Plan of the rest of this talk ..

- **What Mechanics (CM, DEM, SM) tells us about length scales of observed patterns in granular materials**
- Pattern recognition from Complex Systems Theory and what patterns teach us about the nature of complex systems

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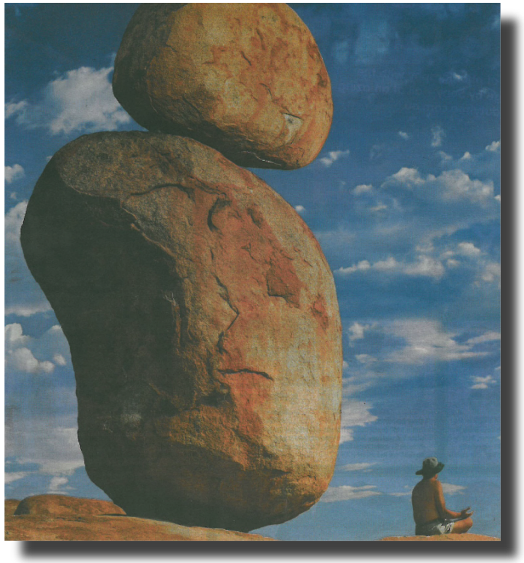
- Extraction of length scales from Grenoble data on Hostun sand
- Results from extraction
- Inception of Hostun sand and the null hypothesis to test length scales are robust, meaningful and **real**
- Results from inception
- Lessons learned and where to next …

Baseline system ...



Testing rolling resistance .. (Devil's marbles, Western Australia)

- 2D DEM biaxial test constant confining pressure (single shear band)
- 5,098 polydisperse particles
- Circular shape with rolling resistance added to regulate rotations
  - rolling resistance (or contact moment) is a spring-slider
  - parameters calibrated against photoelastic disk experiments
  - extensive parametric analysis to check trends are robust
  - battery of tests* check trends are robust w.r.t. photoelastic disk experiments and DEM simulations of particles with varying irregular shapes in 2D and 3D (undertaken with Fernando Alonso-Marroquin, Mark Hopkins, John Peters, Johannes Wibowo).
  - *Tests: biaxial-constant volume, biaxial-constant p, flat punch test, triaxial, simple shear etc

Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Continuum Theory to the first ring ..

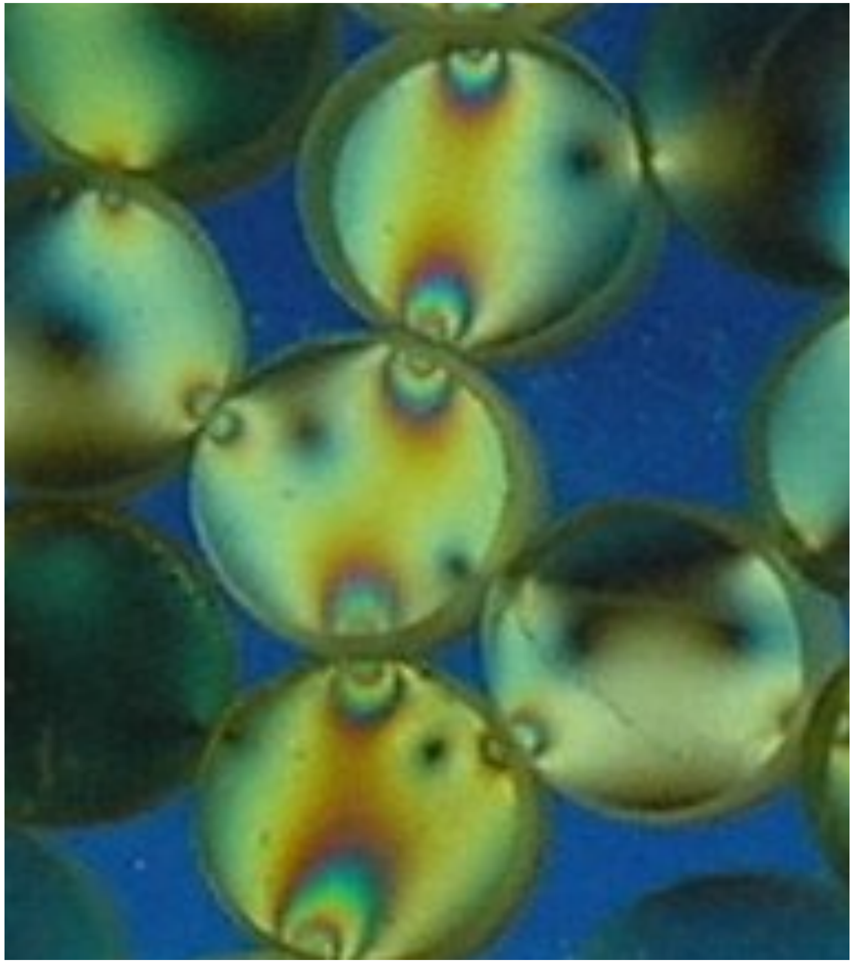
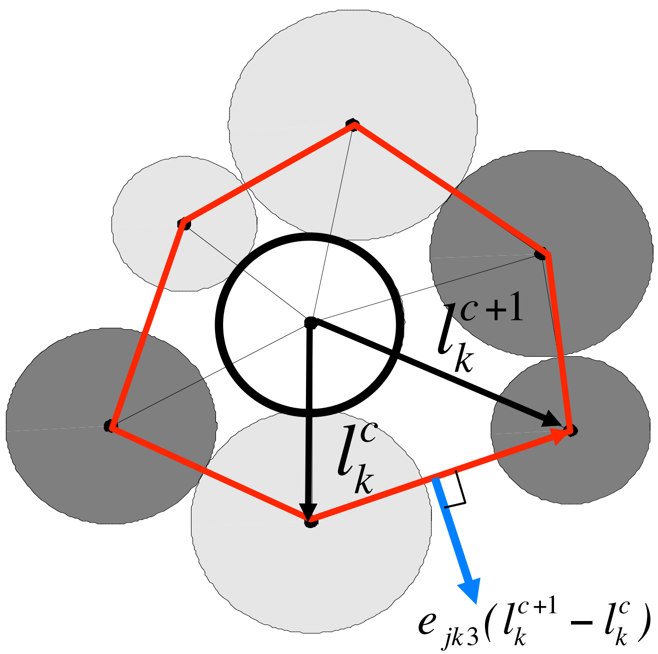


Image from Robert Behringer

❑ Useful for material characterization and modelling: local anisotropy & nonaffine deformation captured.

Deformation from kinematics

- ❑ governed by particle rearrangements
- ❑ rotational - as important as translational - degrees of freedom



Triangulation on final configuration

$$p_i^c = u_i^c - u_i + e_{ij3} l_j^c \omega; \quad \phi^c = \omega^c - \omega$$

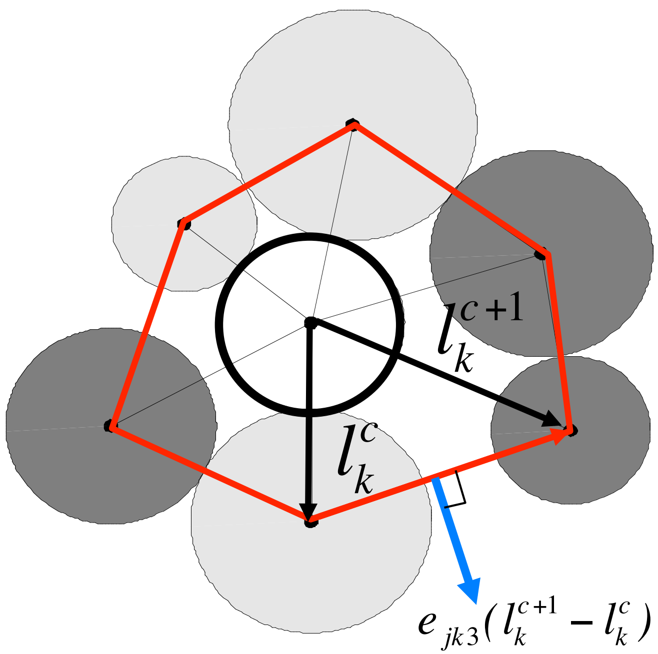
$$\dot{\varepsilon}_{ij} = \frac{1}{V} \sum_{c \in B} \frac{(\dot{p}_i^c + \dot{p}_i^{c+1})}{2} e_{jk3}(l_k^{c+1} - l_k^c)$$

$$\dot{\kappa}_i = \frac{1}{V} \sum_{c \in B} \frac{(\dot{\phi}^c + \dot{\phi}^{c+1})}{2} e_{ij3}(l_j^{c+1} - l_j^c)$$

