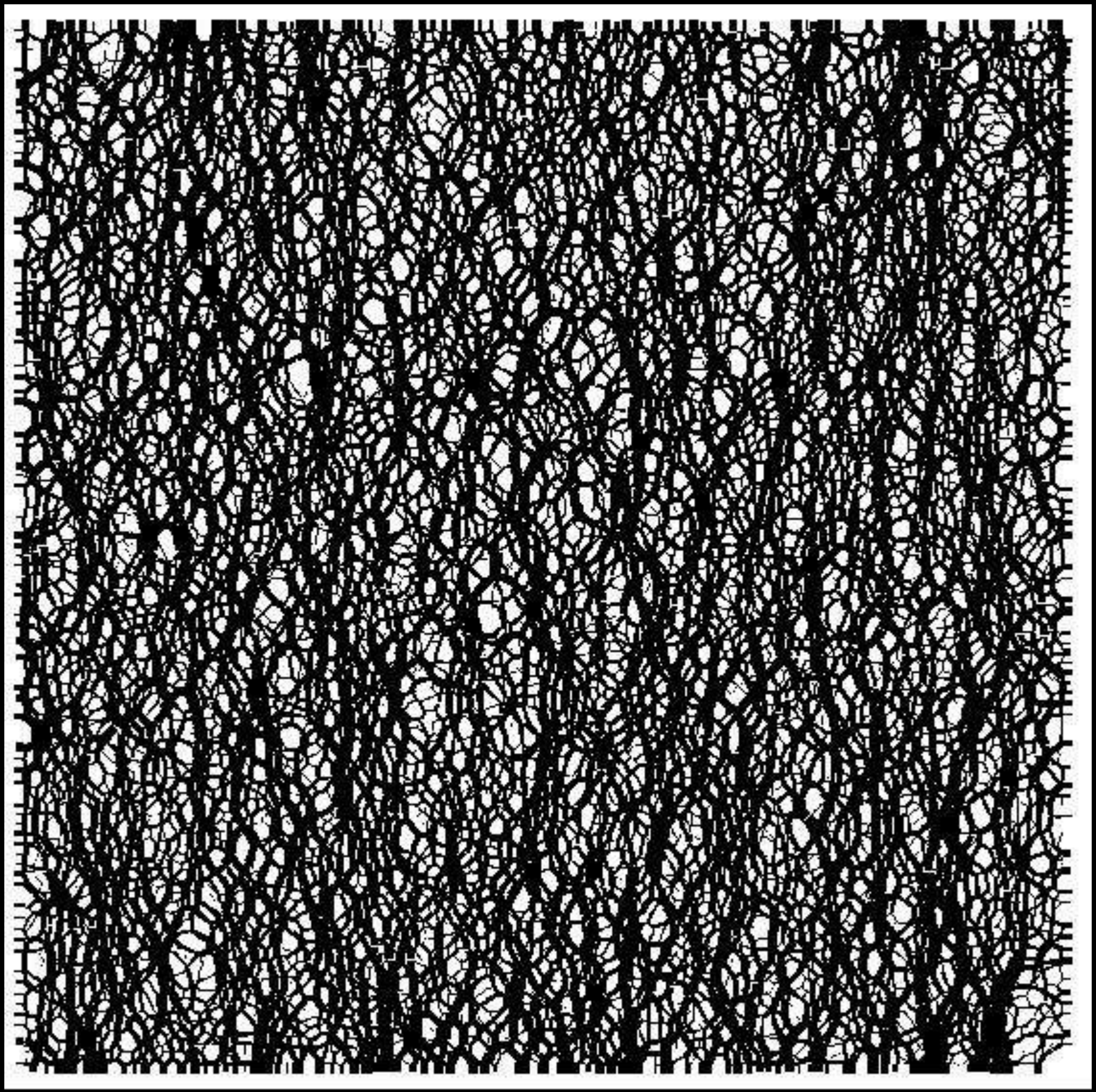


DEM Simulation

2D - diameter gradually reduced - 20% of particles

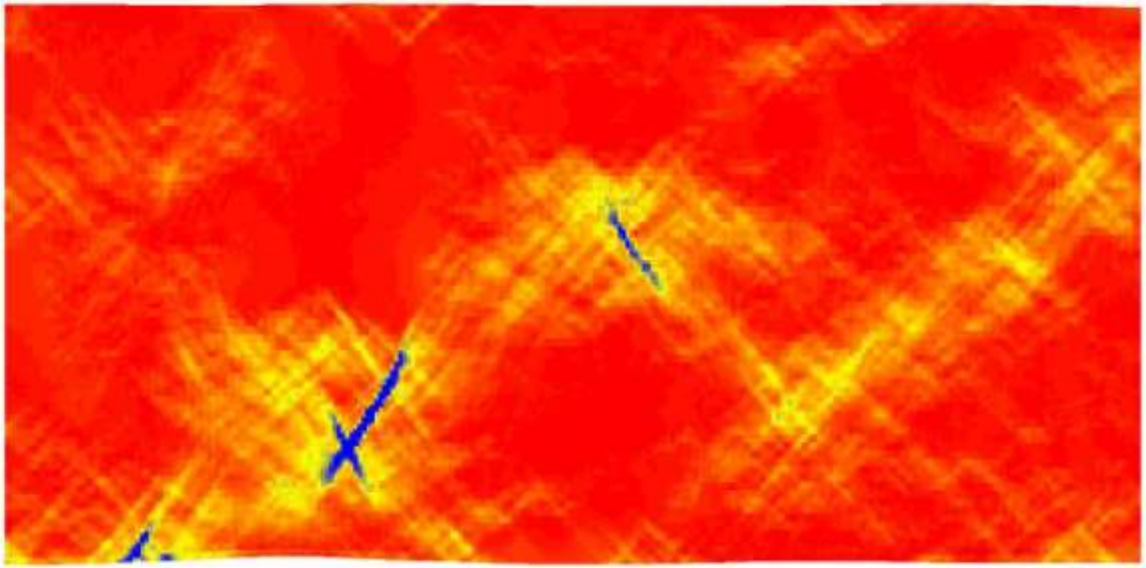


DEM Simulation dR/dt=f(N)



