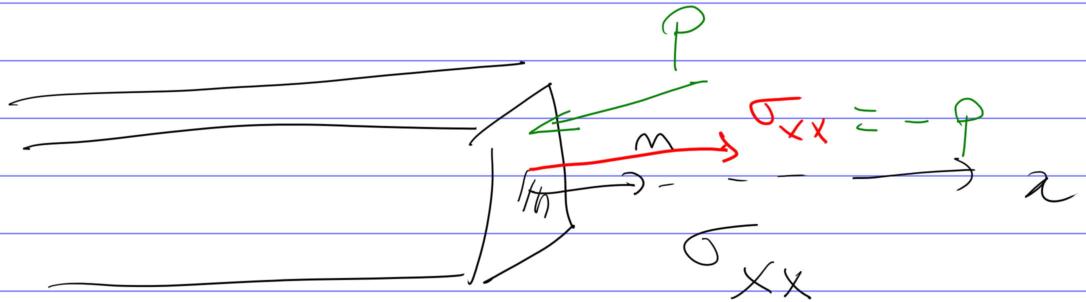
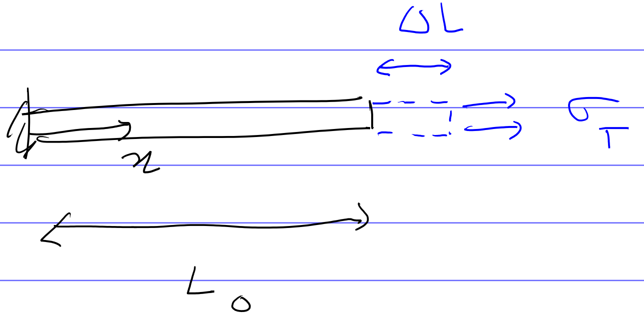

P
σxx = −P
n
x
σxx

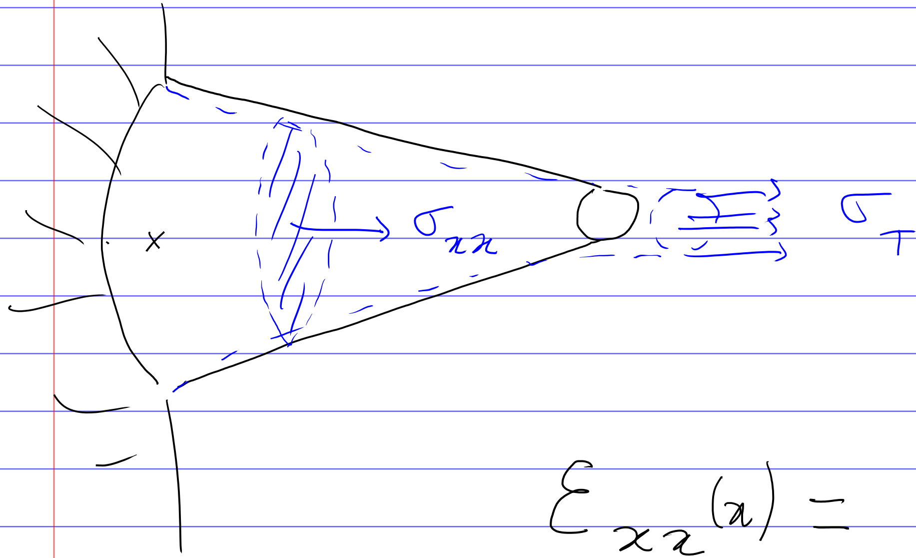
1) Définition de l'allongement

* RDM



$L_0 \rightarrow L_0 + \Delta L$ dans une poutre en traction

$$\varepsilon_{xx} = \frac{\Delta L}{L_0}$$



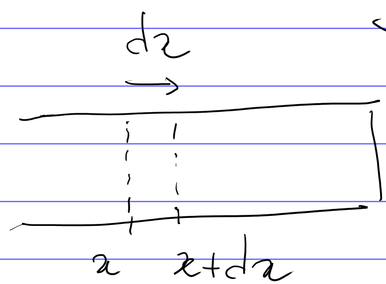
Si section non-constante

$$\sigma_{xx} = \sigma_{xx}(x)$$

$$\varepsilon_{xx}(x) = \frac{\Delta V}{V}$$

$$= \frac{x - x_0}{x}$$

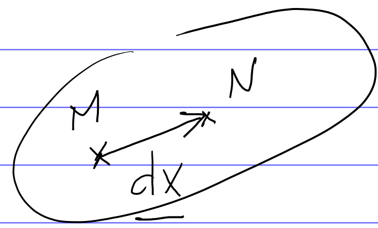
∂u



Soit $u_x = u_x(x)$ le déplacement suivant x

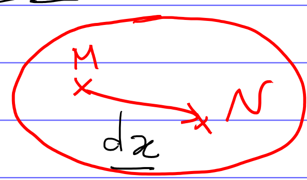
$$= [u(x+dx) - u(x)]/dx = \frac{\partial u_x}{\partial x}$$

