Micromechanics of three-phase granular materials

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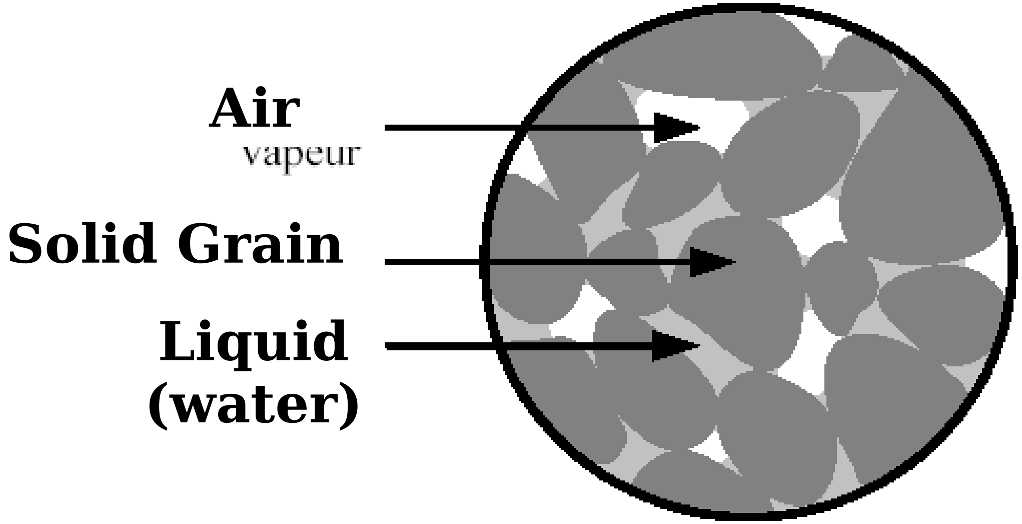
Lab. 3S-R – Grenoble INP

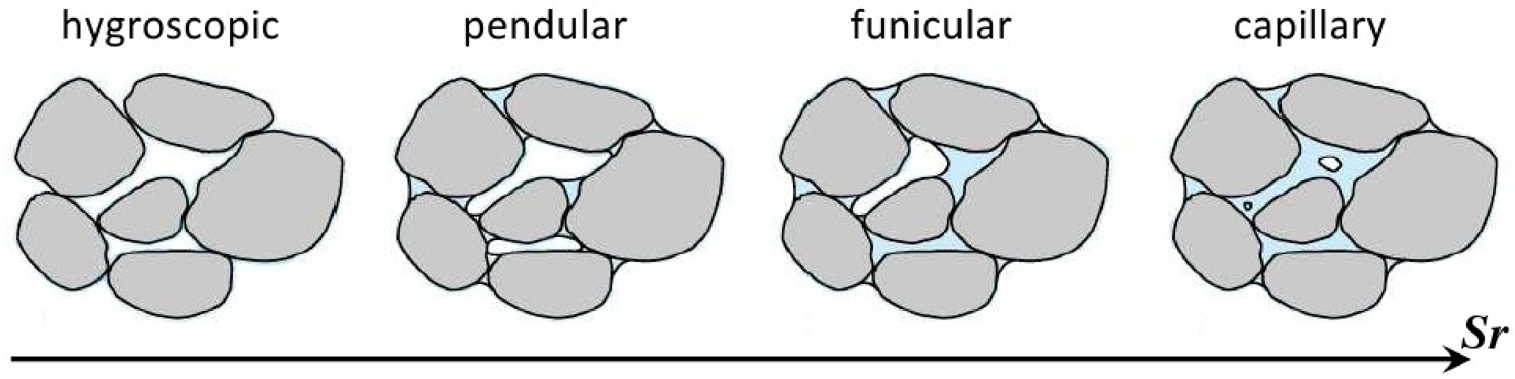
ALERT O.Z. Course, June 2011, Grenoble

Plan

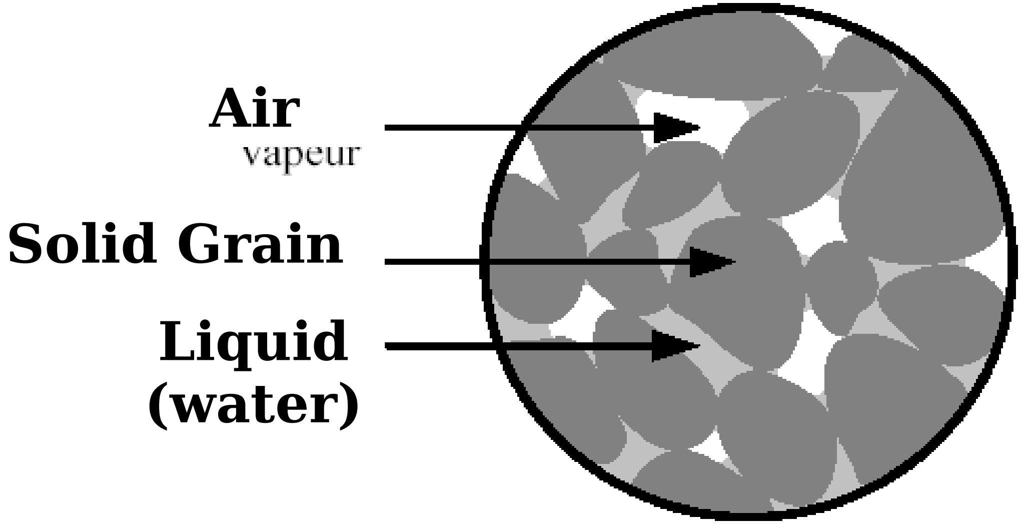
- 1. Wet granulars
- 2. DEM modeling
- 3. Macroscopic results and validation
- 4. Upscaling
- 5. Generalized effective stress
- 6. Discussion

Capillarity in Unsaturated Granular Materials

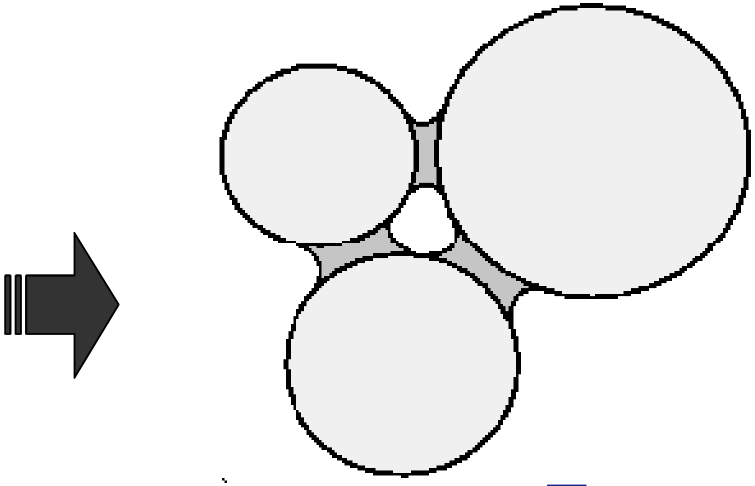




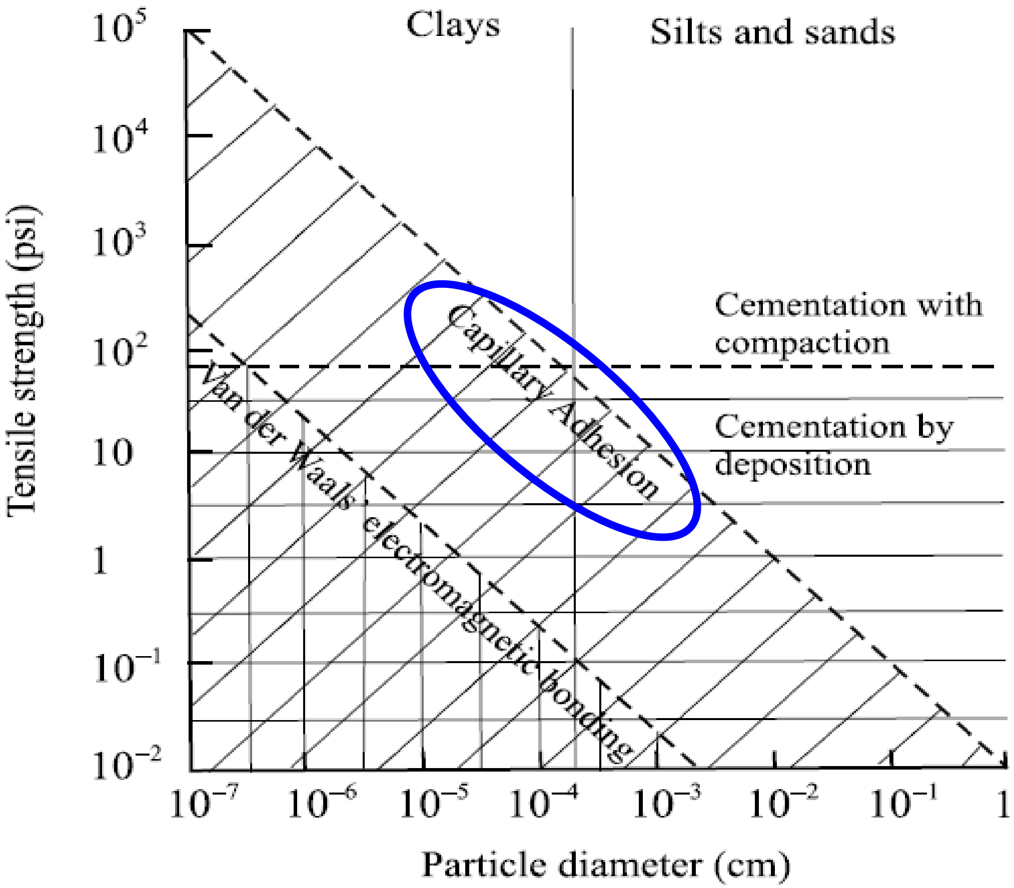
Capillarity in Unsaturated Granular Materials



At **low water content** levels interfacial phenomena lead to **intergranular** water **menisci**



Capillarity in Unsaturated Granular Materials



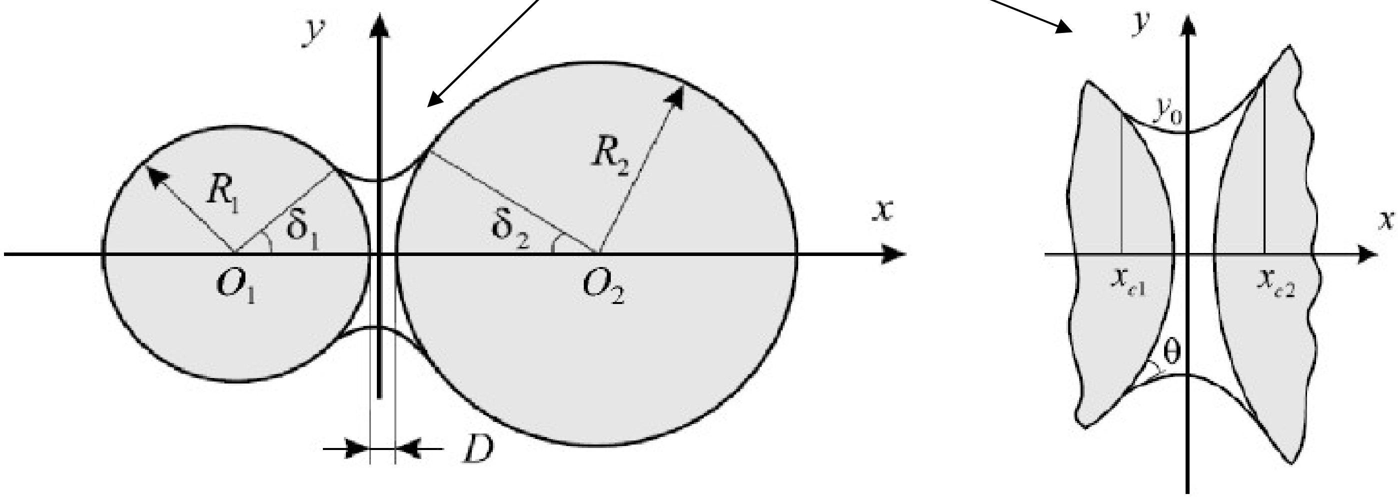
[Mitchell: *Fundaments of soil behavior*, Wiley Inter Science,1993]

In **granular** soils (silts and sands), **capillary effects** are of primary **significance** in the unsaturated induced **strength** increase

Capillarity in Unsaturated Granular Materials

Capillary Theory (Laplace): $\Delta u = u_a - u_w = \sigma C$

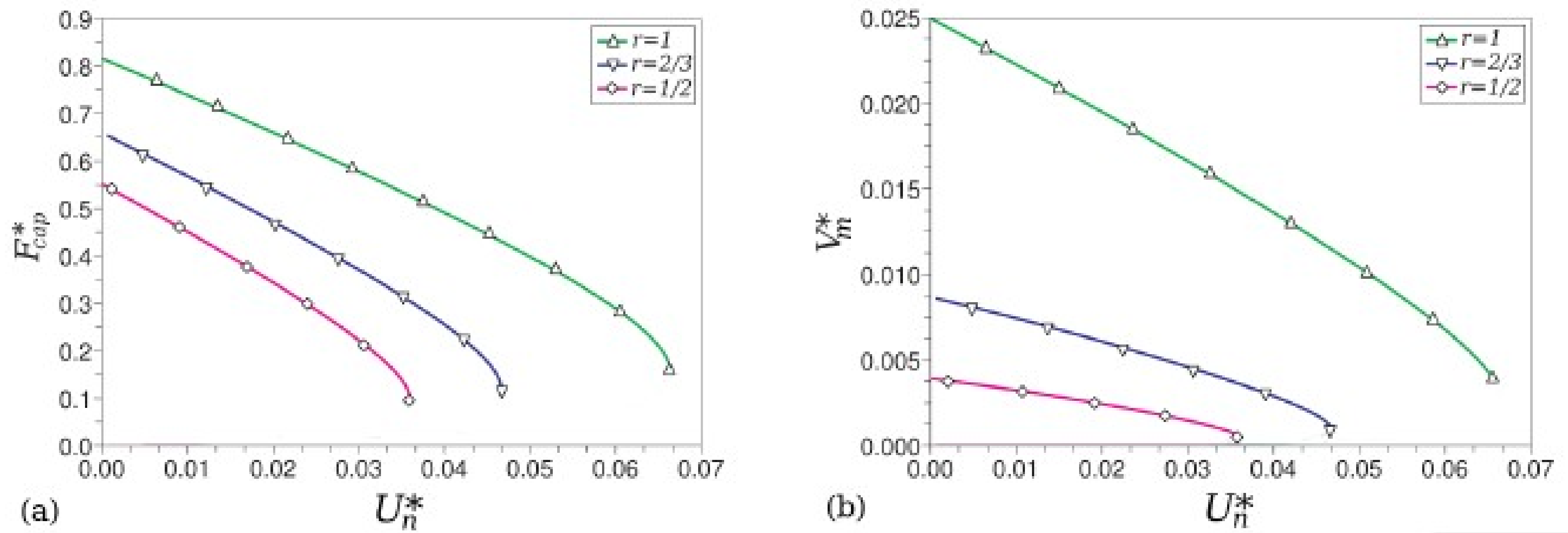
$C = f[y(x)]$; *y(x)* is the interface profile