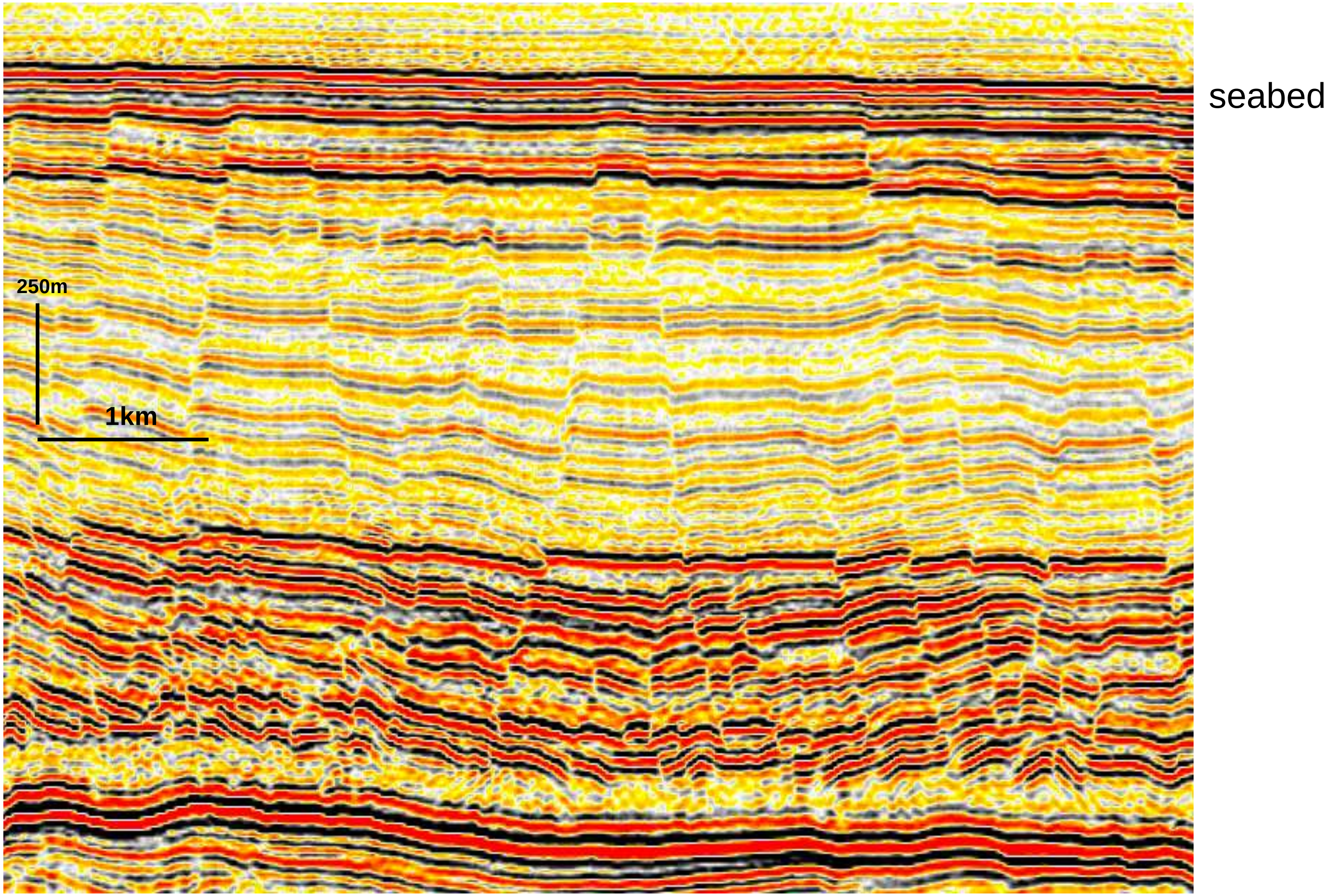
Shear Localization

FEM simulation



natural sediments

Shear Localization: Marine Sediments



Cartwright (2005)