$*$ En 3D : gradient du déplacement.



$$\underset{\text{transformation}}{\phi}: \quad X_M \longrightarrow x_M, \qquad X_N \longrightarrow x_N$$

$$\underset{\text{déplacement}}{u} = \phi - Id: \quad X_M \longrightarrow x_M - X_M, \qquad X_N \longrightarrow x_N - X_N$$



$$\phi(X_N) - \phi(X_M) \simeq \underline{\underline{\nabla \phi}} \cdot \underline{dX} \simeq \underline{\underline{\nabla}}(\underbrace{u + Id}_{x}) \cdot \underline{dX}$$

$$\simeq \left(\underline{\underline{Id}} + \underline{\underline{\nabla u}}\right) \cdot \underline{dX}$$

Série de Taylor pour $\Phi : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$: $\underline{\Phi}(\underline{X} + \underline{dX}) = \underline{\Phi}(\underline{X}) + \underset{\text{gradient de } \Phi}{\nabla \underline{\Phi}}\, \underline{dX} + O(\|\underline{dX}\|^2)$

$$\begin{cases} \Phi_1(\underline{X} + \underline{dX}) = \Phi_1(\underline{X}) + \dfrac{\partial \Phi_1}{\partial x_1} dX_1 + \dfrac{\partial \phi_1}{\partial x_2} dX_2 + \dfrac{\partial \phi_1}{\partial x_3} dX_3 + O(\|dX\|^2) \\ \Phi_2(\underline{X} + \underline{dX}) = \cdots + \dfrac{\partial \Phi_2}{\partial x_1} dX_1 + \cdots \\ \Phi_3(\underline{X} + \underline{dX}) = \cdots \end{cases}$$

$$\underline{\Phi}(\underline{X} + \underline{dX}) = \underline{\Phi}(\underline{X}) + \begin{bmatrix} \partial\phi_1/\partial x_1 & \partial\phi_1/\partial x_2 & \partial\phi_1/\partial x_3 \\ \partial\phi_2/\partial x_1 & & \\ & & \dfrac{\partial \phi_{i \leftarrow \text{ligne}}}{\partial x_{j \leftarrow \text{colonne}}} \end{bmatrix} \times \begin{pmatrix} dX_1 \\ dX_2 \\ dX_3 \end{pmatrix} + O(dX^2)$$

Remarque : $\Phi_1(\underline{X} + \underline{dX}) = \Phi_1(\underline{X}) + \underline{\nabla \Phi_1} \cdot \underline{dX} + O(dX^2)$

$$\underline{\nabla \Phi_1} = \begin{pmatrix} \partial\Phi_1/\partial x_1 \\ \partial\Phi_1/\partial x_2 \\ \partial\Phi_1/\partial x_2 \end{pmatrix}$$

* Si translation : $\underline{u} = \underline{u}_0$ en tout point.

$$\underline{\underline{\nabla}}\,\underline{u} = \underline{\underline{0}}$$

* Si translation + rotation : $\underline{u} = \underline{u}_0 + \overbrace{(\underline{\underline{R}} - \underline{\underline{Id}})}\cdot\underline{X}$

$$\underline{\underline{\nabla}}\,\underline{u} = (\underline{\underline{R}} - \underline{\underline{Id}})$$

$\underline{\underline{\sim}}$ matrice antisymmétrique

* Si translation + rotation + déformation :

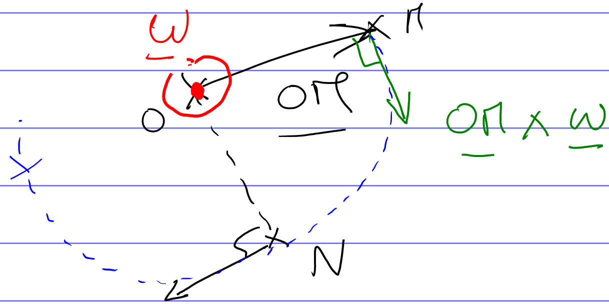
$$\underline{u} = \underline{u}_0 + \underline{\underline{\omega}}\cdot\underline{X} + \underline{\tilde{u}}(\underline{x}) = \underline{u}_0 + \underline{\omega}\wedge\underline{X} + \underline{\tilde{u}}(\underline{x})$$

anti-symmétrique (rotation)

déformation.