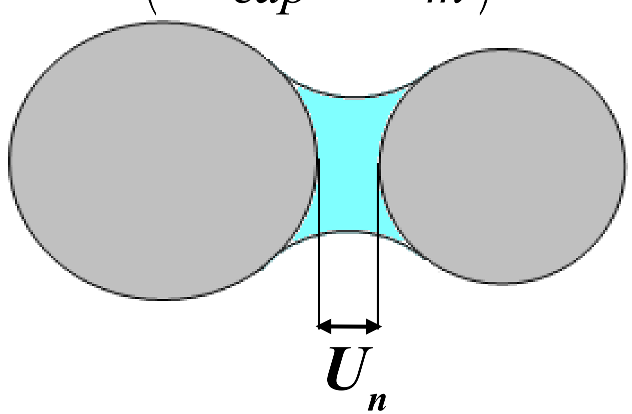
$$\begin{cases} F_{capillary} = 2\pi\sigma y_0 + \pi \Delta u y_0^2 \\ V_{meniscus} = \pi \int y^2(x) . dx \end{cases}$$

Capillarity in Unsaturated Granular Materials

Evolution of the capillary force at constant suction :



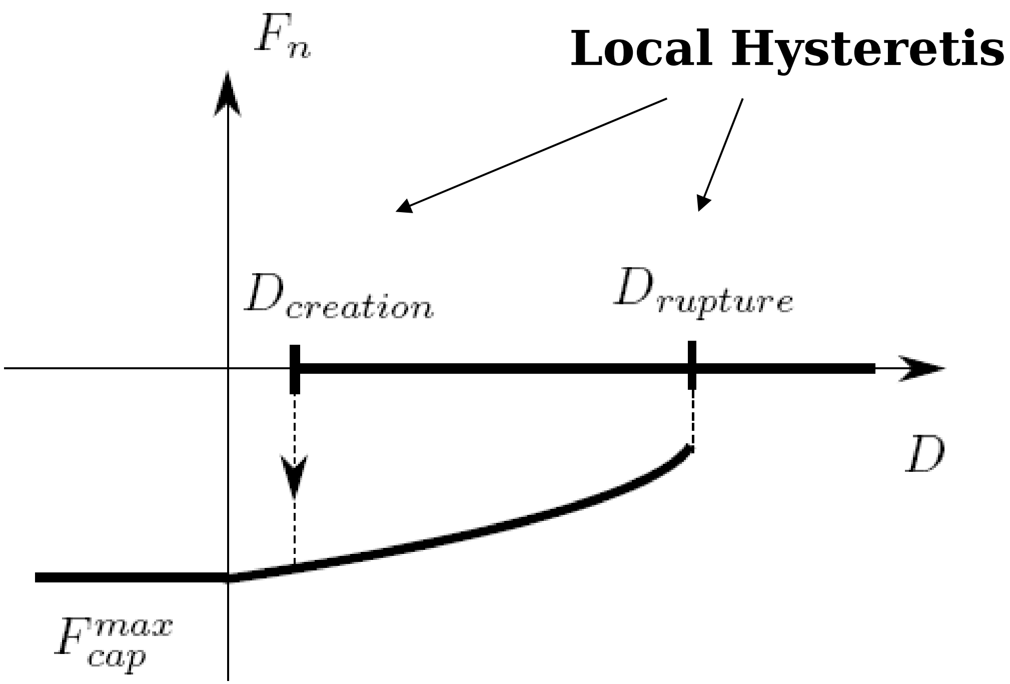
$$\left(F_{cap}, V_m\right)$$

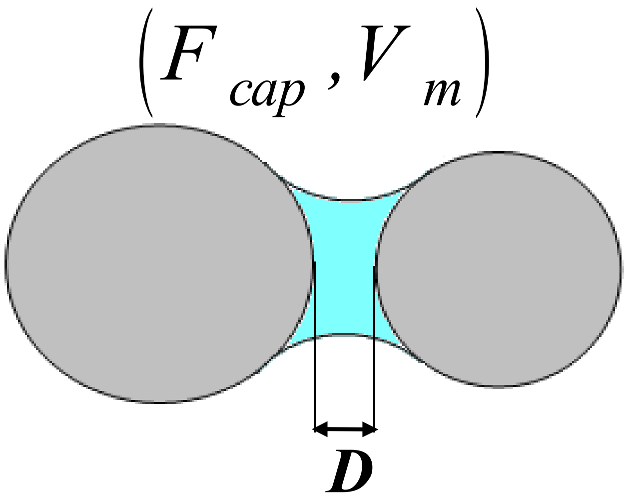


Capillarity in Unsaturated Granular Materials

Evolution of the capillary force at constant suction :

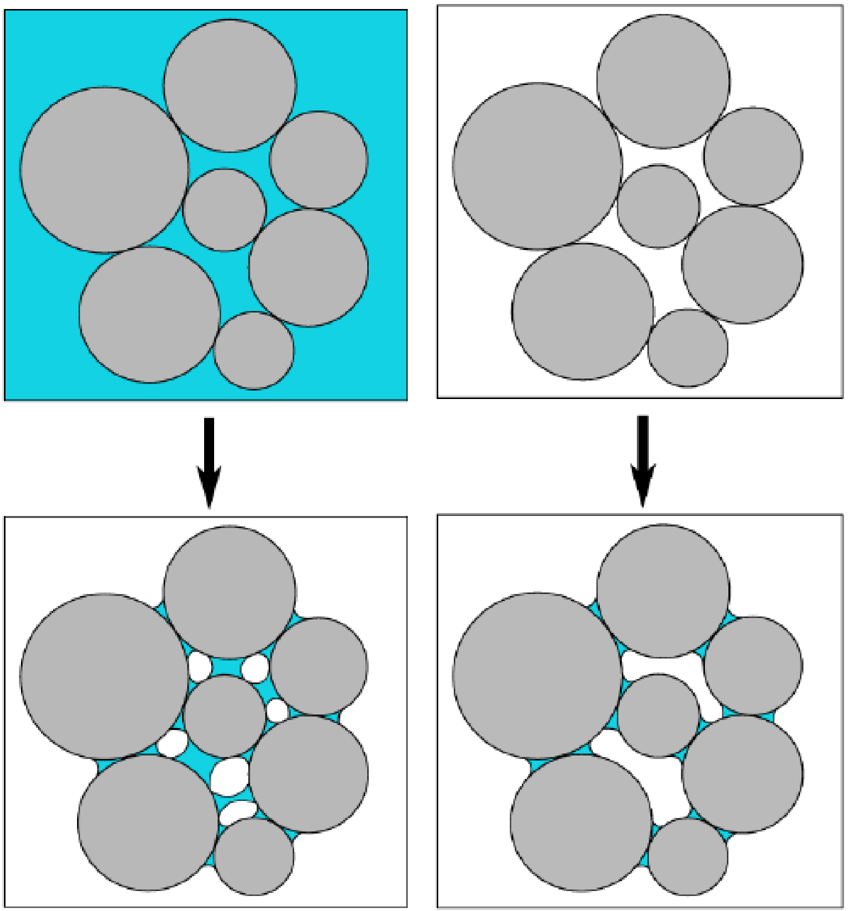
$$\Delta u = u_a - u_c = cst$$

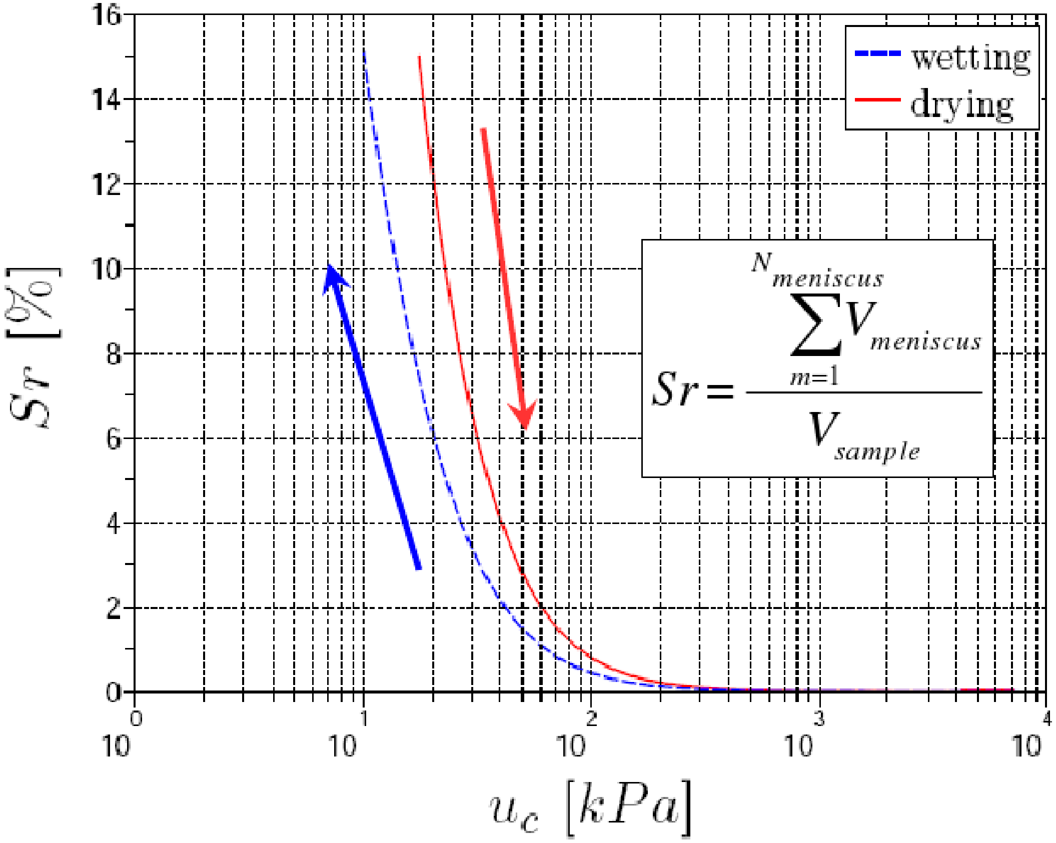




DEM simulations and results : suction variation

Water retention hysteresis:





Local hysteresis → « ink bottle effect »

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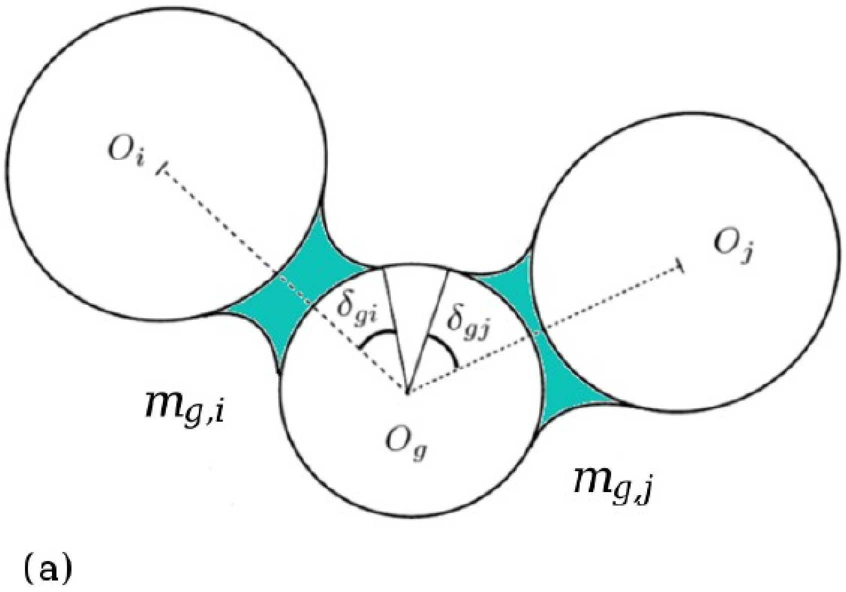
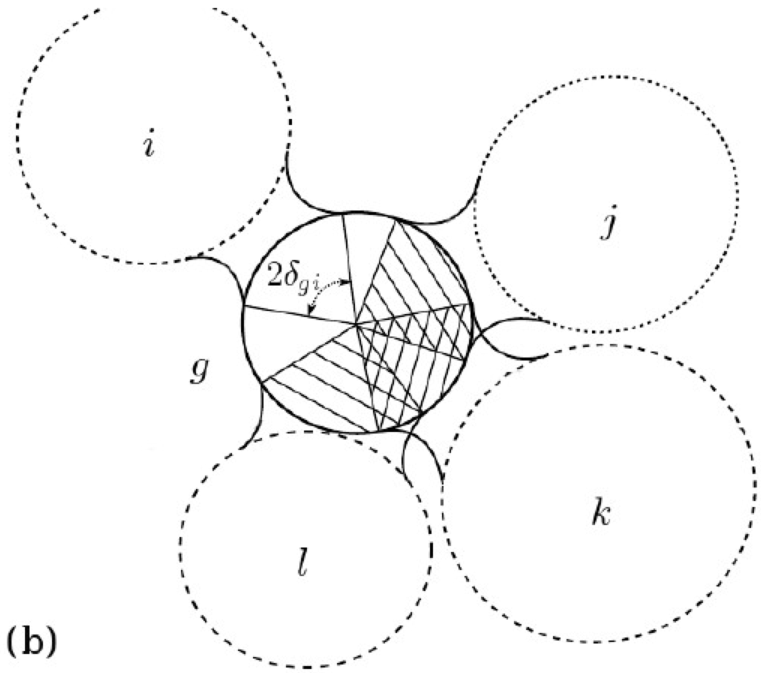
DEM simulations and results : suction variation

The range of simulated saturation degree





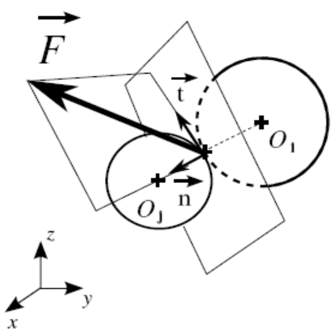
DEM modelling

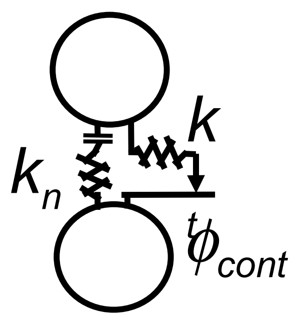
using YADE – open DEM ***(http://yade-dem.org)***

(based upon the pioneering work of Cundall and Strack, 1979)

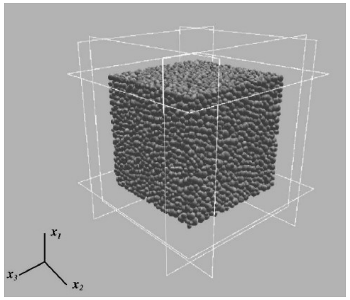
Resultant forces and moments

Force-Displacement Law
applied to each interaction





Law of Motion
(Newton's 2^{nd} Law)
applied to each particle



Positions and contacts update
(finite difference scheme)

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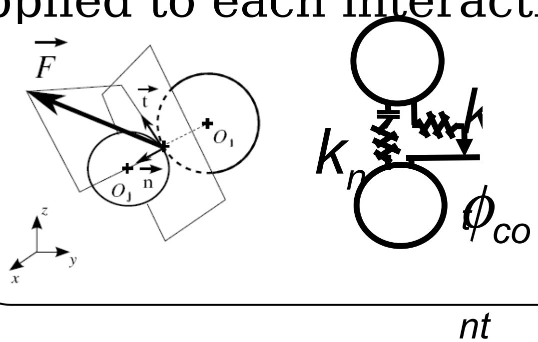
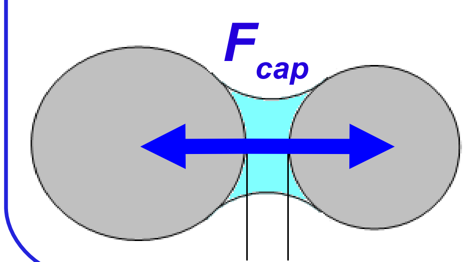