Tordesillas et al Mathematics and Mechanics of Solids 08

Deformation from kinematics

- ❑ governed by particle rearrangements
- ❑ rotational - as important as translational - degrees of freedom
- ❑ **highly nonaffine (tied to dissipation)**



Triangulation on final configuration

$$\Delta\dot{p}_i = \dot{p}_i^c - \dot{\varepsilon}_{ij} l_j^c; \quad \Delta\dot{\phi}^c = \dot{\phi}^c - \dot{\kappa}_i l_i^c$$

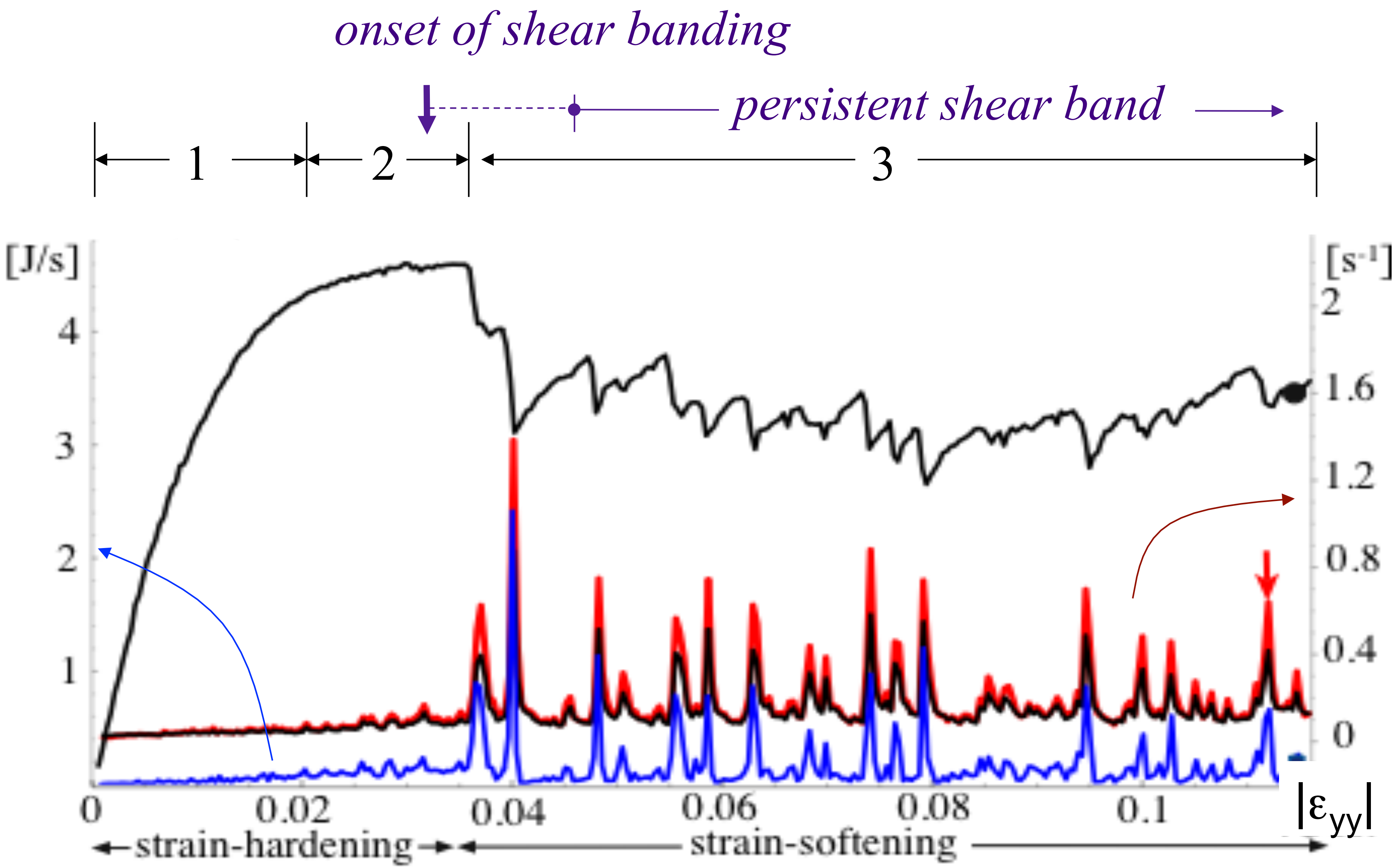
$$\Delta_{ij}^{\varepsilon} = \frac{1}{V}\sum_{c\in B}\frac{(\Delta\dot{p}_i^{c+1} + \Delta\dot{p}_i^c)}{2} e_{jk3}(l_k^{c+1} - l_k^c)$$

$$\Delta_i^{\kappa} = \frac{1}{V}\sum_{c\in B}\frac{(\Delta\dot{\phi}^{c+1} + \Delta\dot{\phi}^c)}{2} e_{ij3}(l_j^{c+1} - l_j^c)$$

Tordesillas et al Maths Mech Solids 08

See also papers of K.C. Valanis

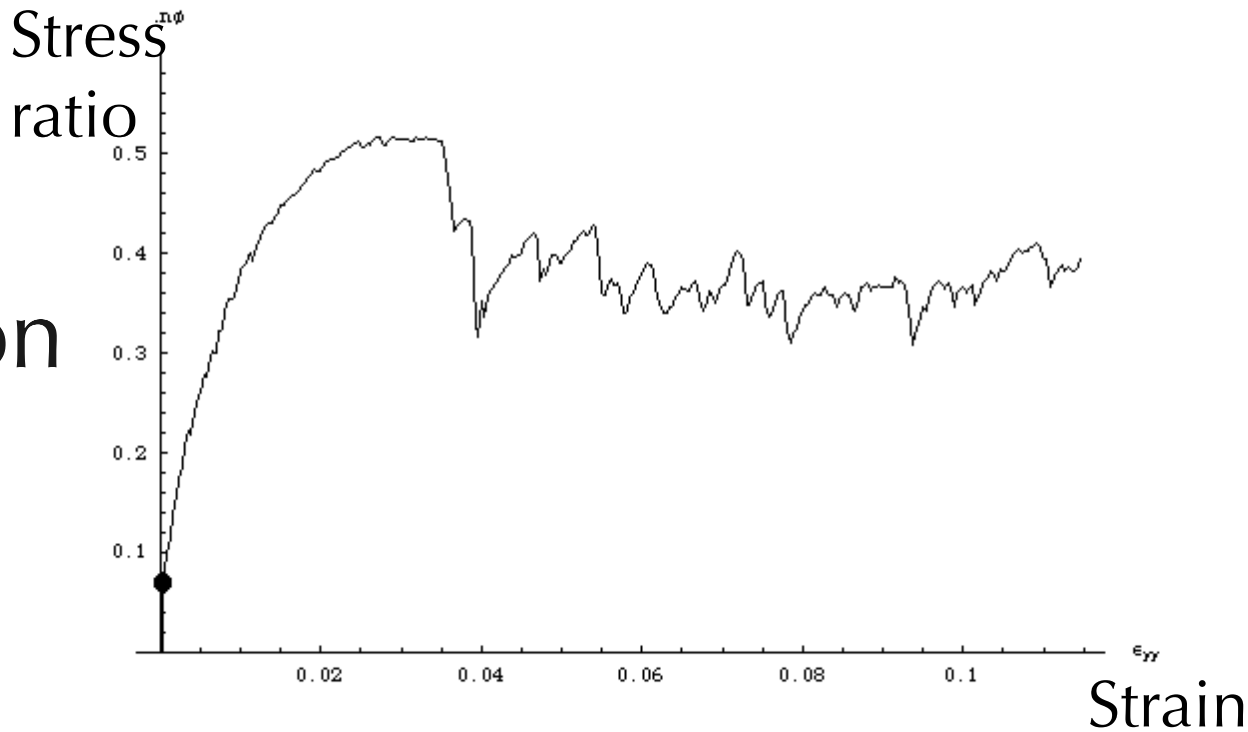
Dissipation, nonaffine strain, nonaffine curvature (& stress ratio)