Size (F=ma)

Shape

Strength: $\tau=\sigma' \tan\phi$

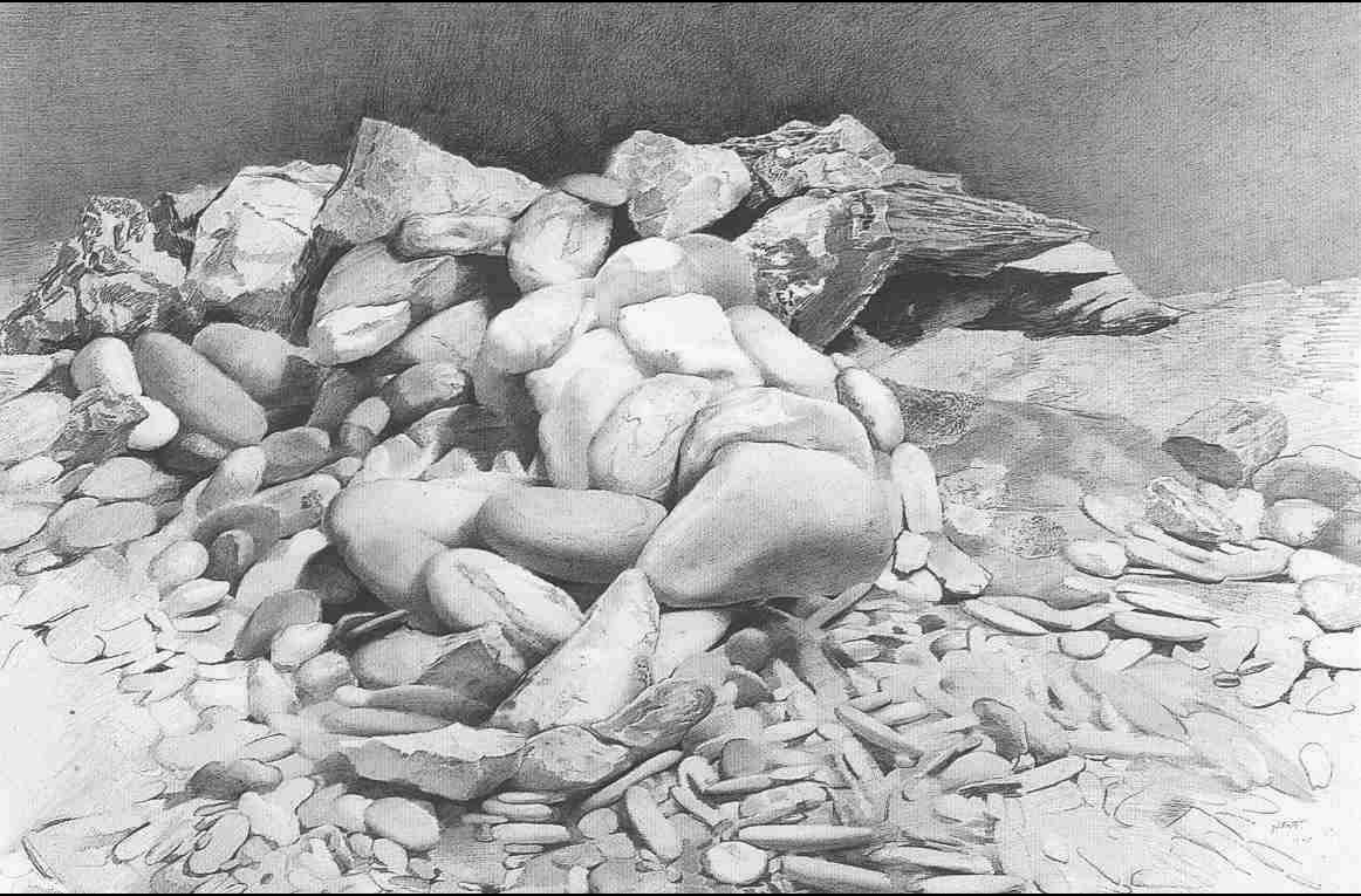
Stiffness: $G=\alpha(\sigma'/kPa)^{\beta}$ … Cementation