Capillary force



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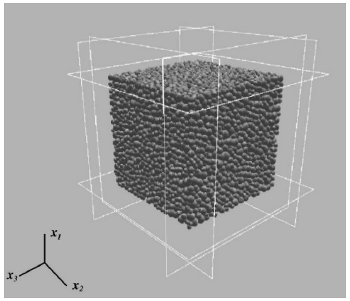
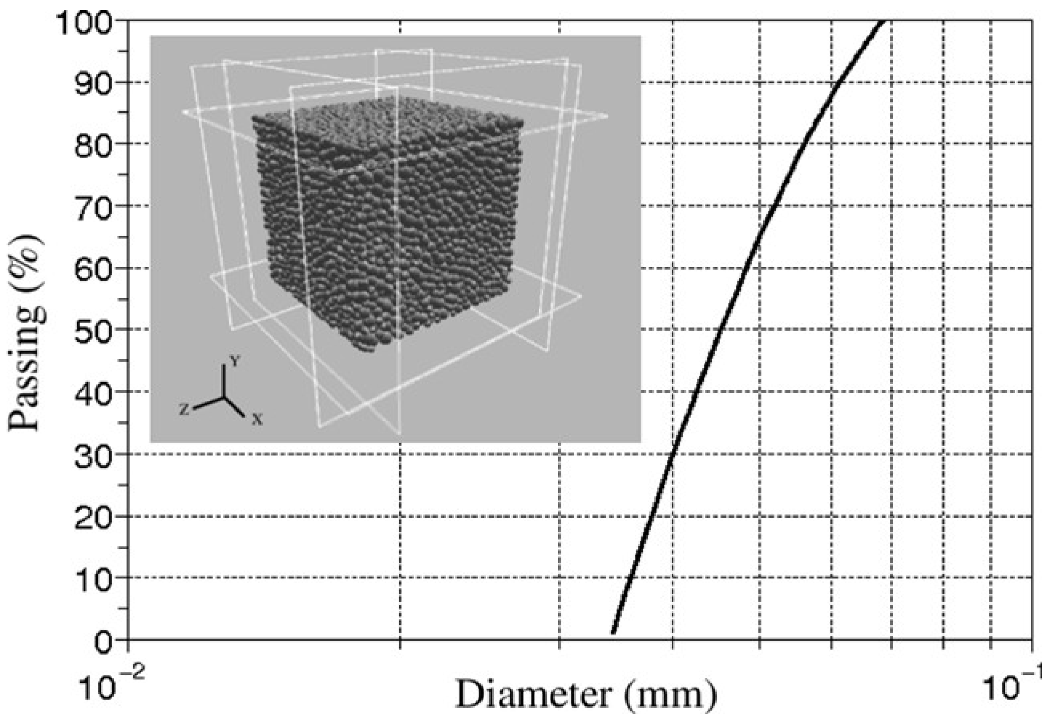
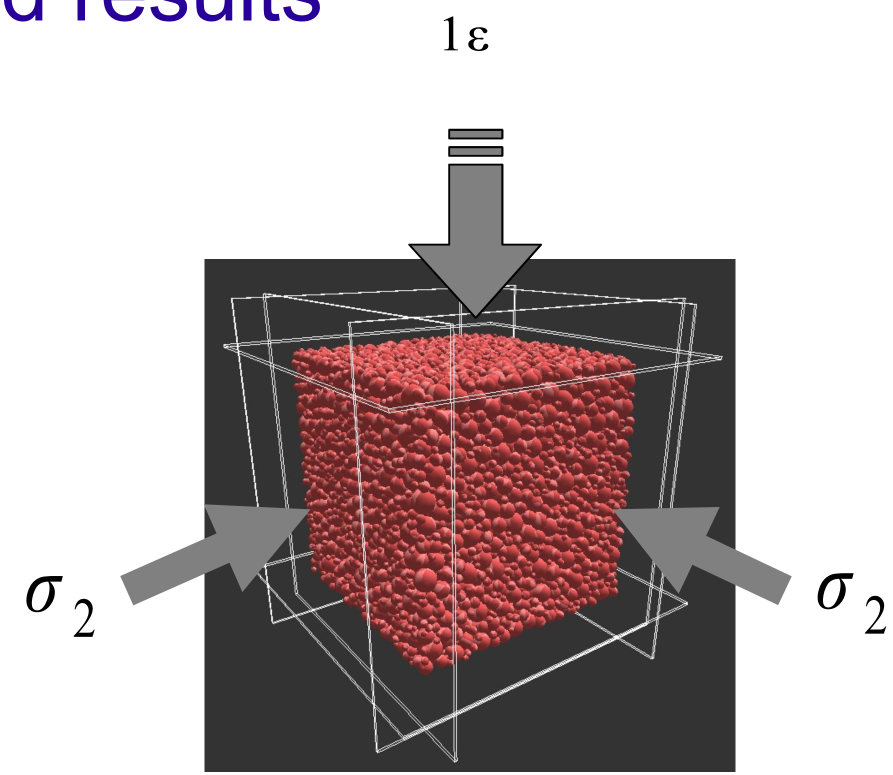


Positions and contacts update (finite difference scheme)

DEM simulations and results

DEM Sample :





- **10 000** spherical particles randomly positionned into a cubic box
- A **unique value of succion** in the sample (thermodynamic equilibrium)
- compacted through radius expansion to ensure the **isotropy** of the packing
- rigid frictionless boundary walls guarantee the **homogeneity** of the loading

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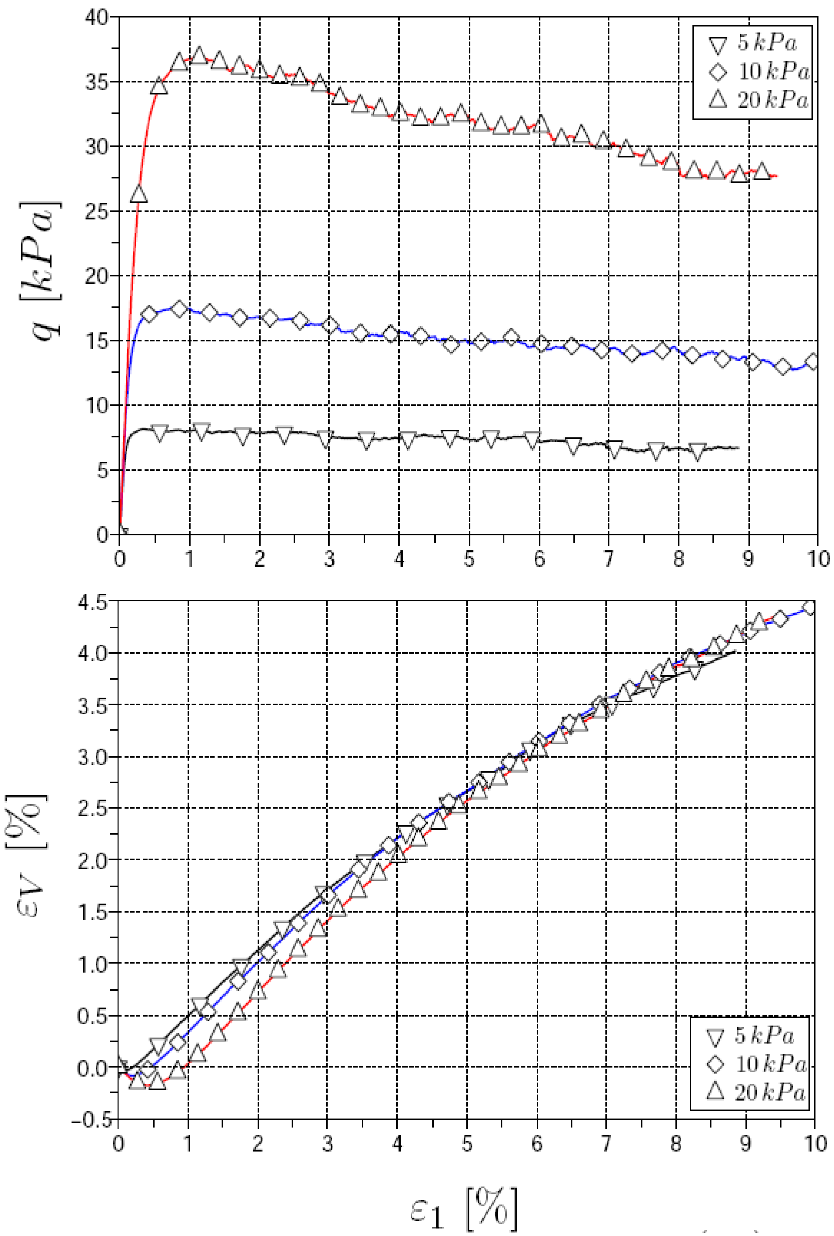
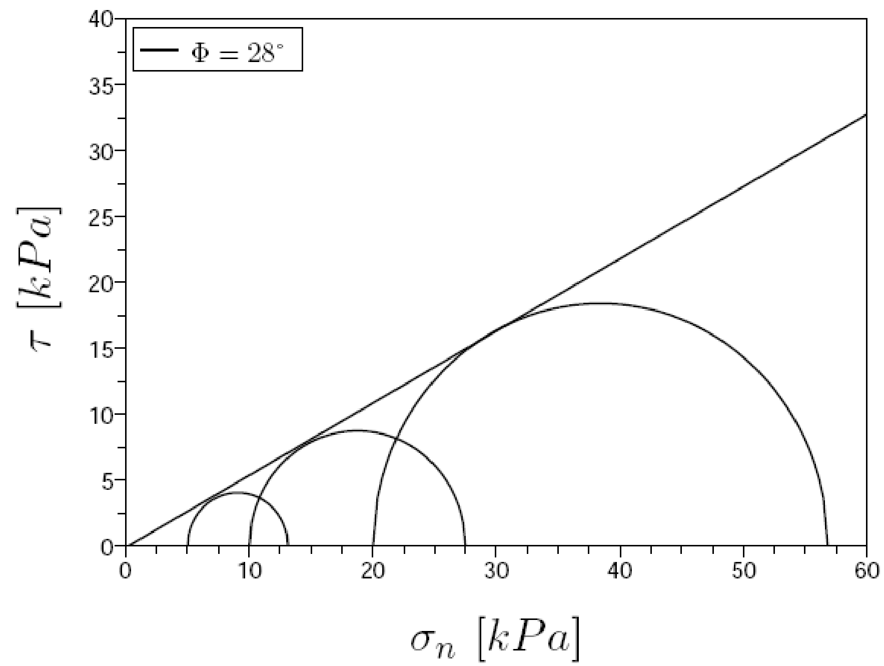
Triaxial loading : dry sample



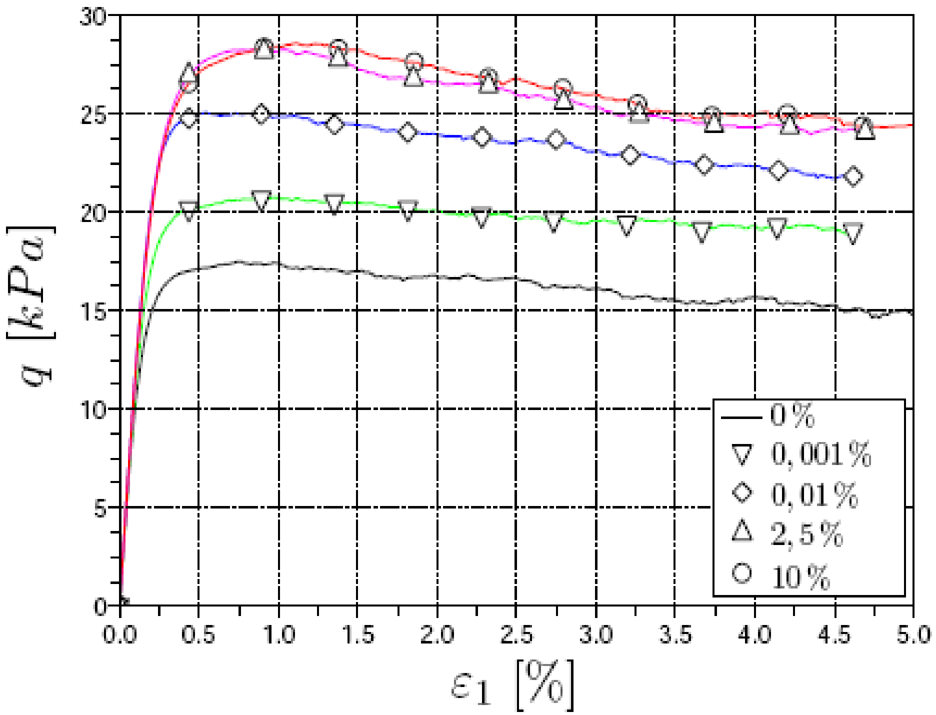
Nombre de grains	E_{global} (Pa)	k_n/k_t	ϕ_c (deg.)
10000	5.10^7	0.5	30

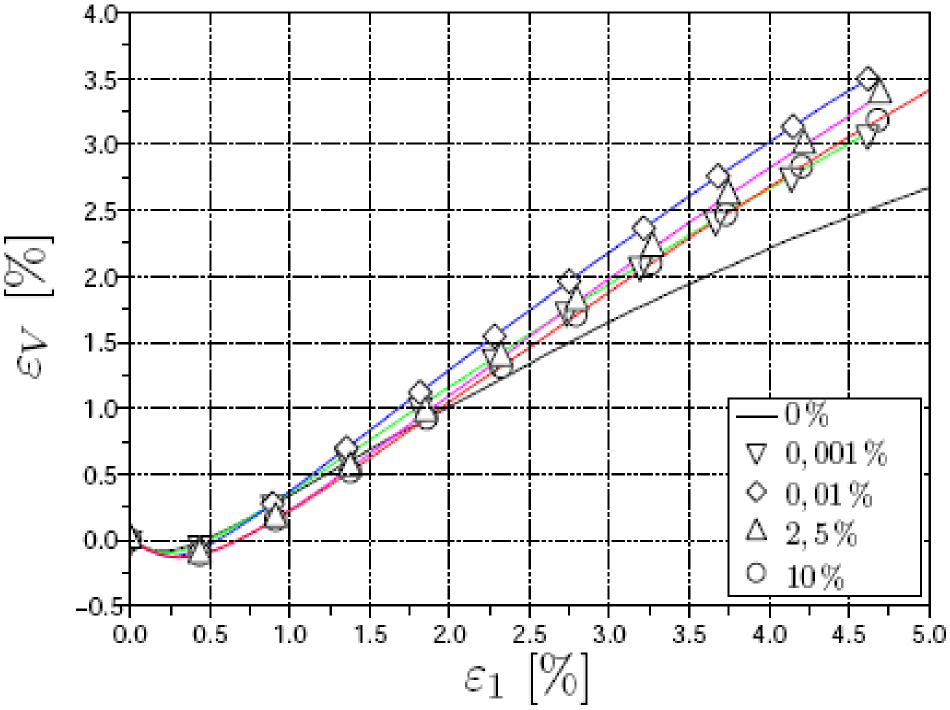


Triaxial loading : wet sample

For several capillary pressure $\left(u_a - u_w\right)$ in the pendular regime (0 < Sr < 12%)

u_c (kPa)	5000	3000	50	20
Sr_{init} (%)	0,001	0,01	2,5	10
w_{init} (%)	0,0006	0,006	1,5	6,0