Tordesillas, Philosophical Magazine 07

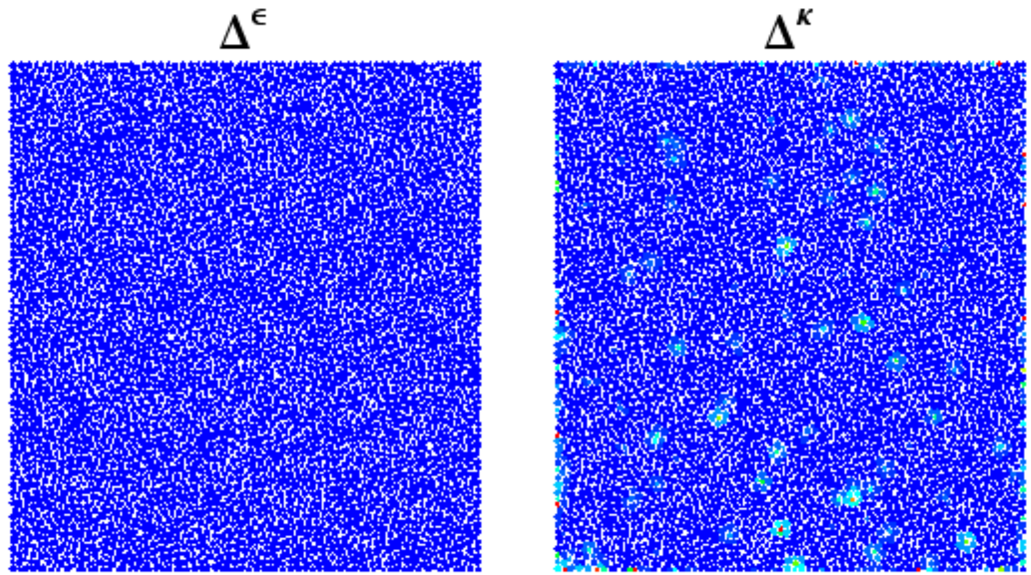
Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Spatial distribution of nonaffine motion



Nonaffine strain/curvature correlate spatially
- with each other
- with dissipation

Patterns captured
- rattlers (stage 1)
- micro(slip) bands (stage 2)
- shear band (stage 3)

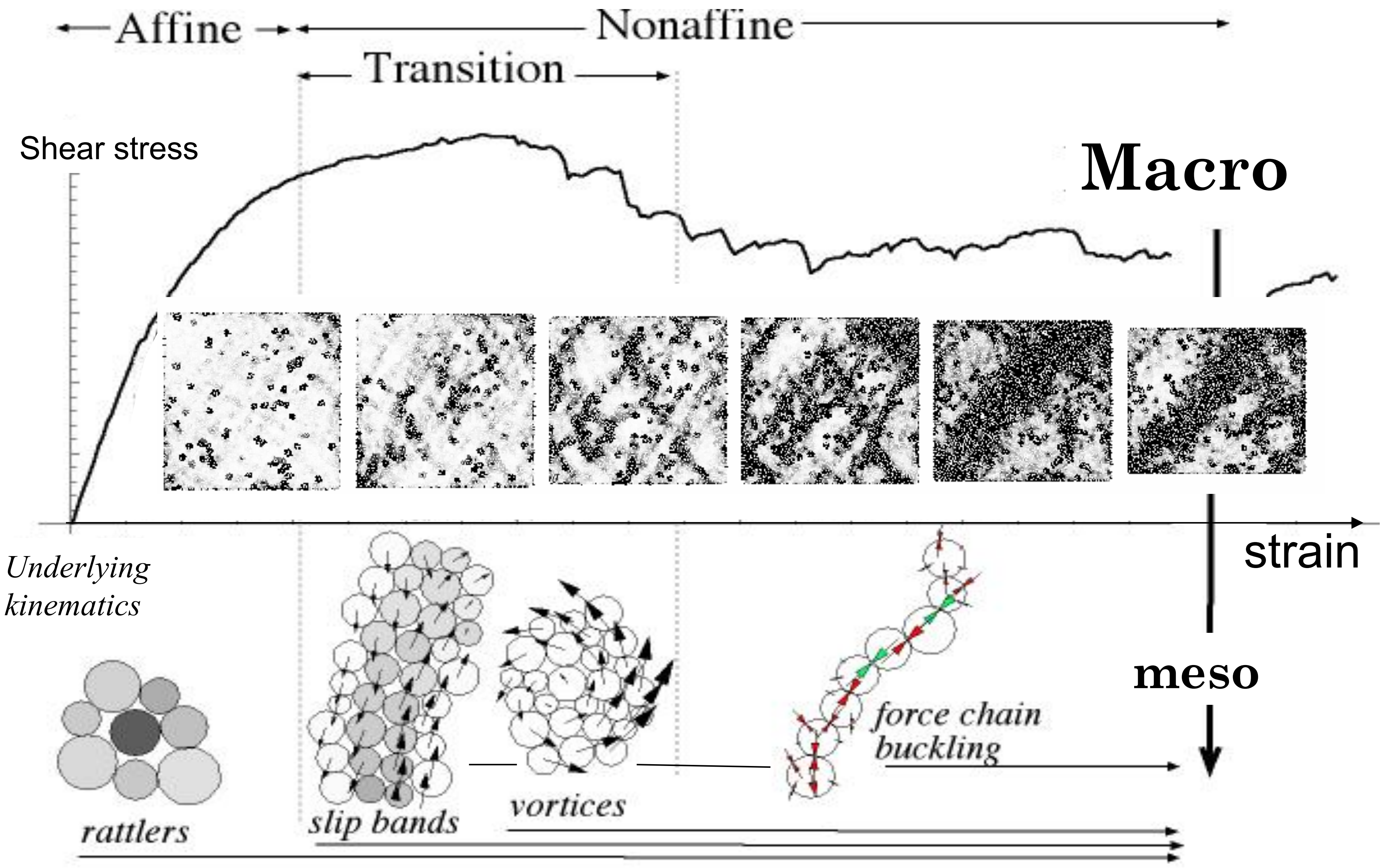


Tordesillas et al

JEM ASCE 08

Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Patterns on the meso-macro scale (DEM)



Plan of the rest of this talk ..

- **What data from DEM/EXP tell us about length scales of observed patterns from contact forces & topology in granular materials**
- Pattern recognition from Complex Systems Theory and what patterns teach us about the nature of complex systems

---

- Extraction of length scales from Grenoble data on Hostun sand
- Results from extraction
- Inception of Hostun sand and the null hypothesis to test length scales are robust, meaningful and **real**
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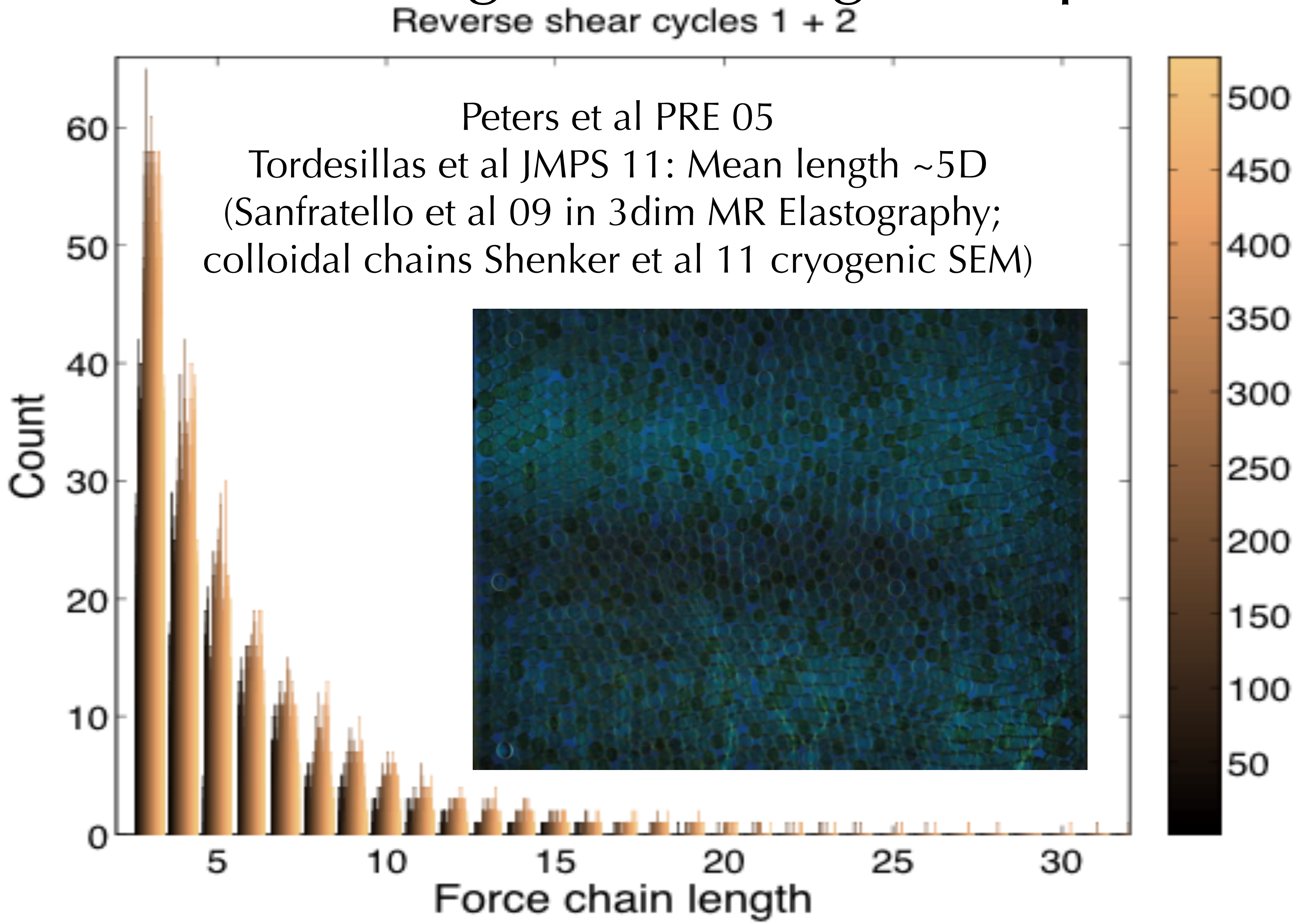
Force chain lengths (Behringer's experiment)