Pores

Mixed fluids (Unsaturated Soils)

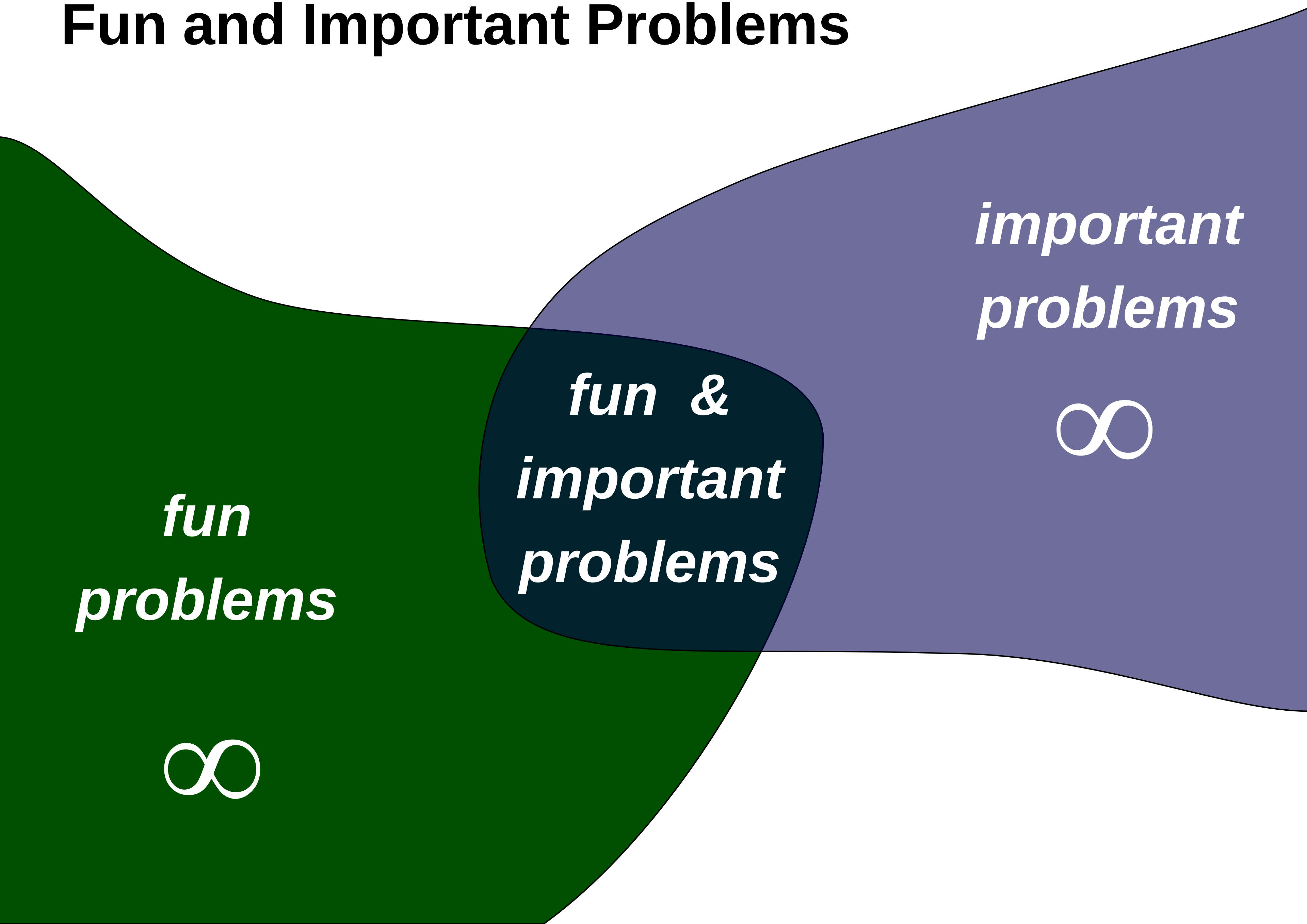
Reactive Fluids

Closing Thoughts