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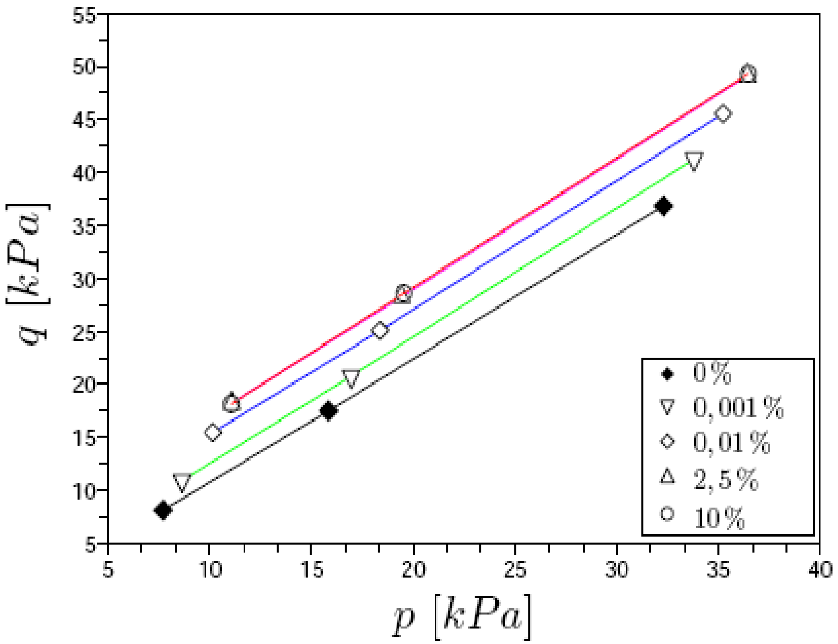
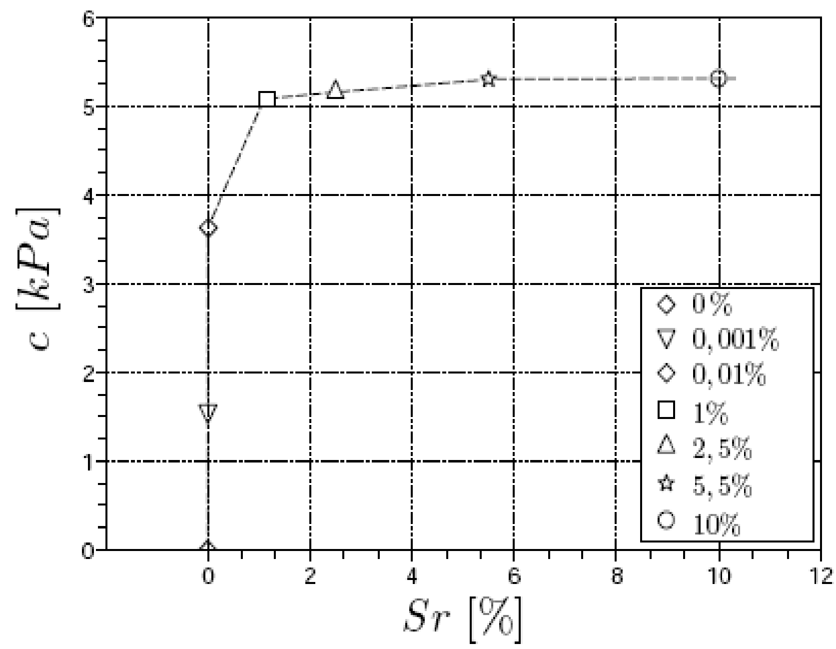
Triaxial loading : wet sample

Yield surfaces in the (q,p) plane



No significant changes
in the internal friction angle

Cohesion vs Saturation degree



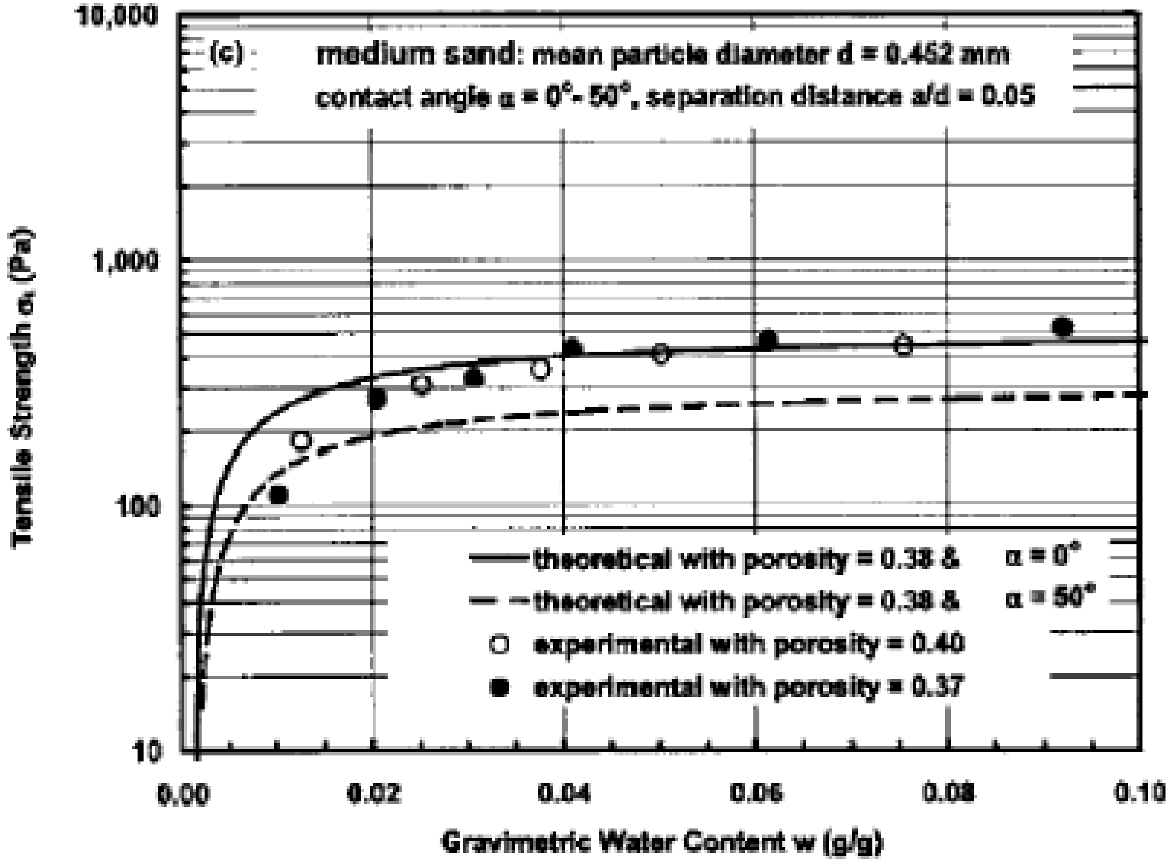
Compares well with experiments and
simulations done in Montpellier
(Richefeu and al.)

for <*D*> = *0.045* mm

c^{DEM} = *5 kPa*

Triaxial loading : wet sample

Quantative validation :

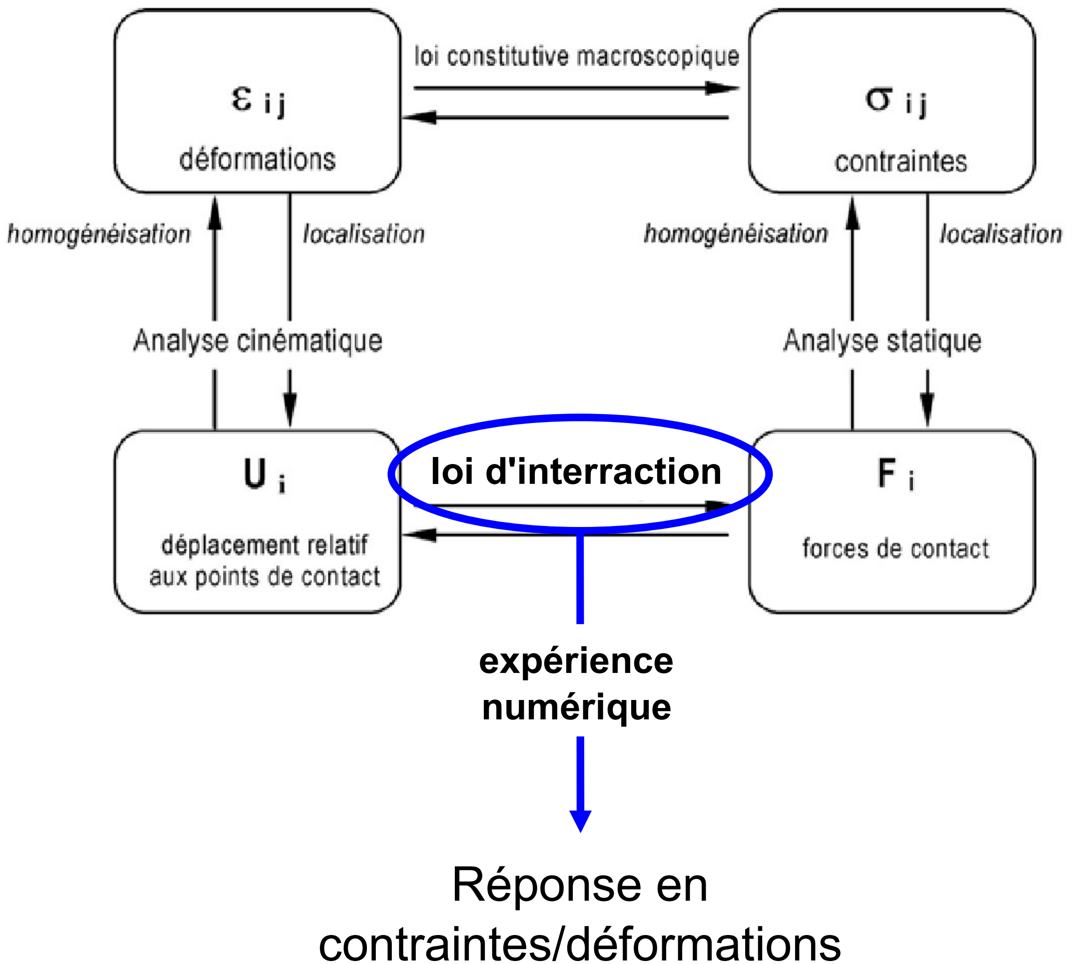


Lu et al., *Tensile strength of unsaturated sands*, J. of Geotech. and Geoenv. Eng. (2007)

$$\frac{\sigma_t(DEM)}{\sigma_t(Sand)} = \frac{\bar{D}(Sand)}{\bar{D}(DEM)} = \frac{0.45}{0.045}$$

A DEM model, so what?

Rôle dans le développement de modèles micromécaniques d'homogénéisation



En particulier : modèle micro-directionel de F. Nicot *(Sholtès et al. 2009a)*, modèle micro-structurel de Chang et Hicher *(Sholtès et al. 2009b)*

Grenoble INP

Effective stress

Effective stress tensor in saturated granular materials (Terzaghi 1936):

« All measurable effects of a change of stress of the soil, that is, compression, distorsion, and change of shearing resistance, are exclusively due to changes in the effective stress. »

$$\sigma_{ij} = \sigma'_{ij} + u_w . \delta_{ij}$$

Generalization for porous elastic materials Biot (1955) :

$$\sigma'_{ij} = \sigma_{ij} - \left(1 - \frac{C_s}{C}\right) u_w \delta_{ij}$$

Standard sand : E_y = 100 Mpa, ν = 0.2 – 0.4

Silice : E_y = 100 Gpa, ν = 0.16

Biot's alpha coefficient : 0.999 ~ 1

Effective stress

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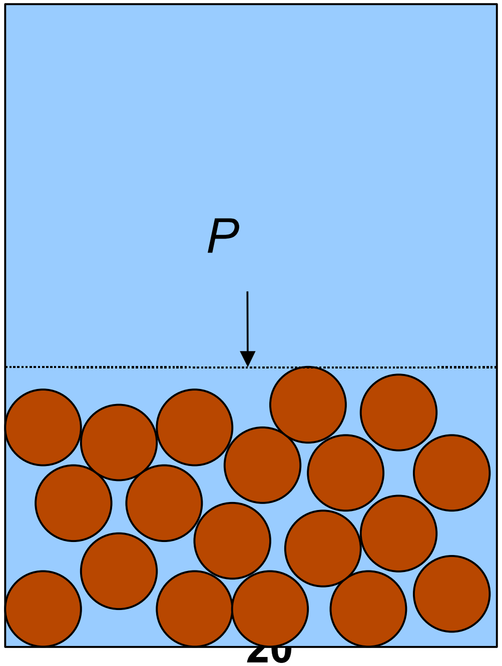


Generalized effective stress

Effective stress tensor in saturated granular materials (Terzaghi 1936):

« All measurable effects of a change of stress of the soil, that is, compression, distorsion, and change of shearing resistance, are exclusively due to changes in the effective stress. »

$$\sigma_{ij} = \sigma'_{ij} + u \, . \, \delta_{ij}$$

« Seating solely in the solid skeleton, the effective stress enter the constitutive equations of the soil matrix, linking a change in stress to strain-like quantity of the skeleton. [...] A unique stress is necessary and sufficient to describe the mechanical behaviour. »

[Nuth and Laloui , *"Effective stress concept in unsaturated soils: Clarification and validation of a unified framework"*, IJNAMG 2007]

Generalised effective stress

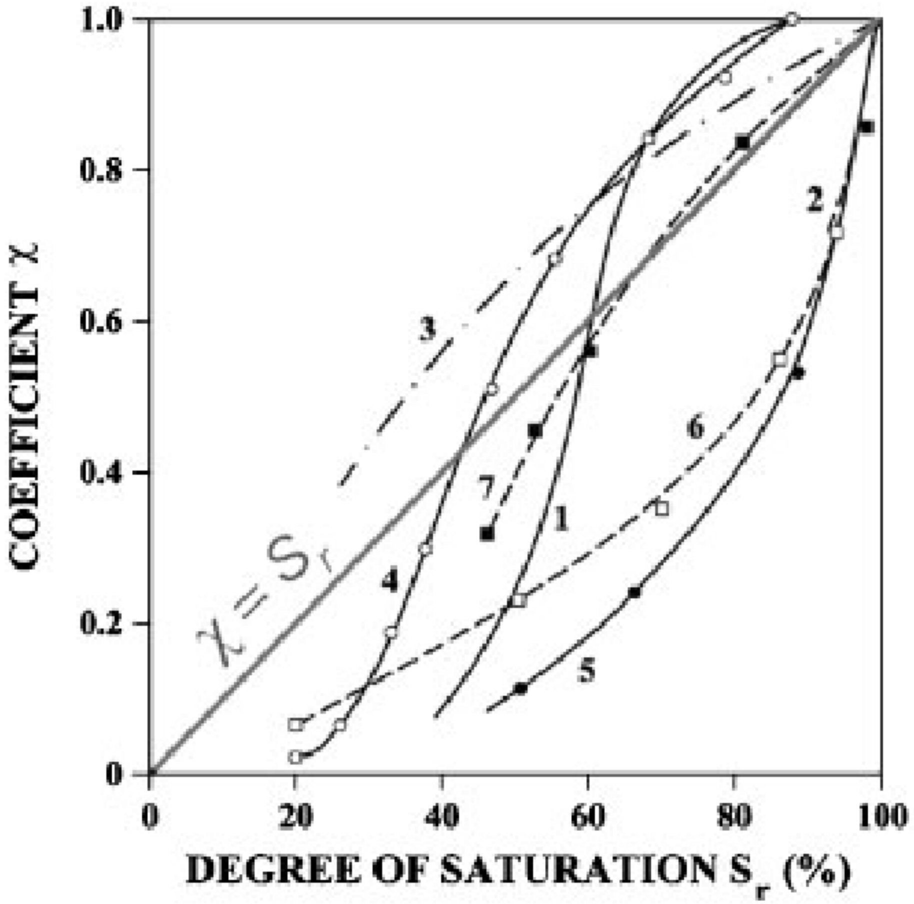
Partial saturation :
Bishop and Blight, Géotechnique (1963)

$$\sigma_{ij}' = \sigma_{ij} - \left(u_a + \chi\left(u_a - u_w\right)\right)\delta_{ij}$$

?