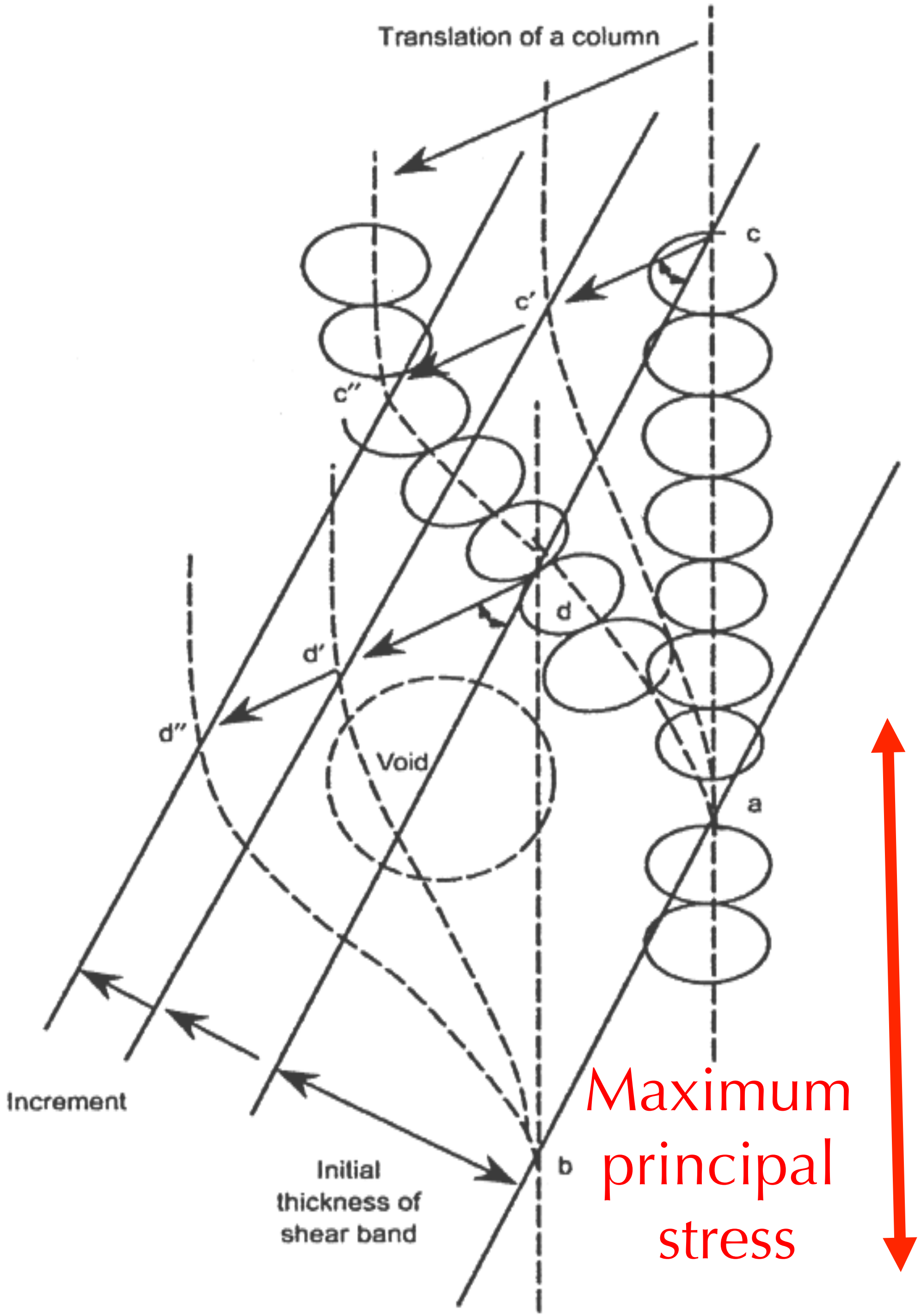


Peters et al PRE 05
Tordesillas et al JMPS 11: Mean length ~5D
(Sanfratello et al 09 in 3dim MR Elastography;
colloidal chains Shenker et al 11 cryogenic SEM)

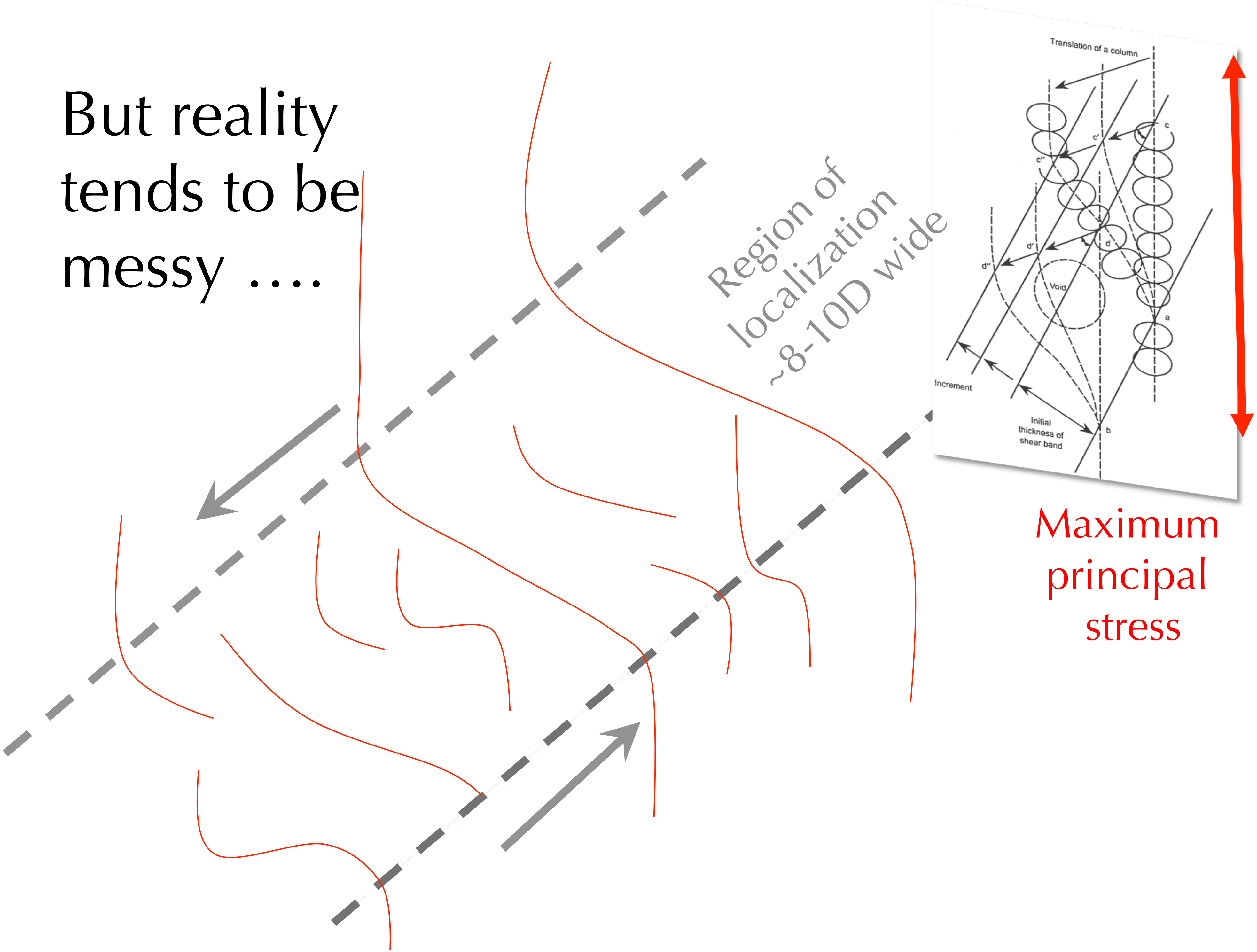
Oda's view of route to shear banding:

Oda & Kazama 98, Geotechnique, **48** (fig 15)

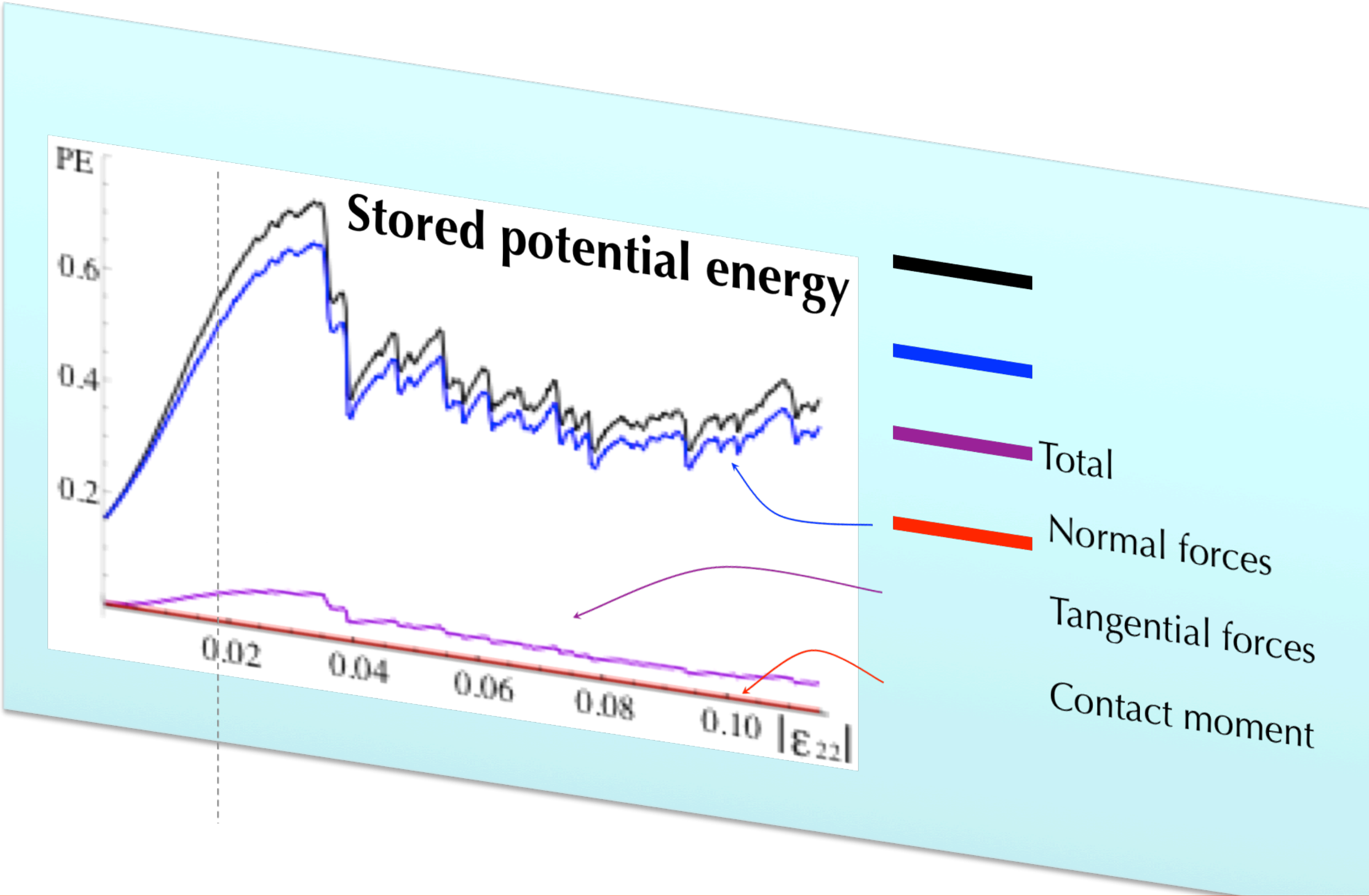
Oda's hypothesis: "… columns extending parallel to the major principal stress direction. The columns start buckling at the peak stress, and the buckling columns tend to concentrate in shear bands during the strain softening process …."



But reality tends to be messy ....



Follow the energy ..



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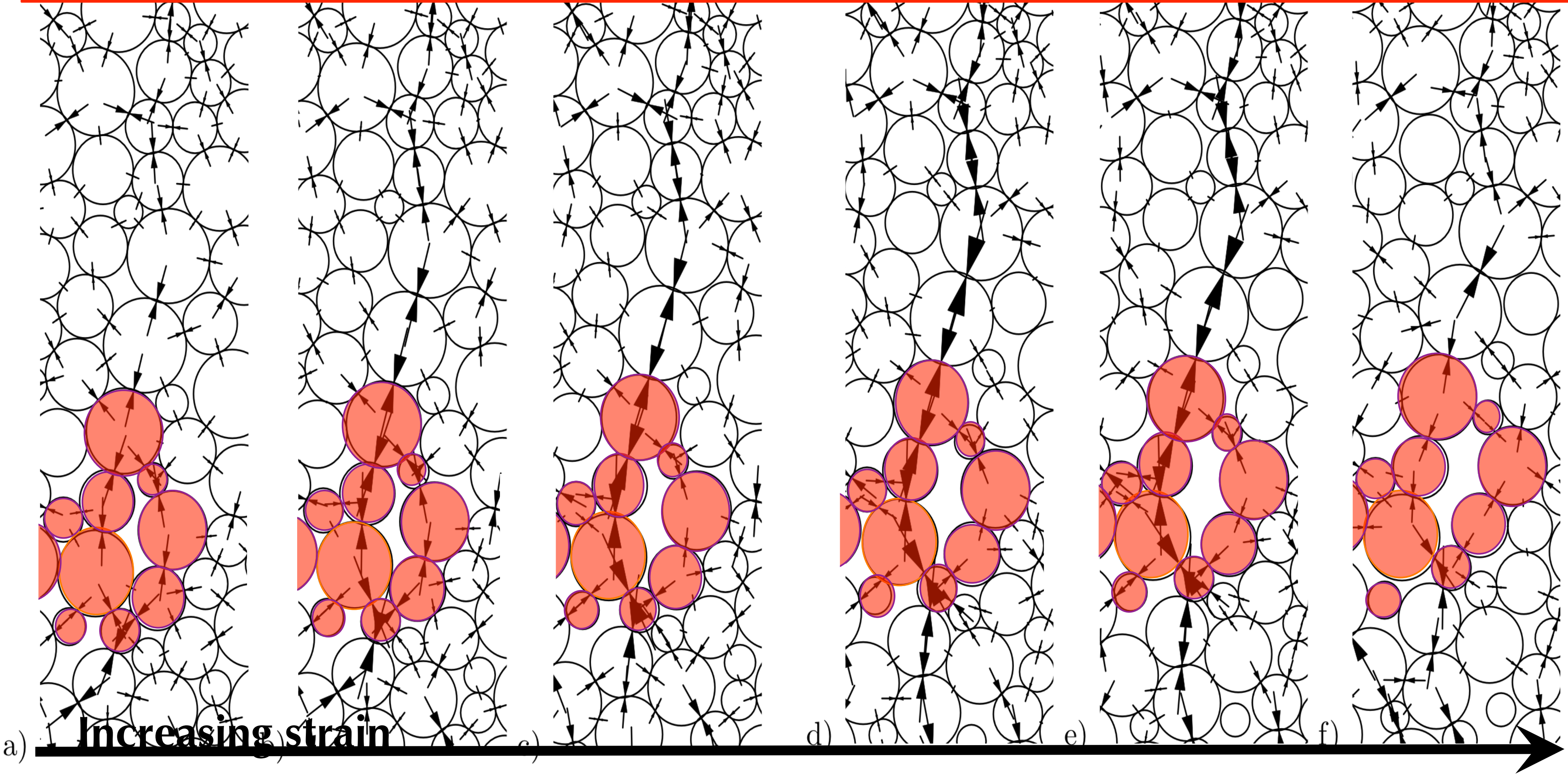


Key mechanism for release of energy stored in force chains?