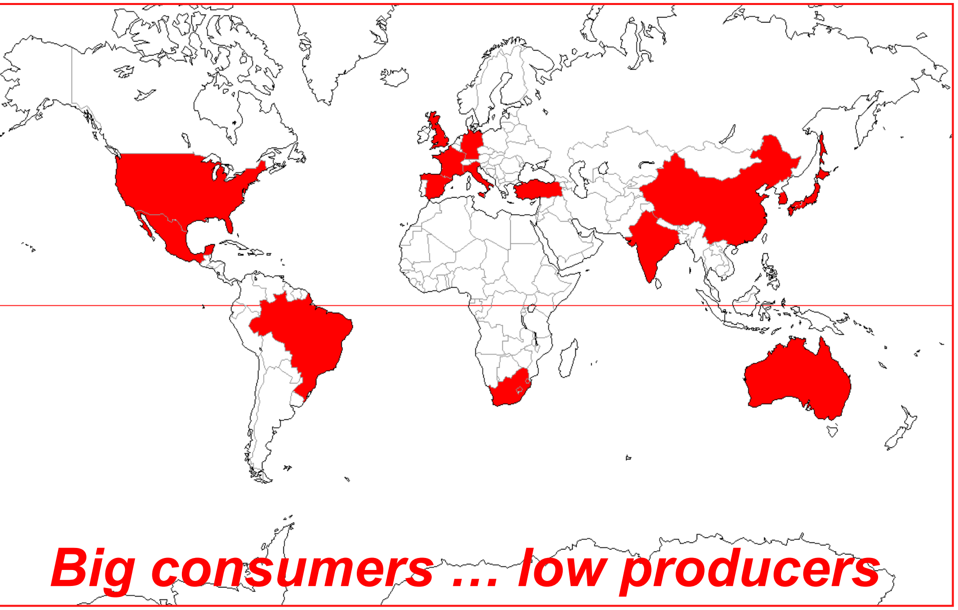


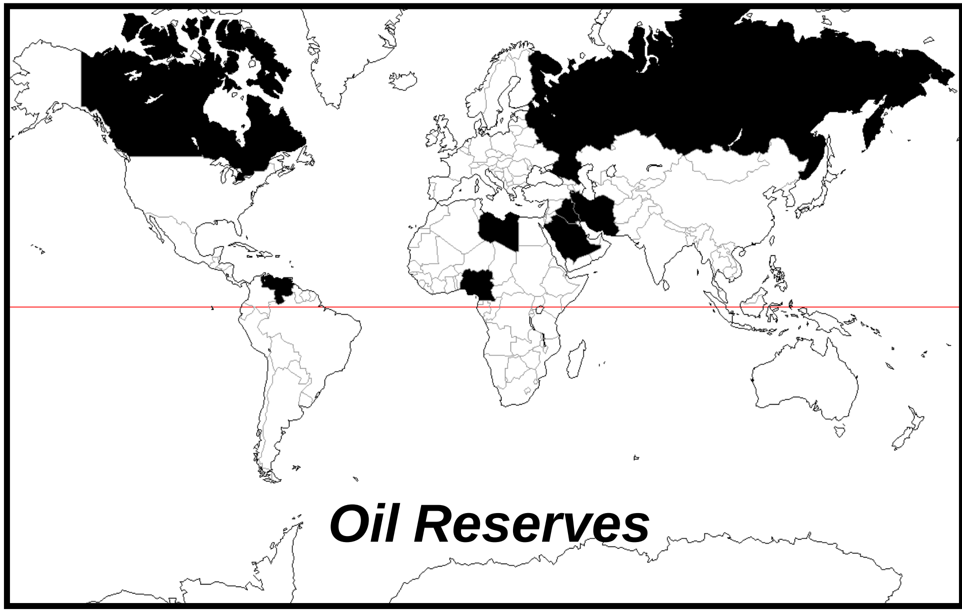
Sleeping Beach – Antoni Pitxot – Museu Dali