Generalised effective stress

A common assumption : $\chi = S_r$



1 COMPACTED BOULDER CLAY (Bishop et al, 1960)
2 COMPACTED SALE (Bishop et al, 1960)
3 BREAHEAD SILT (Bishop & Donald, 1961)
4 SILT (Jennings & Burland, 1962)
5 SILTY CLAY (Jennings & Burland, 1962)
6 STERREBEEK SILT (Zerhouni, 1991)
7 WHITE CLAY (Zerhouni, 1991)

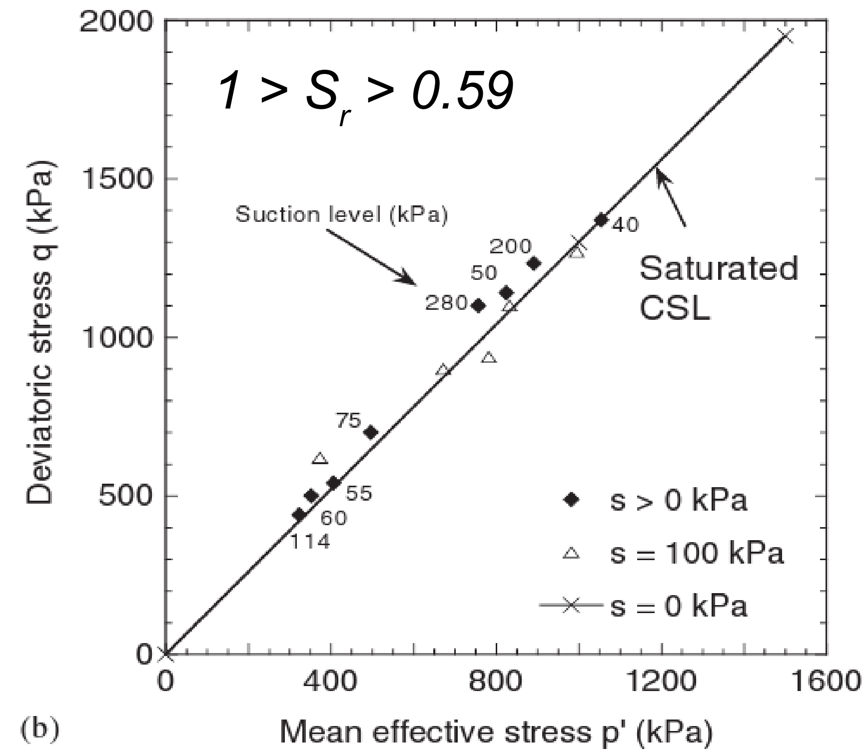
Generalised effective stress

Yield surfaces in the (p',q') plane :

$$\chi = \begin{cases} \left(\dfrac{s}{s_e}\right)^{-0.55} & \text{if } s > s_e \\ 1 & \text{if } s \leqslant s_e \end{cases}$$

Khalili and Khabbaz, Géotechnique (1998)
A unique relationship for χ...

The advantages of an effective stress is pointed out.

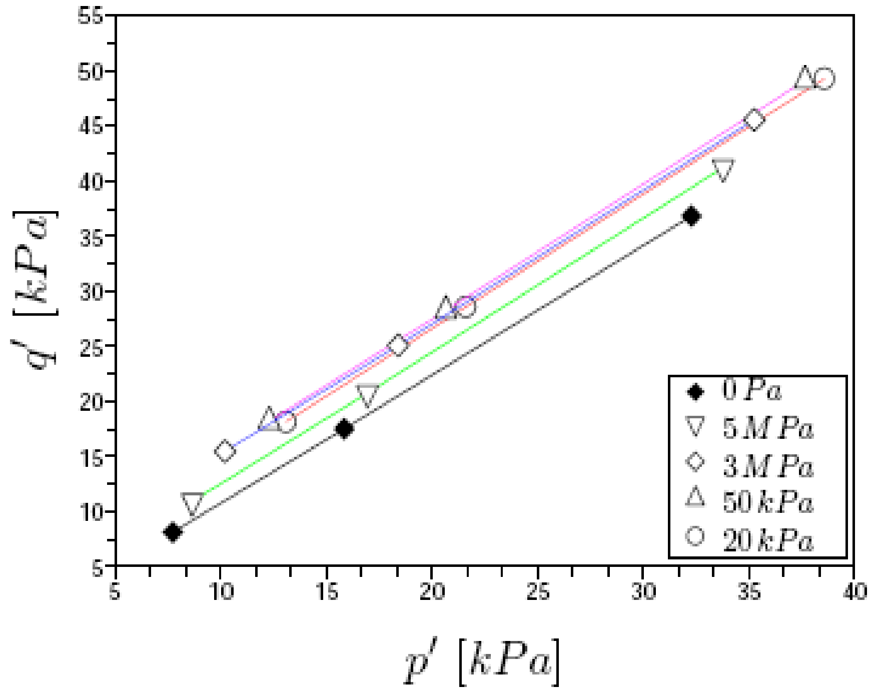


After Nuth and Laloui (2007)

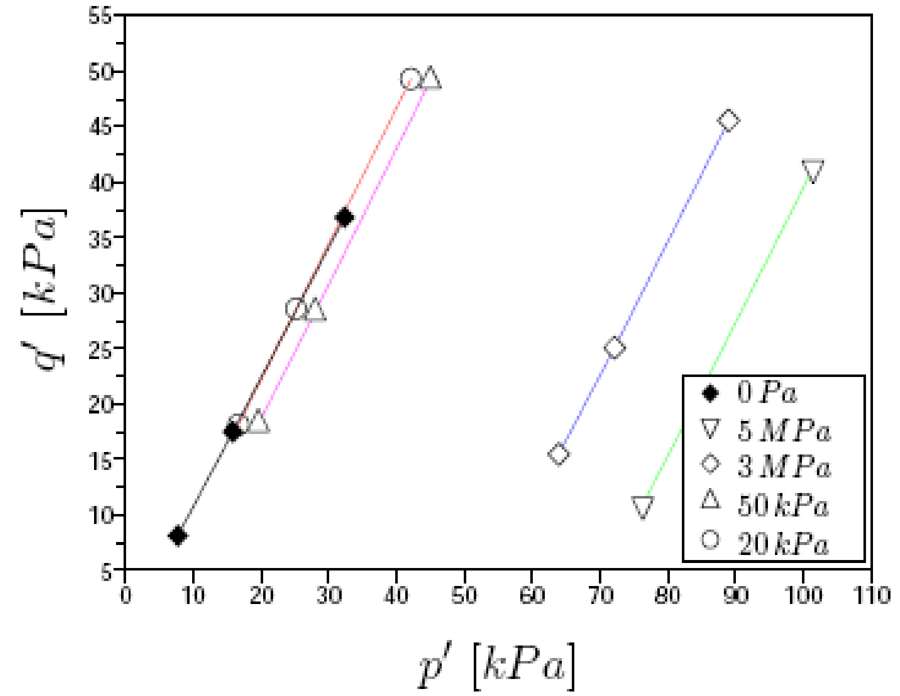
Generalised effective stress

Yield surfaces in the (p',q') plane for simulated low saturations :

$\chi = Sr$



Khalili and Khabbaz (1998)



None of the common definitions will result in a unique yield surface...

Generalised effective stress

Interpretation of the suction term as an additional confinement :

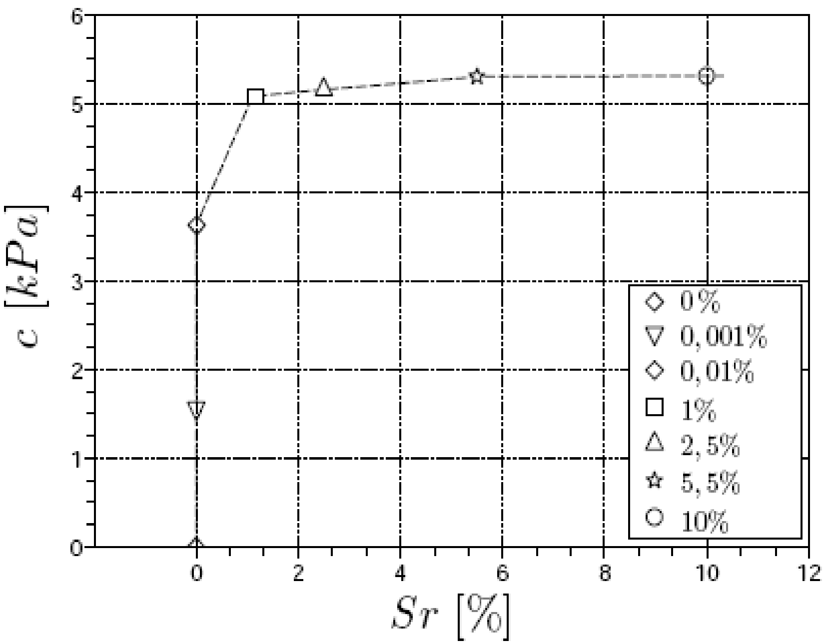
$$\sigma_{ij}{}' = \sigma_{ij} - \left(u_a + \chi\left(u_a - u_w\right)\right)\delta_{ij}$$

$$\chi = Sr$$

$$\chi = \begin{cases} \left(\dfrac{s}{s_e}\right)^{-0.55} & \text{if } s > s_e \\ 1 & \text{if } s \leqslant s_e \end{cases}$$

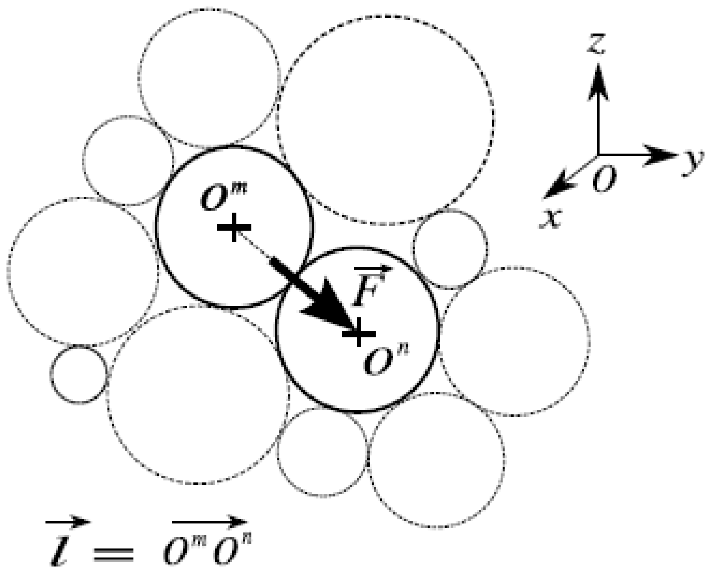
Khalili and Khabbaz, Géotechnique (1998)
A unique relationship for χ…

Cohesion vs Saturation degree

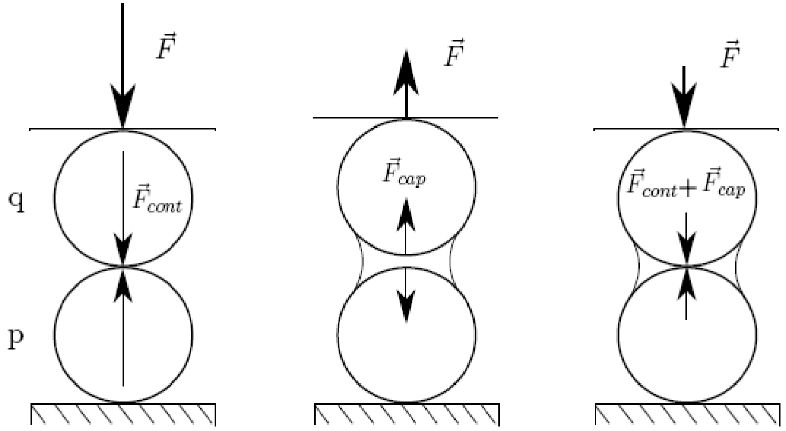


→ 10 kPa < s < 10^4 kPa !!

Generalised effective stress : micromechanical definition



on each particle n of the assembly :



$$\vec{F} = \vec{F}_{cont} + \vec{F}_{cap}$$

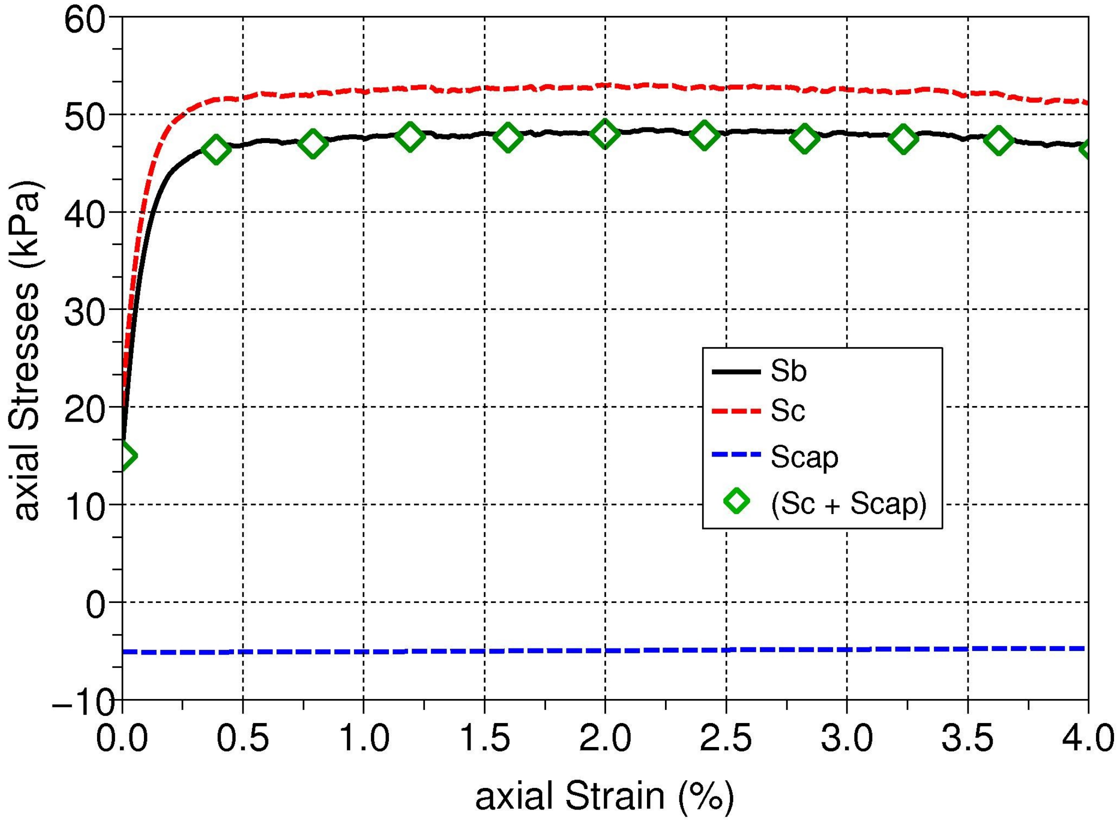
Cauchy stress tensor by homogenisation : [Love, 1927]