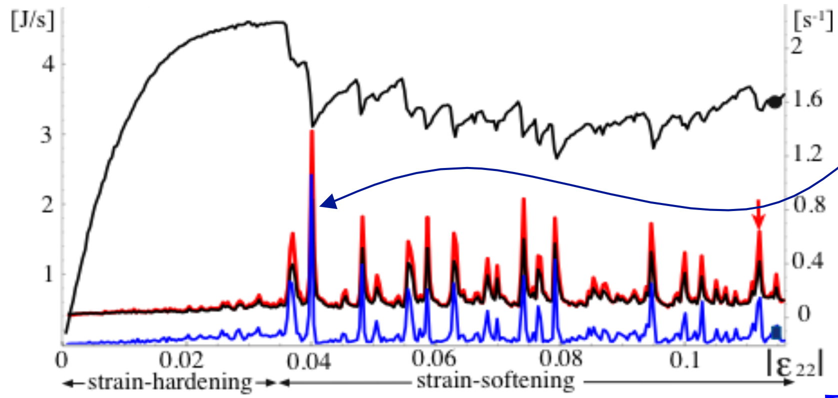
Slip or sliding limits the growth of tangential forces.

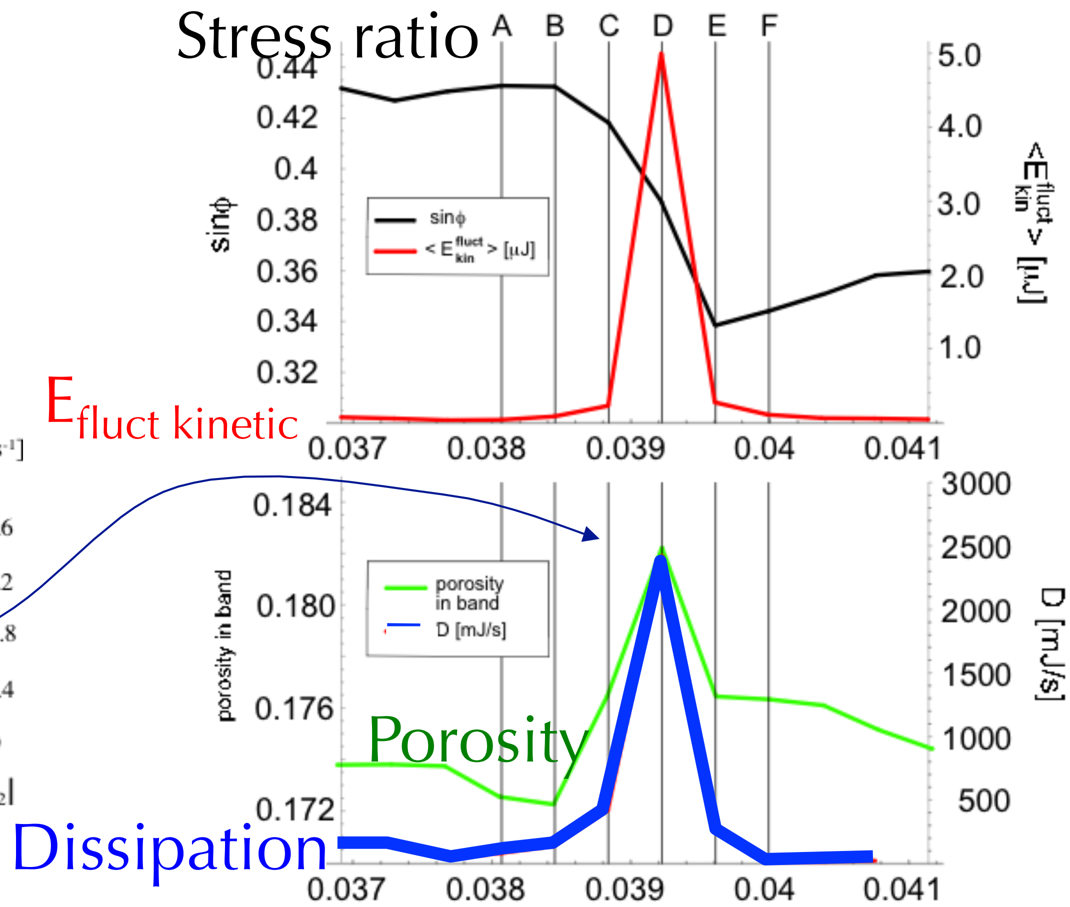
Rolling limits the growth of contact moments.

What limits the growth of normal forces?

Granular Matter 07

Dissipation in a single unjamming event
- 2nd drop in stress ratio





❑ Follow the energy trail ....

Where is the energy stored ?

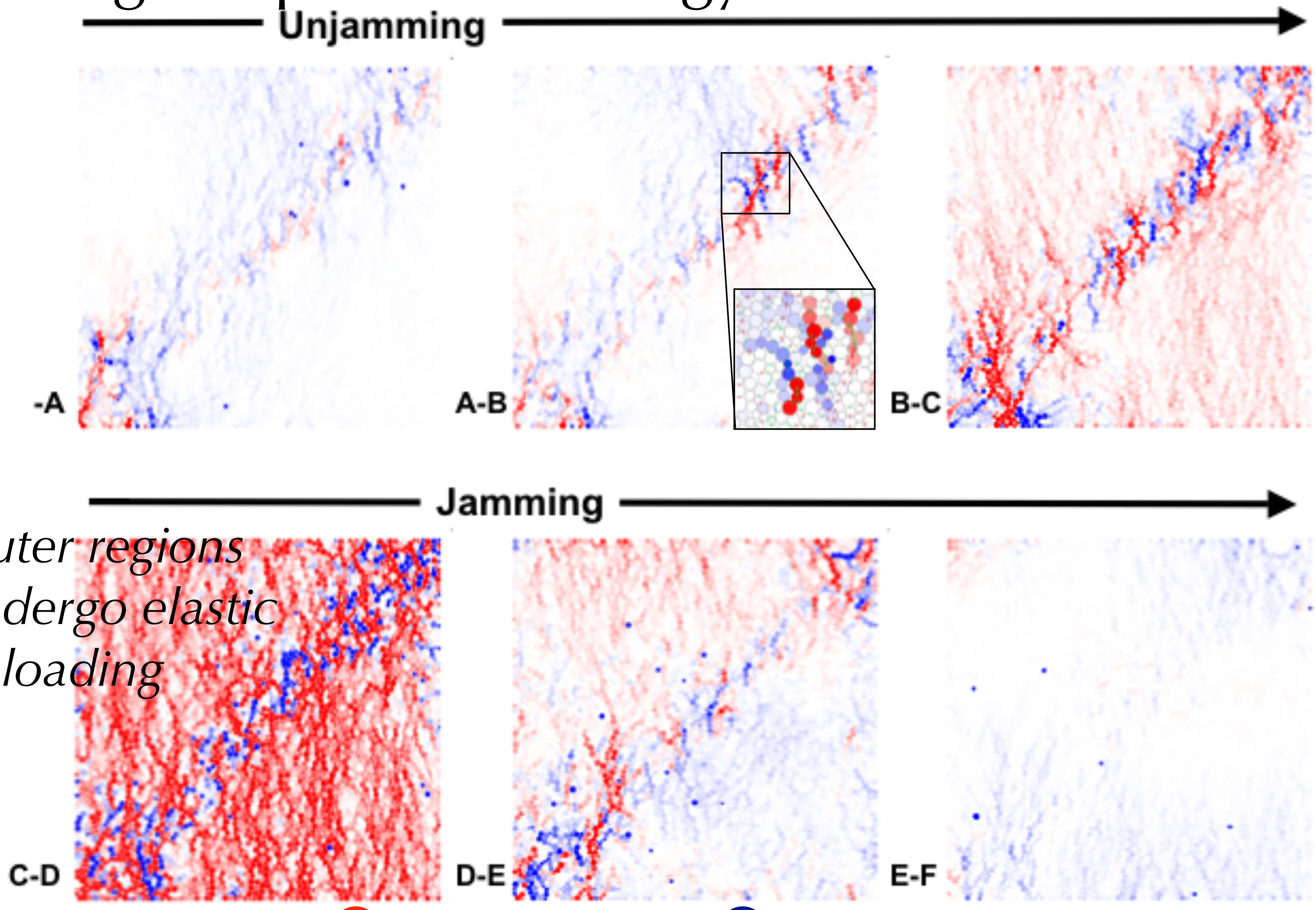
What triggers energy release?

Tordesillas Phil Mag 07

Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Change in potential energy

Phil Mag 07



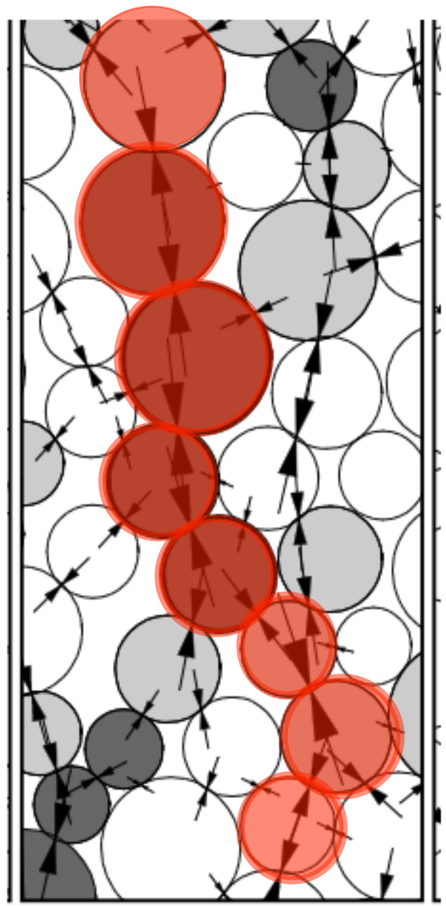
Outer regions undergo elastic unloading

Decrease in Epot Increase in Epot

Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Key mechanism for energy release?