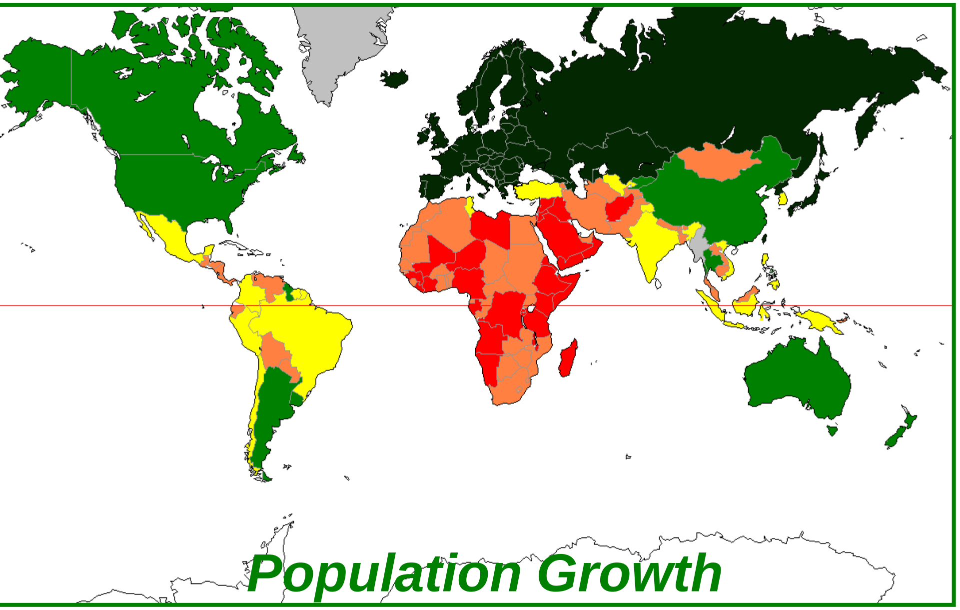
Fun and Important Problems

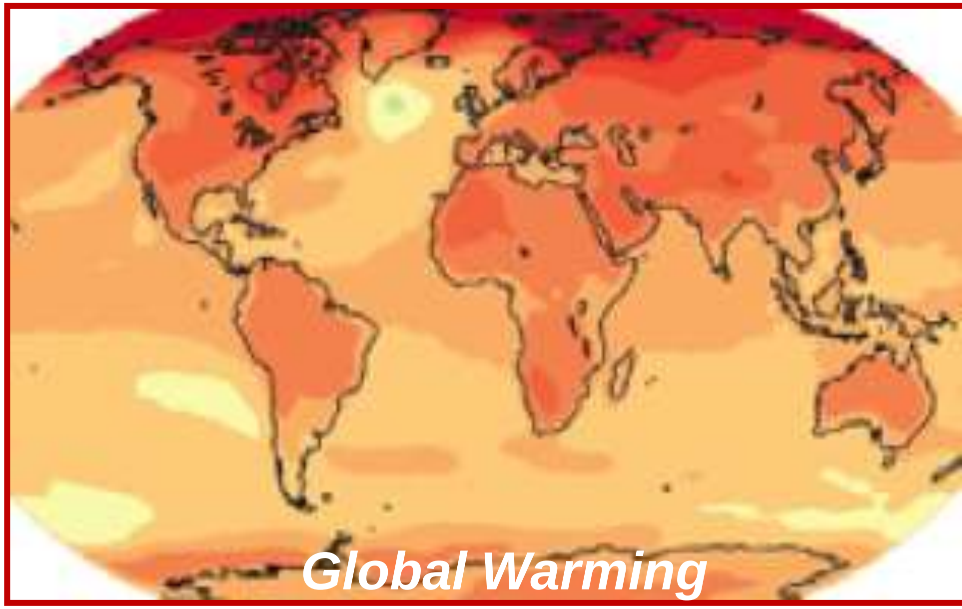


Fun and Important Problem: Energy









Hobby

D. Carbajal Solsona

Hobby

D. Carbajal Solsona

Hobby

D. Carbajal Solsona

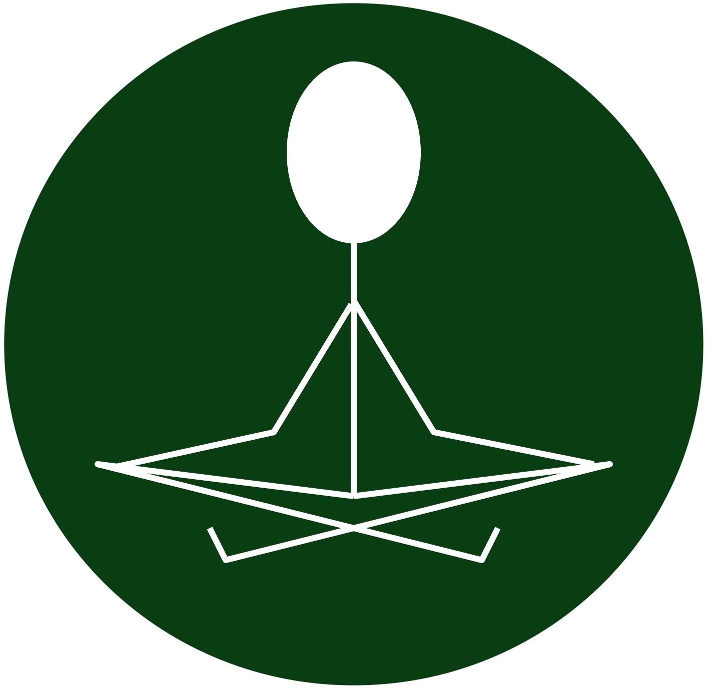
Hobby

D. Carbajal Solsona

Potential – Attitude – Dedication – Impact

P intellectual potential

A attitude



D focused dedication

I impact

Potential – Attitude – Dedication – Impact

$$I = 0.04P + 0.18A + 0.73D$$ cc≈0.89

$$I = P^{0} A^{0.18} D^{0.94}$$ cc≈0.89

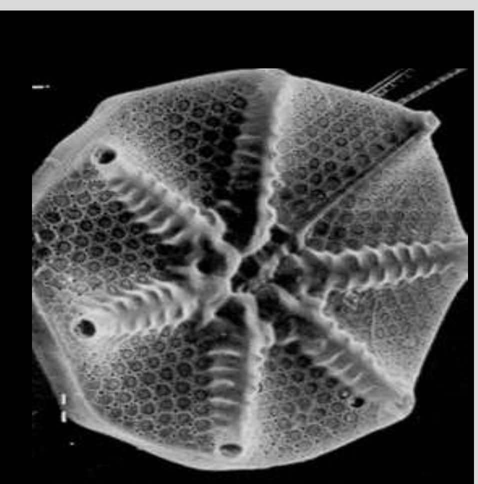
$$I = [\min(P,A)]^{0.18} D^{0.92}$$ cc≈0.88