$$\sigma_{ij} = \frac{1}{V} \sum_{c=1}^{N_{contacts}} F_i^{cont} l_j + \frac{1}{V} \sum_{m=1}^{N_{menisci}} F_i^{cap} l_j$$

$$\Rightarrow \quad \sigma = \sigma_{contact} + \sigma_{capillary}$$

Generalised effective stress : micromechanical definition

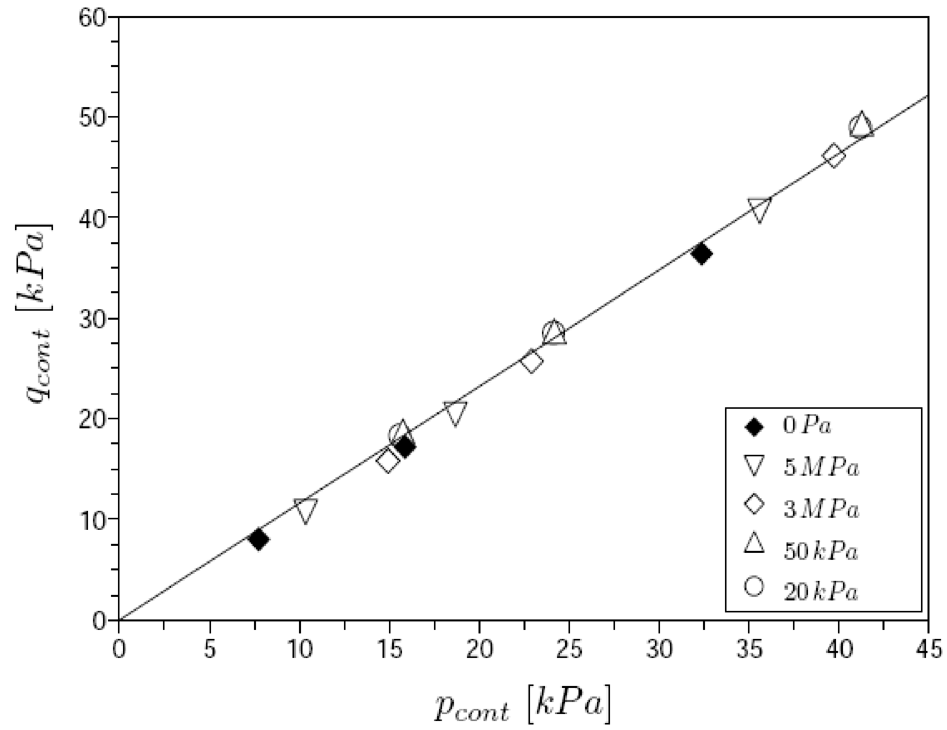
$$\sigma_{ij} = \sigma_{ij}^{c} + \sigma_{ij}^{cap}$$



Generalised effective stress : numerical results

Possible definition of the effective stress : $\sigma_{ij}^{contact} = \frac{1}{V} \sum_{c=1}^{N_{contacts}} F_i^{cont} \, l_j$

Yield surfaces in the (p^{cont}, q^{cont}) plane :



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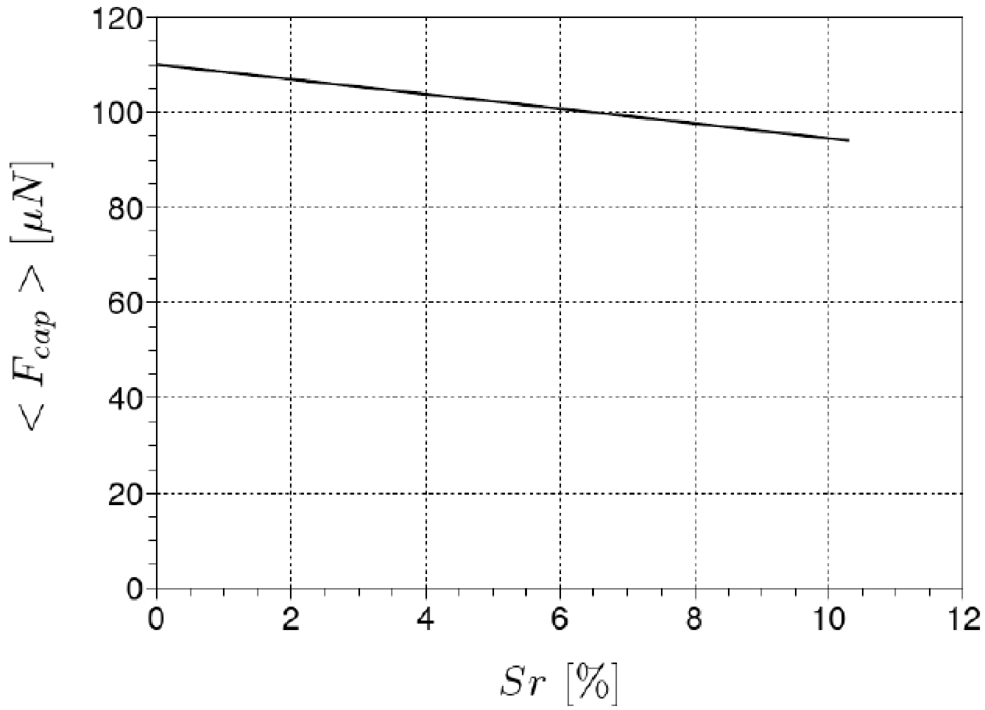
Generalised effective stress : numerical results

$$\sigma_{ij}^{cap} = \frac{1}{V} \sum_{m=1}^{N_{menisci}} F_i^{cap} l_j$$

Provides an explanation of the plateau in *c* vs. S_r curves. As suggested in e.g. Richefeu et al. (2006), the magnitude of capillary effects scales like :

$$\sigma_t = \frac{3}{4\pi} \frac{s\kappa\Theta z_m}{D_{grains}}$$



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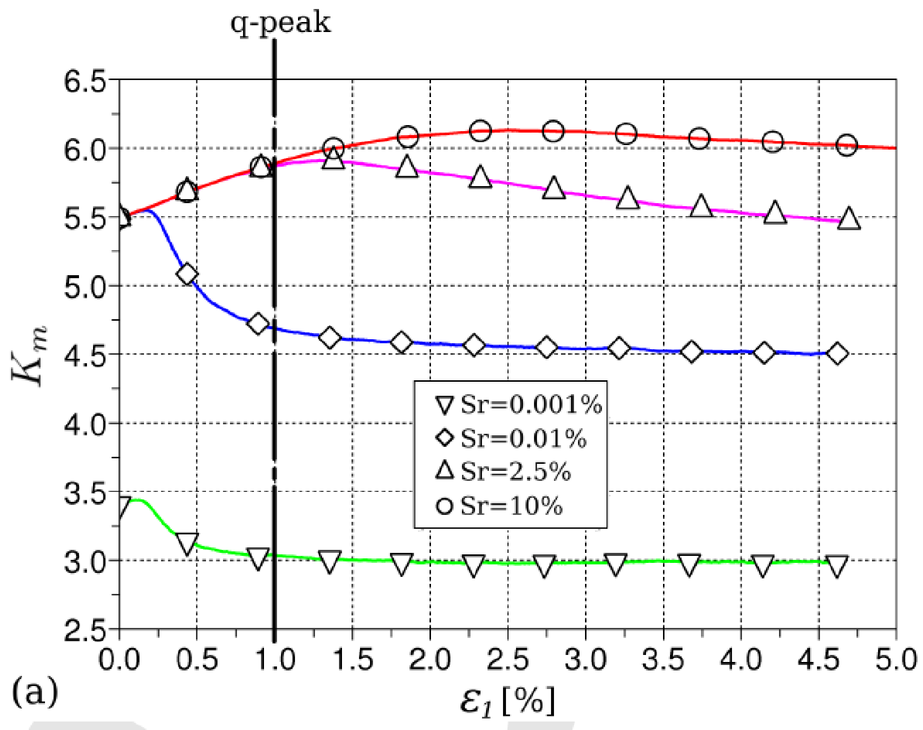
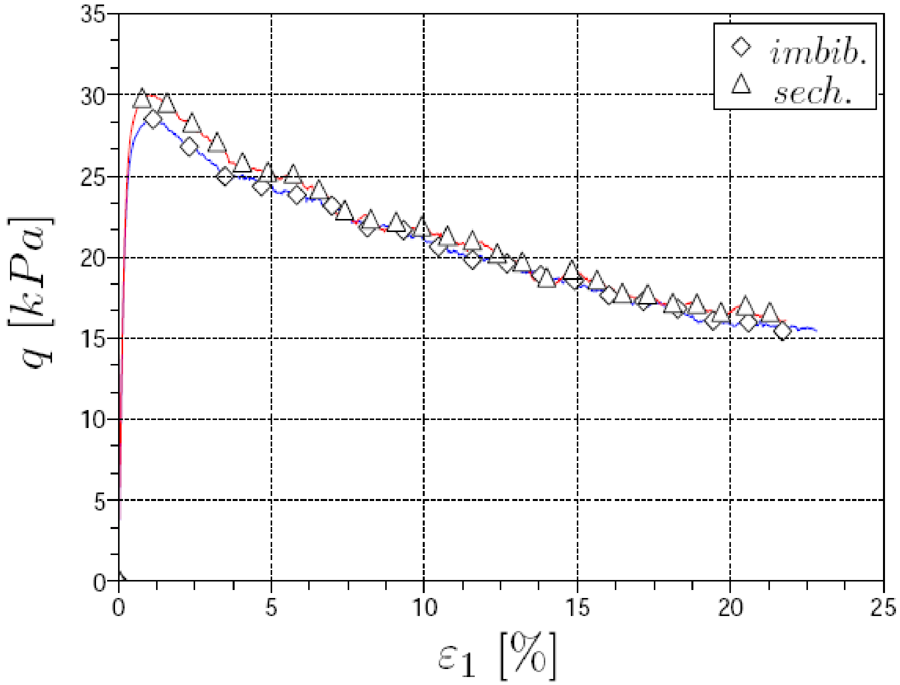


(a)

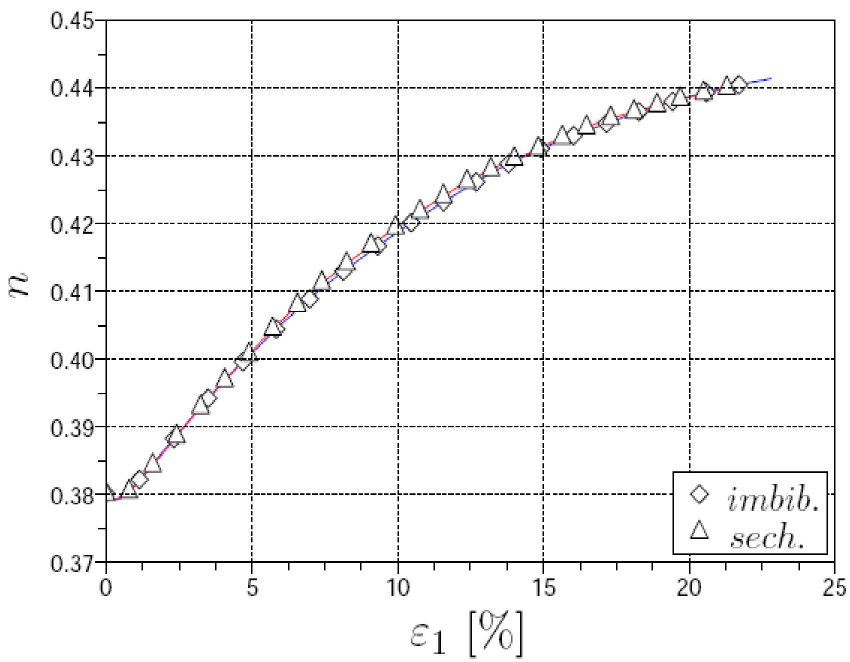
Generalised effective stress : numerical results

« Wetting » vs. « drying » initial states :



The shear strength is larger in the drying phase than in the wetting one

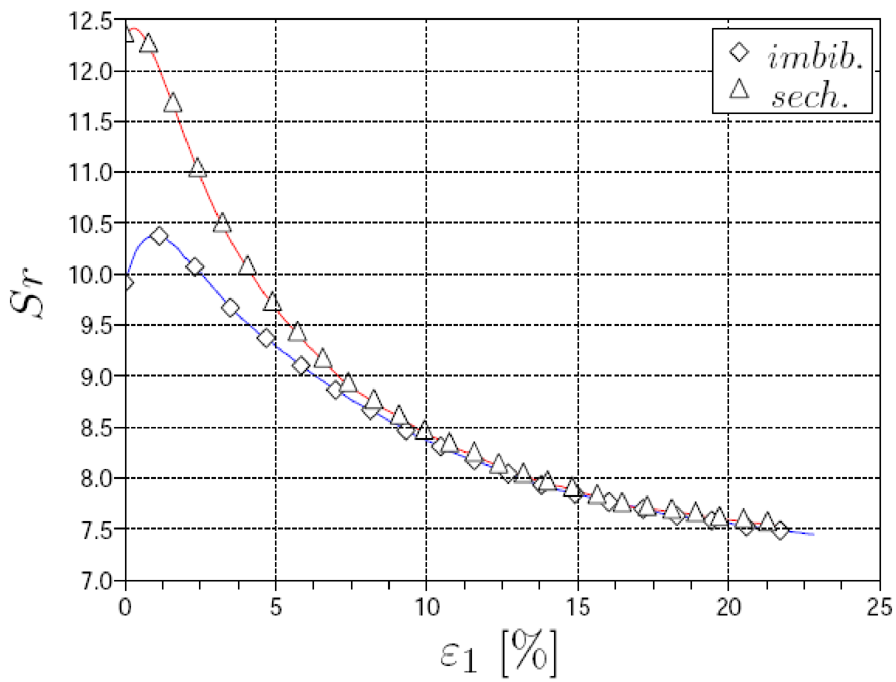
Common residual stress state



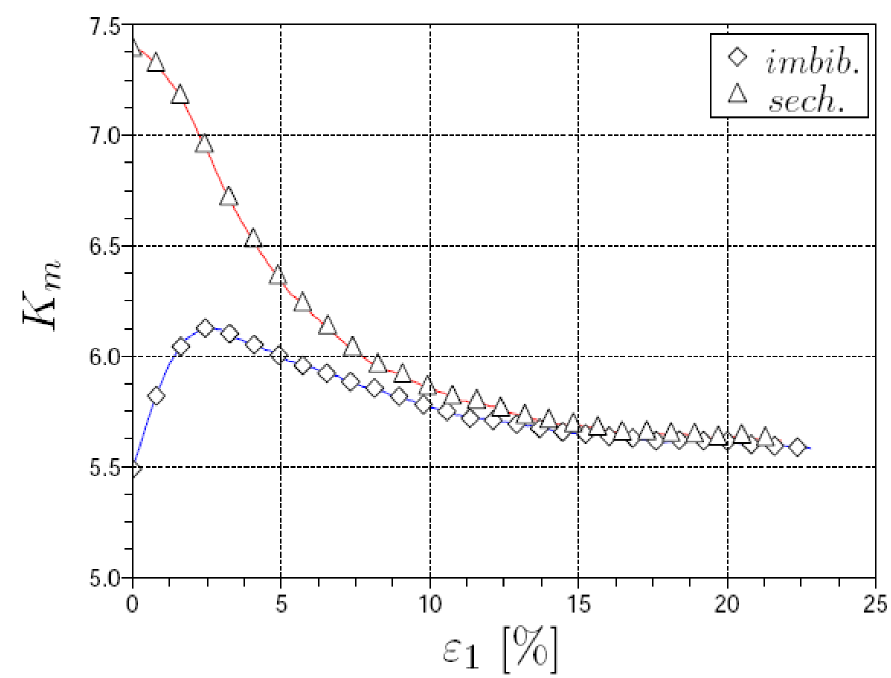
No significant changes in the volumetric strain
(nor in the internal friction angle)