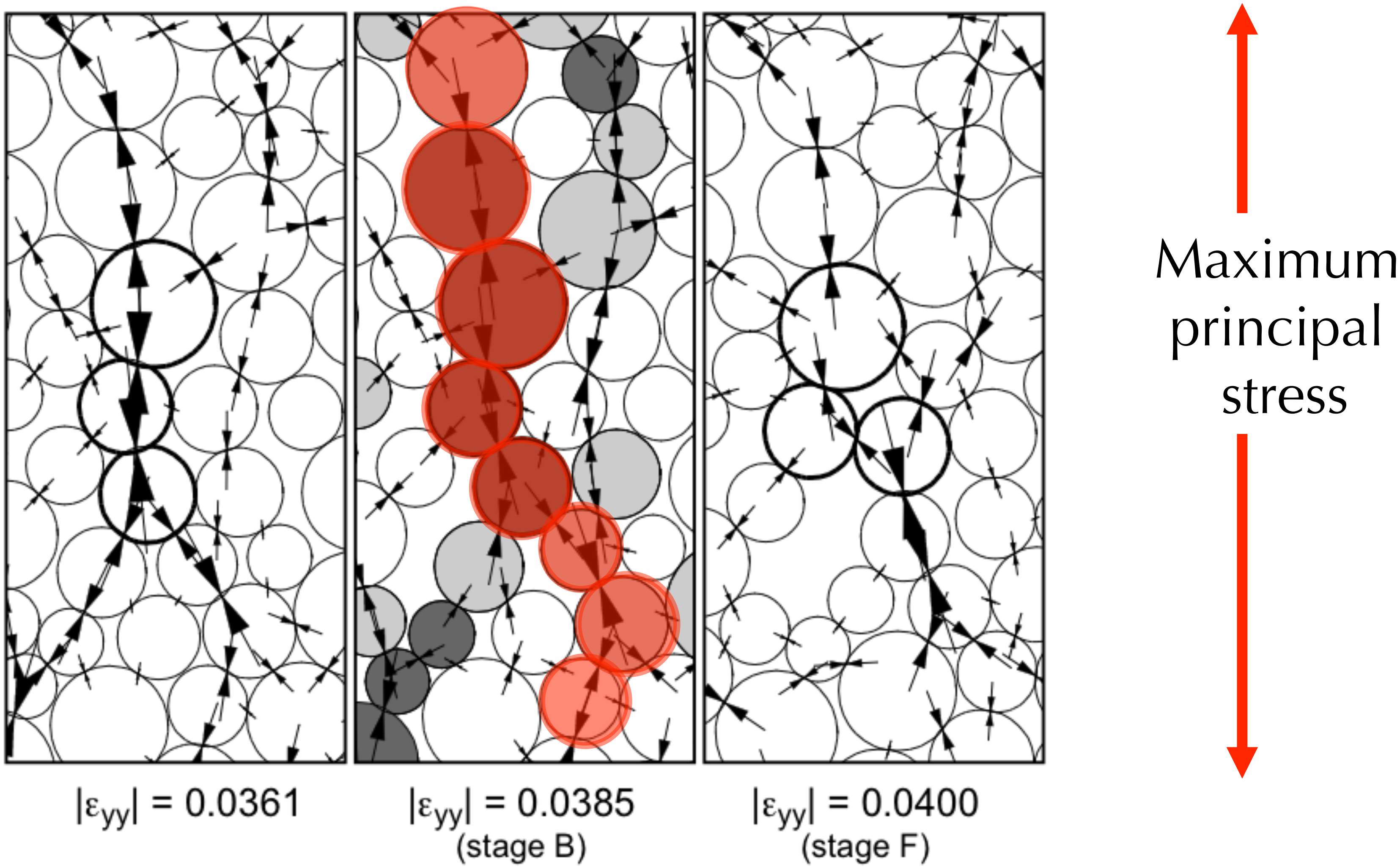
❑ Since force chains are where energy is mainly stored, the prime suspect is the mechanism for force chain failure.



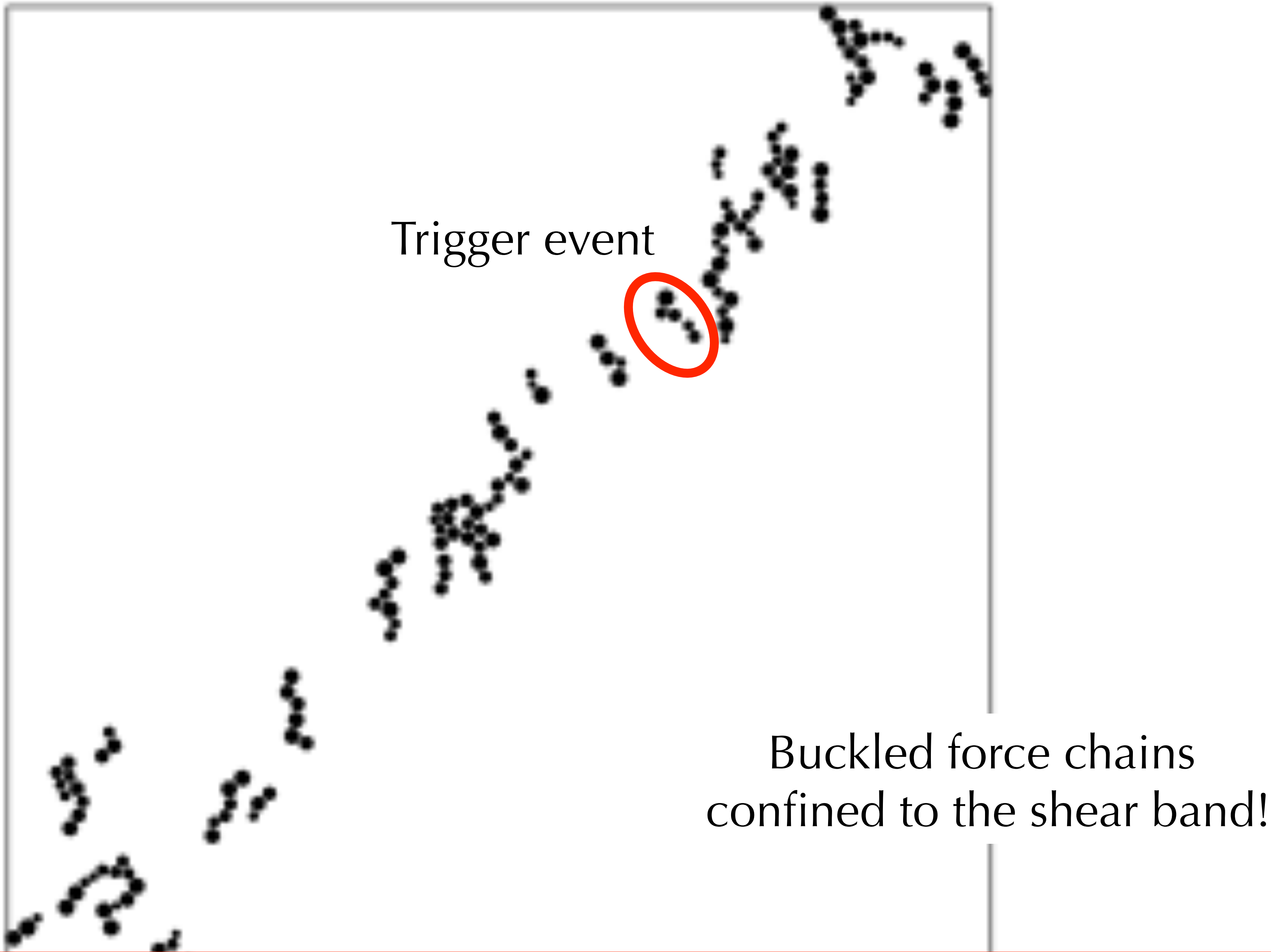
$|\varepsilon_{yy}| = 0.0385$
(stage B)