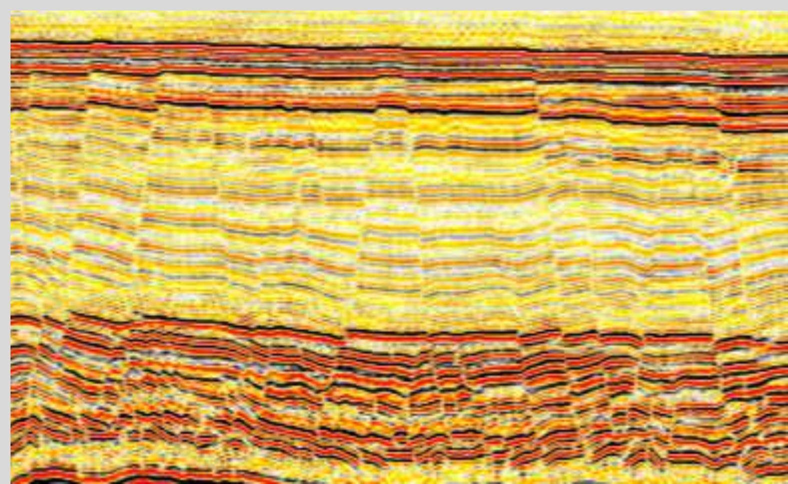
"per ardua ad astra" *through struggle to the stars*











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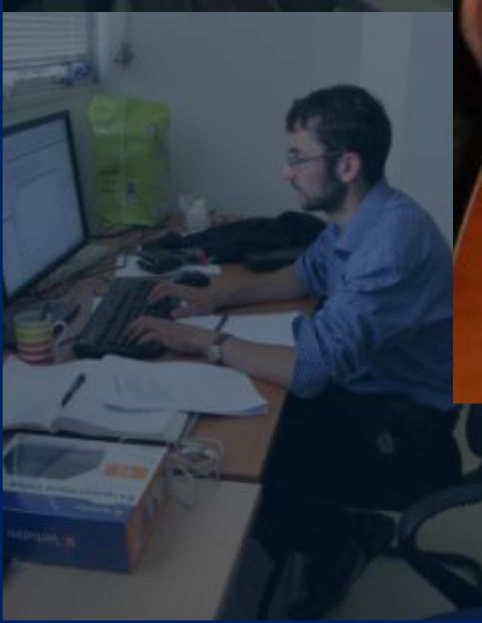

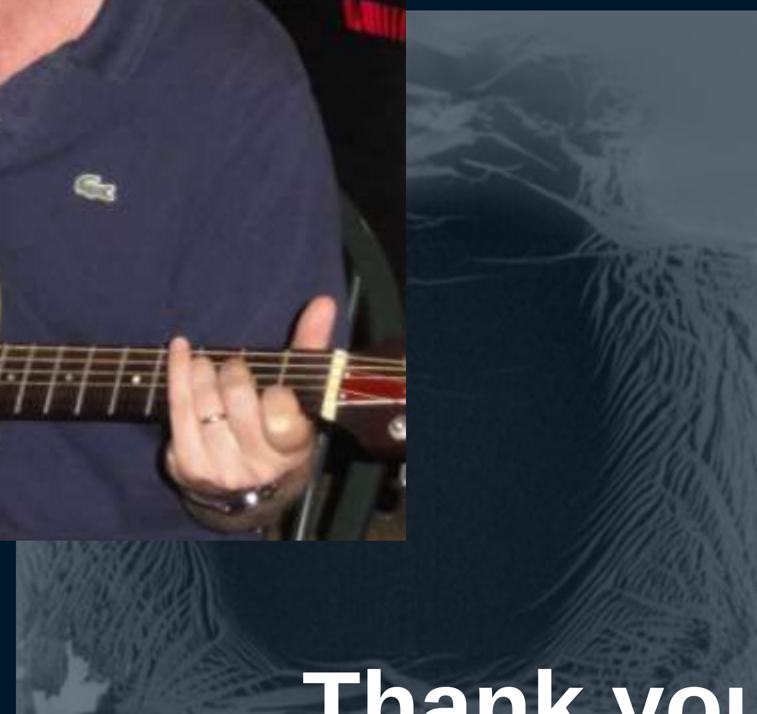

Thank you !