Generalised effective stress : numerical results

« Wetting » vs. « drying » initial states :



The difference in the shear strength is linked to the number of liquid bridges inside the sample



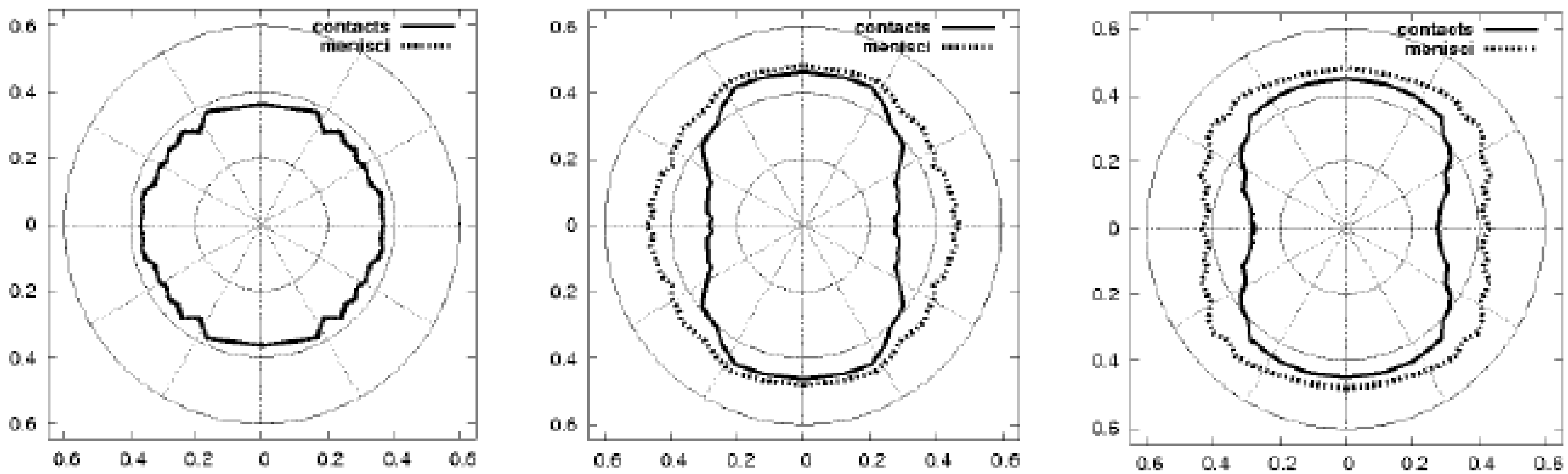
The liquid bridges tend to the same distribution with the deformations

Generalised effective stress : numerical results

The usual formalism fail to describe the anisotropy of the fluid contribution :

$$\sigma_{ij}^{contact} = \sigma_{ij} - \sigma_{ij}^{capillary} \quad \Leftrightarrow \quad \sigma_{ij}' = \sigma_{ij} + \chi (u_a - u_w) \delta_{ij}$$

Contacts and Menisci orientation distributions



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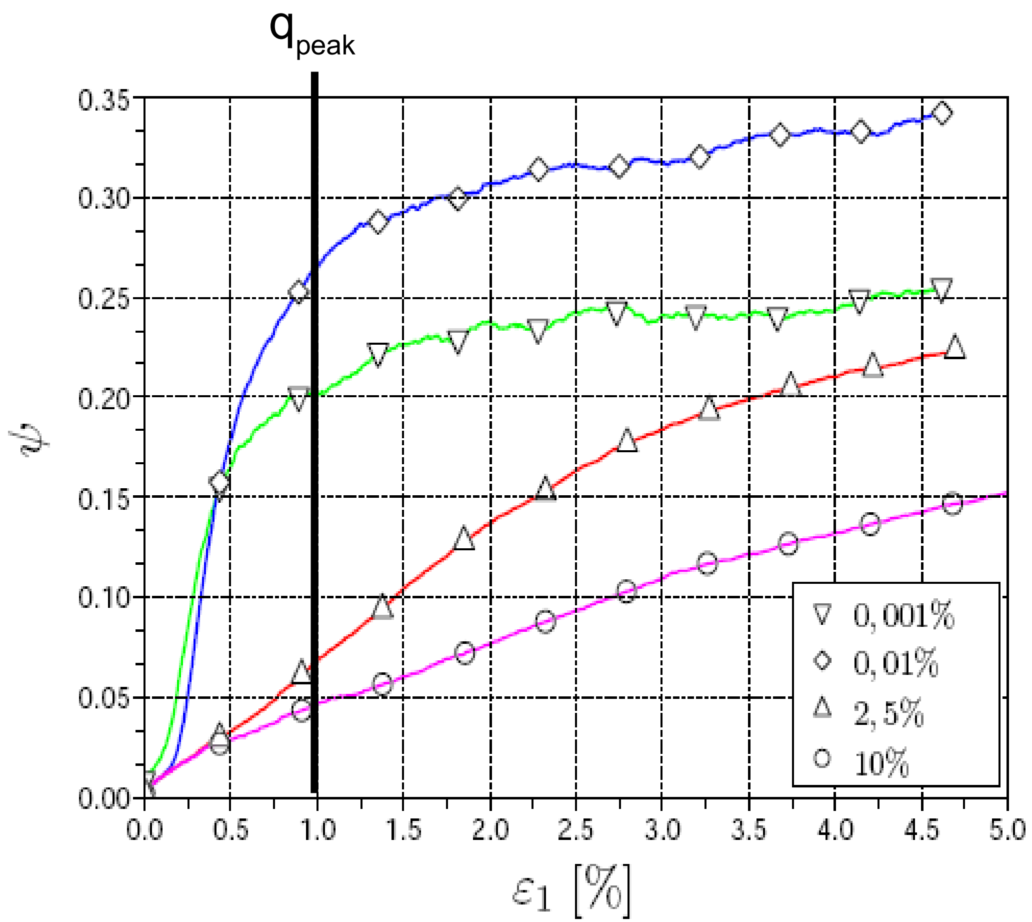


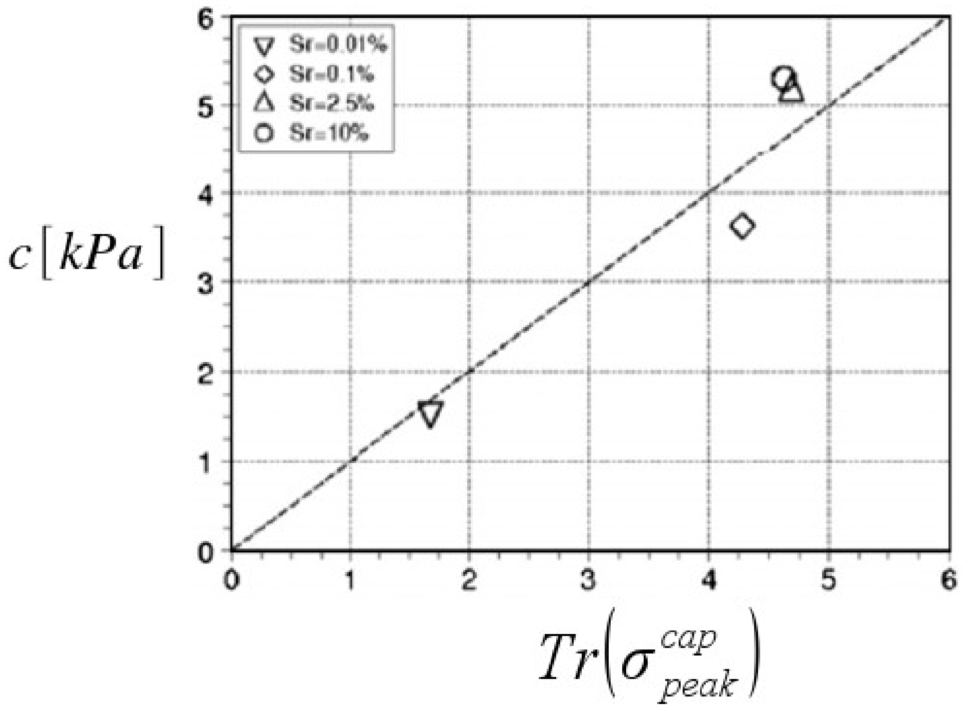
$$\Psi = 2\frac{\left(\sigma_1^{cap} - \sigma_2^{cap}\right)}{\left(\sigma_1^{cap} + \sigma_2^{cap}\right)}$$

$$\Psi = 0 \quad if \quad \sigma_{cap} = \chi\left(u_a - u_w\right)$$

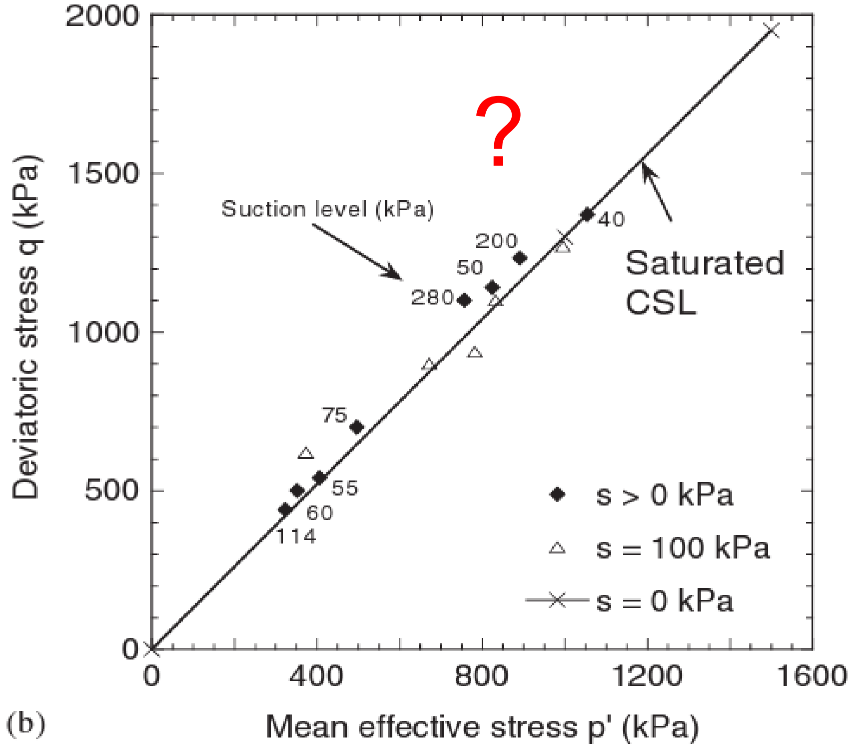
Generalised effective stress : numerical results

The usual formalism fail to describe the anisotropy of the fluid contribution :

$$\sigma_{ij}^{contact} = \sigma_{ij} - \sigma_{ij}^{capillary} \Leftrightarrow \sigma_{ij}' = \sigma_{ij} + \chi \left(u_a - u_w \right) \delta_{ij}$$



Khalili and Khabbaz (1998)



Generalised effective stress : discussion

After the thermodynamical approach of Gray and Schrefler (2006) :

$$\mathbf{t}^{\text{Total}} = \left(1 - \frac{K_T}{K_S}\right) (\mathbf{n}_s \cdot \mathbf{t}_s \cdot \mathbf{n}_s)^s + \epsilon^s \boldsymbol{\tau}^s$$

with :

$$-(\mathbf{n}_s \cdot \mathbf{t}_s \cdot \mathbf{n}_s)^s = x_s^{ws} p_{ws}^w + (1 - x_s^{ws}) p_{ns}^n - \frac{l^{wns}}{a^s} \gamma_{wns}^{wn} \sin \psi^w$$

- x_s^{ws} : solid-liquid surface ratio (not volume fractions);
- last term reflects direct effects of surface tension
- still, the effect of the fluids are isotropic

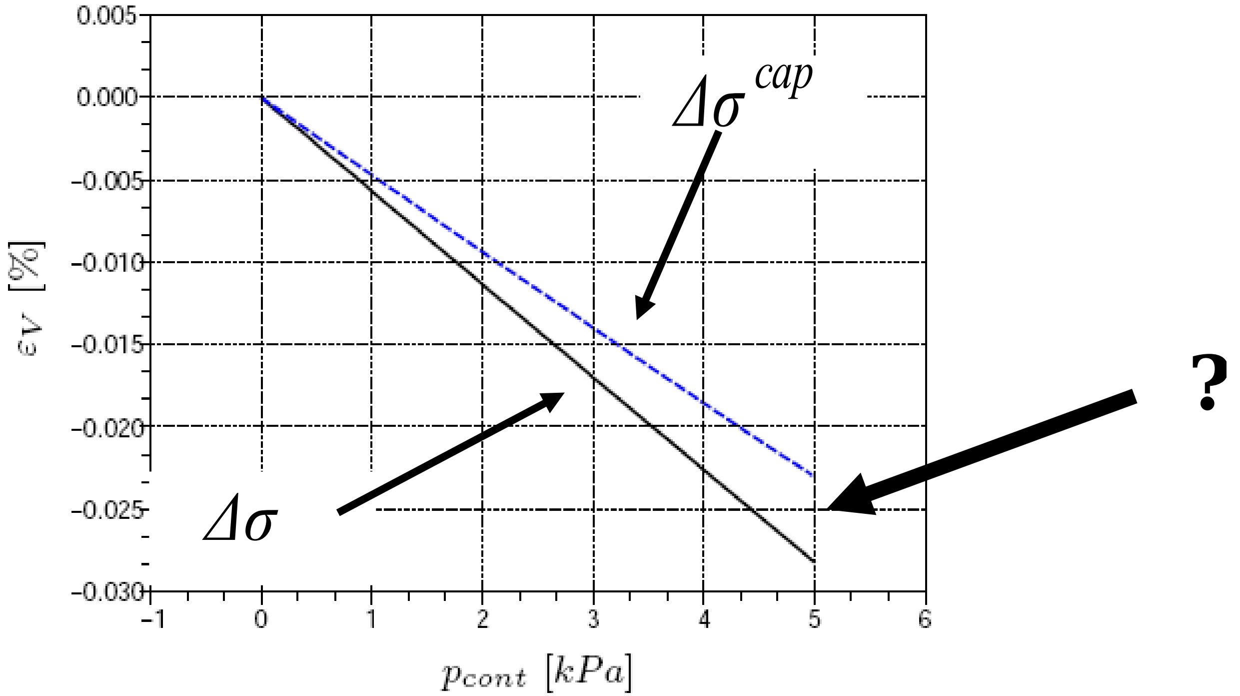
- ✓ In the same way as frictional phenomena, water effects at low saturation degrees are adequately modelised at the grains scale.
- ✓ Capillary forces generate an apparent cohesion, which compares well with measured values.
- ✓ The mechanical behaviour of the sample is almost constant on the range of saturation degree [2%,10%].
- ✓ The contribution of the liquid in the effective stress is anisotropic.
- ✓ Capillary forces homogeneized using Love-Weber stress provides a relevant quantity to describe the effect of capillary forces.