Maximum principal stress

Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

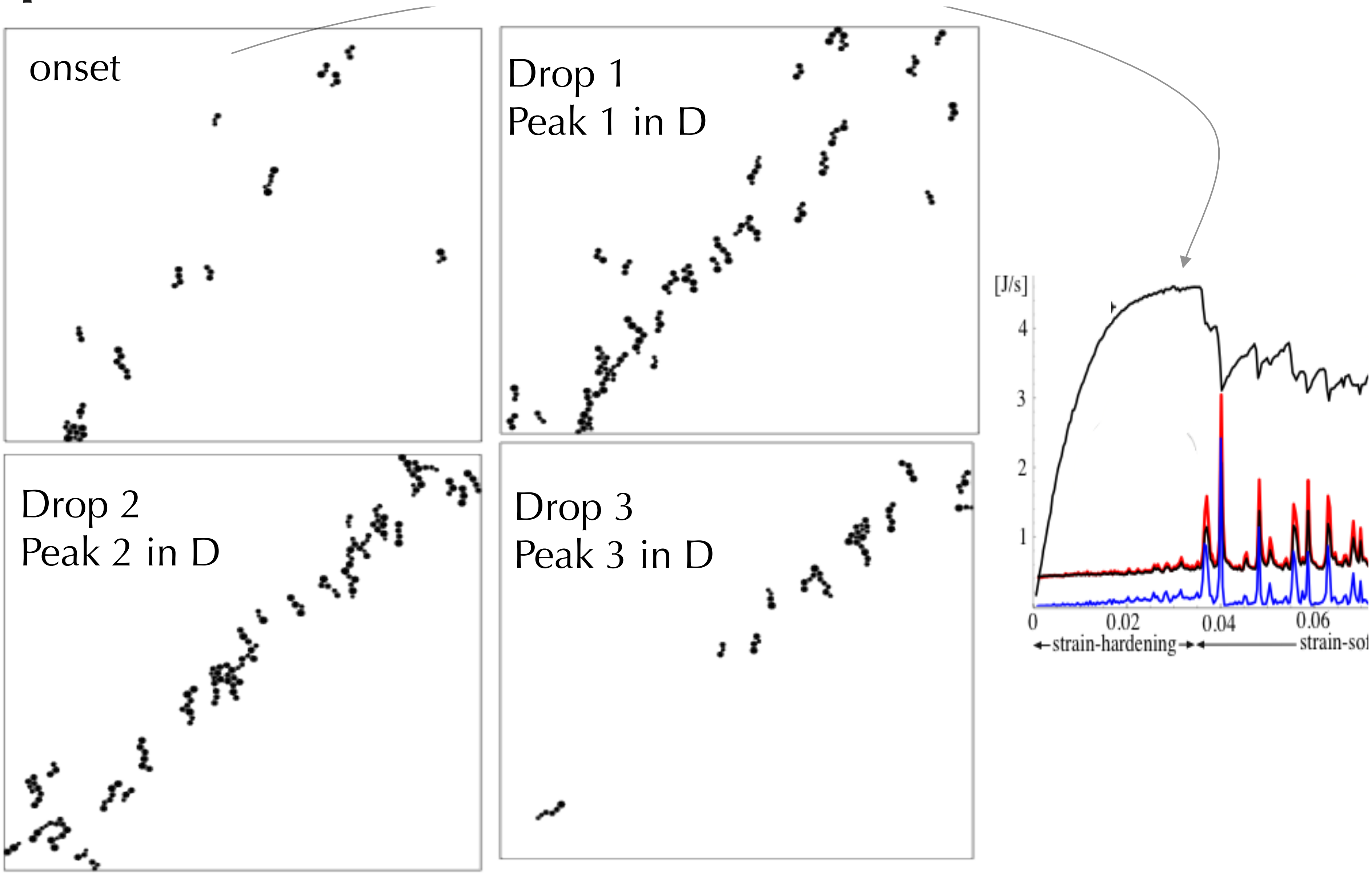


Oda's hypothesis: "…columns extending parallel to the major principal stress direction. The columns start buckling at the peak stress, and the buckling columns tend to concentrate in shear bands during the strain softening process …."



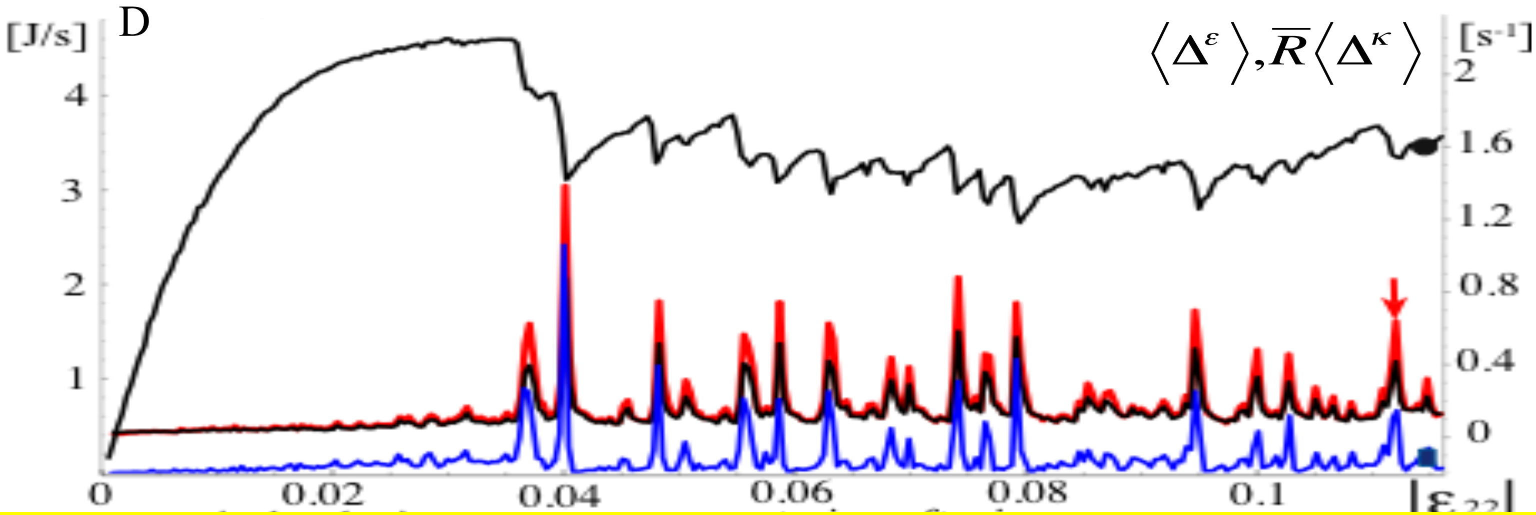
Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Spatial distribution of buckled force chains

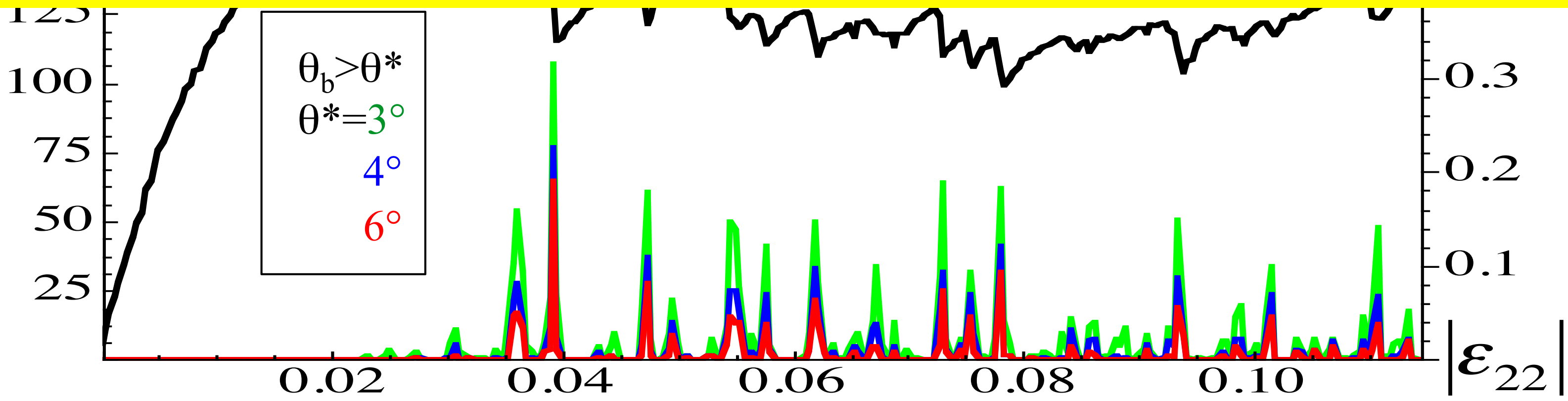


Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles

Dissipation & force chain buckling



But where does the ~8D come from?



Baseline system: 2D DEM, Biaxial test with constant confining pressure, 5098 particles