Generalised effective stress : micromechanical results

However…

$$\sigma_{ij}^{contact} = \frac{1}{V} \sum_{c=1}^{N_{contacts}} F_i^{cont} \, l_j = \sigma_{ij} - \sigma_{ij}^{cap}$$

Elastic response to isotropic compression ($\Delta\sigma$) vs Wetting ($\Delta\sigma^{cap}$)

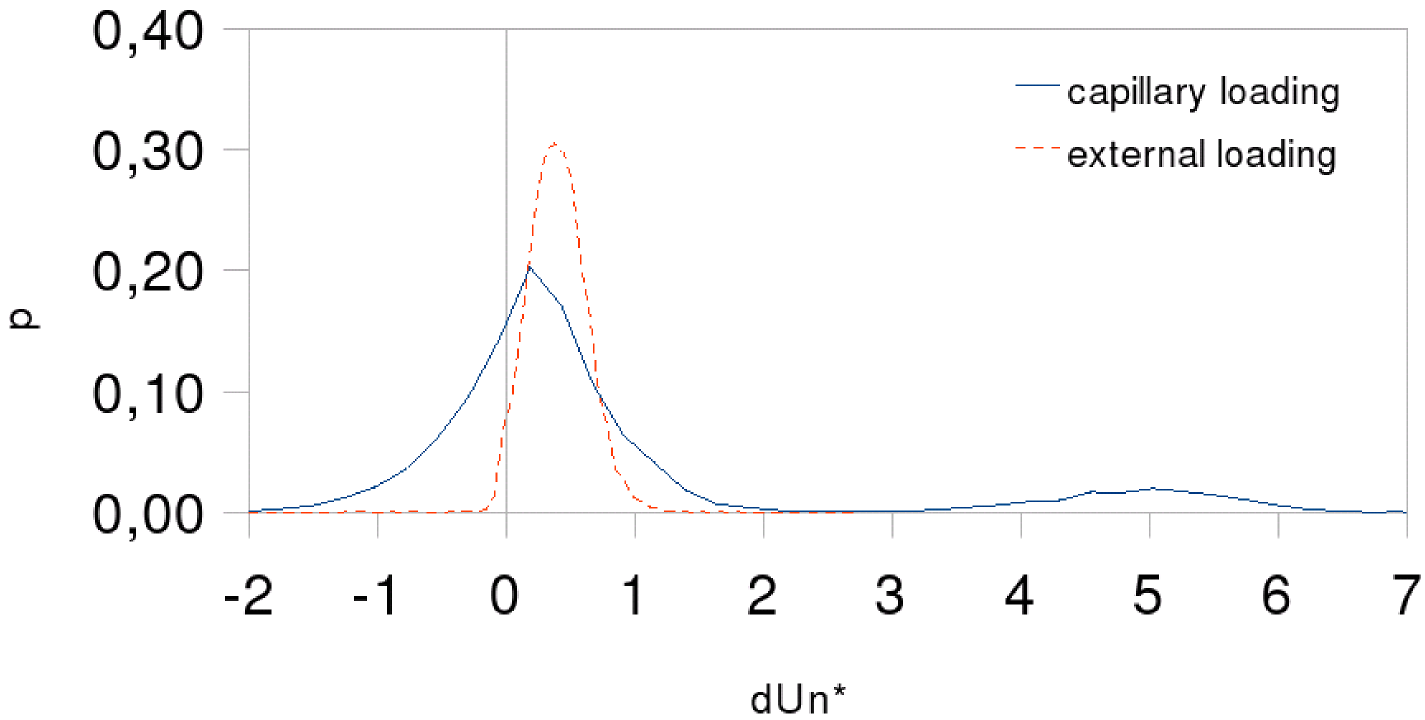


Generalised effective stress : micromechanical results

$$\sigma_{ij}^{contact} = \frac{1}{V} \sum_{c=1}^{N_{contacts}} F_i^{cont} \, l_j = \sigma_{ij} - \sigma_{ij}^{cap}$$

Local kinematics are different :

P.D.Fs. of normal displacement at contacts



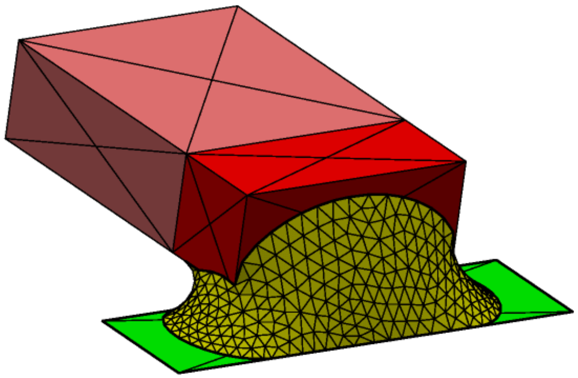
Capillarity in Unsaturated Granular Materials

In the same as frictional phenomena, water effects at low saturation degrees are adequately modelised at the grains scale as a result of capillary menisci

A multi-scale approach to analyse water induced phenomena then appears as a pertinent tool for critical examination of constitutive models.

Capillarity in Unsaturated Granular Materials

Challenges:



http://www.susqu.edu/facstaff/b/brakke/evolver/evolver.html

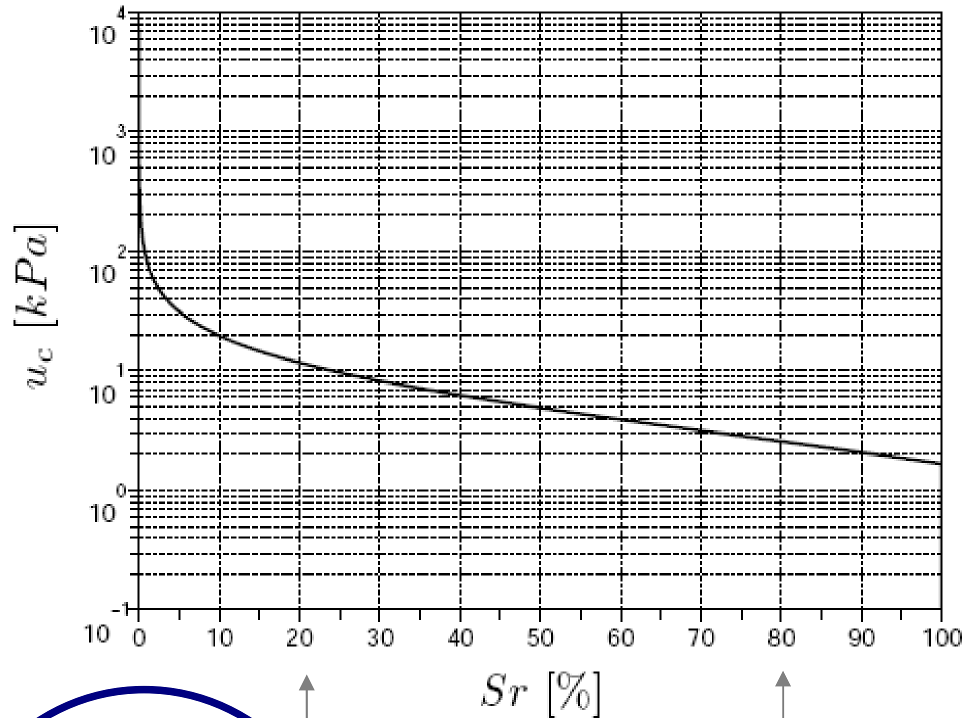
- range of water content: shape of the liquid bridges between 3, 4, N particles....

- kinetics: interfaces, transfers, variable wetting angle.

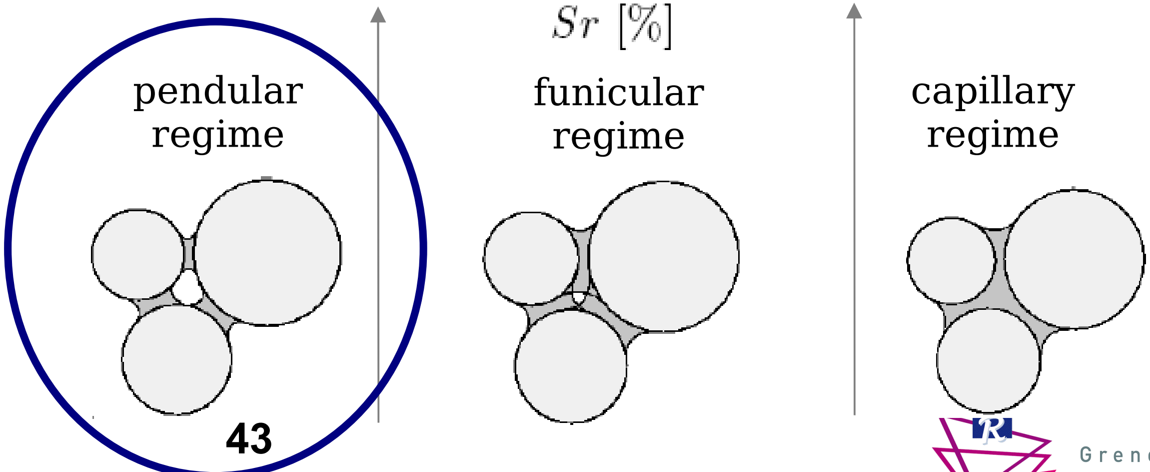
- constitutive macro-modeling

Suction Variation under isotropic loading : Wetting

$$Sr = \frac{\sum_{m=1}^{N_{meniscus}} V_{meniscus}}{V_{sample}}$$



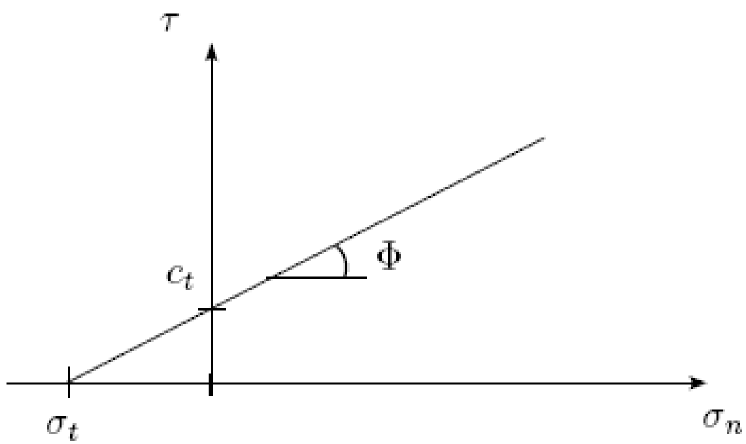
Hydric Domains:



Quantitative validation :

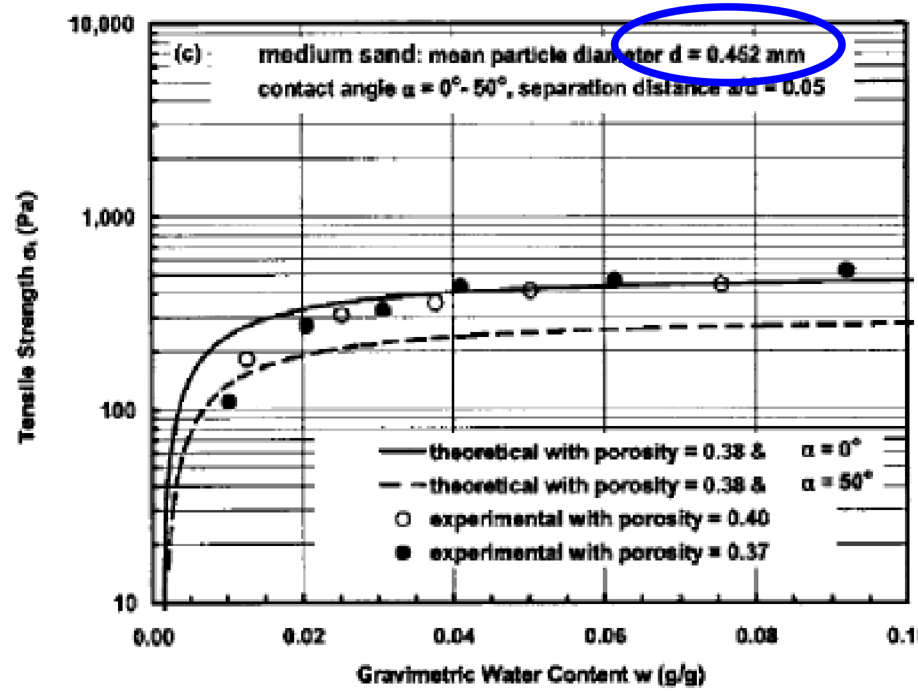
Tensile strength :

$$\sigma_t = \frac{c}{\tan\varphi} \propto \frac{1}{D_{grains}}$$



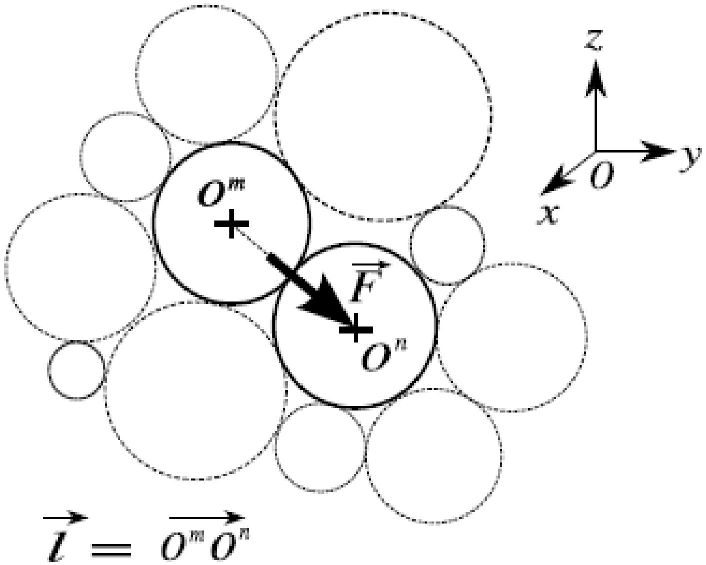
$$\sigma_t = \frac{3}{4\pi} \frac{s\kappa\Theta z_m}{D_{grains}}$$

Richefeu et al., Physical Review E (2006)
Shear strength properties
of wet granular materials,

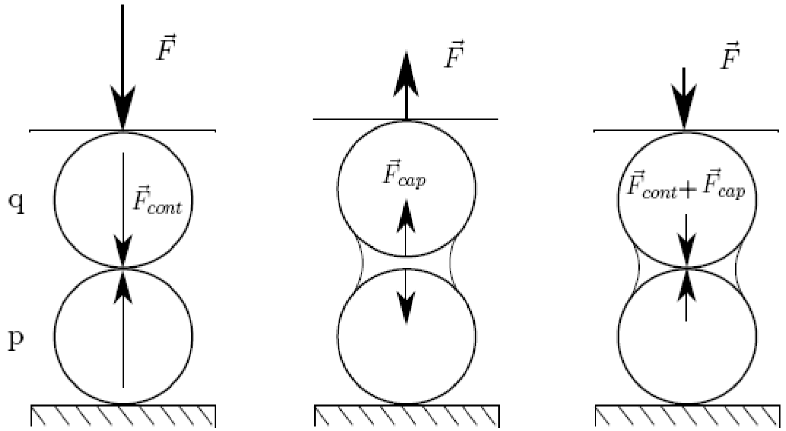


$$\frac{\sigma_t(DEM)}{\sigma_t(Sand)} = \frac{\bar{D}(Sand)}{\bar{D}(DEM)} = \frac{0.45}{0.045}$$

Generalised effective stress



on each particle *n* of the assembly :



$$\vec{F} = \vec{F}_{cont} + \vec{F}_{cap}$$

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