Equivalence $\underline{\underline{\omega}}$ / $\underline{\omega}$:

$$\begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \wedge \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



$\underline{\tilde{u}}$ est le déplacement qui déforme :

$$\boxed{\underline{\underline{\varepsilon}} = \underline{\underline{\nabla \tilde{\underline{u}}}} = \underline{\underline{\nabla \underline{u}}} - \text{sym}(\nabla \underline{u}) = \frac{\nabla \underline{u}^T + \nabla \underline{u}}{2}}$$

Tenseur de déformation

∇ du déplacement réel

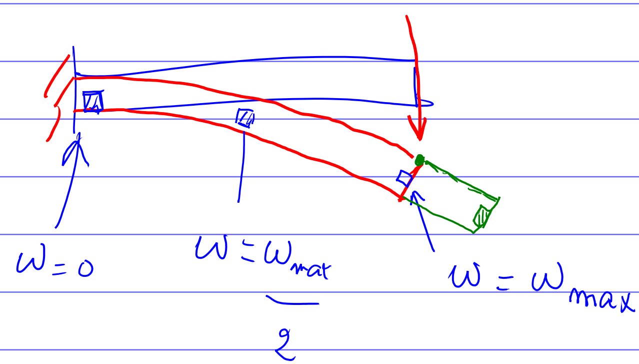
partie anti-symmétrique.

On décomposera classiquement :

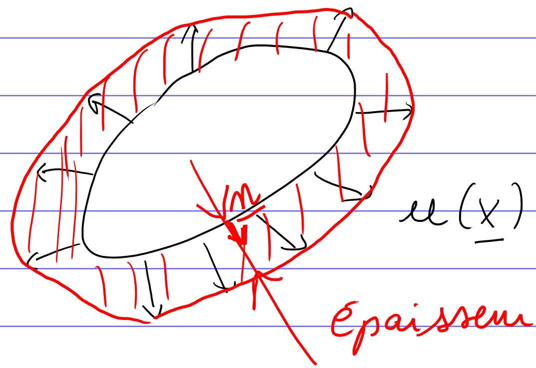
$$\underline{\underline{\nabla \underline{u}}} = \underline{\underline{\omega}} + \underline{\underline{\varepsilon}}$$

petite rotation — tenseur de déformation

⚠ $\underline{\underline{\omega}}$ n'est pas la rotation du solide rigide mais une rotation de la matière que dépends de $\underline{x}$ (potentiellement)



* Variation de volume



$$\Delta V = \int_{\partial\Omega} \underline{u} \cdot \underline{n} \, ds$$

$$= \int_{\Omega} \nabla \cdot \underline{u} \, dv$$

$$= \int_{\Omega} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) dv$$

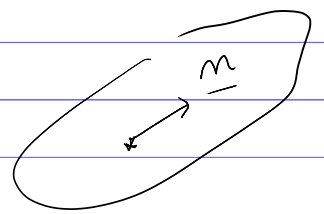
$$= \int_{\Omega} tr(\underline{\underline{\varepsilon}}) \, dv$$

$$\varepsilon_V = \frac{\Delta V}{V} = tr(\underline{\underline{\varepsilon}})$$

invariant de $\underline{\underline{\varepsilon}}$

(ind^{t} du repère)

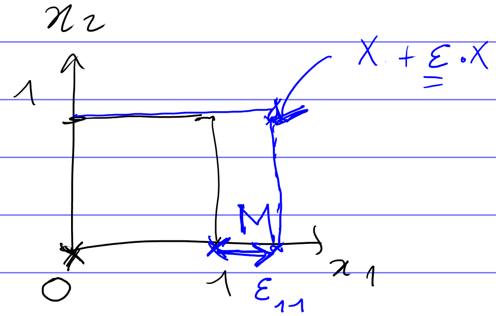
* Variation de longueur.



ε_{nn} : allongement dans la direction $\underline{n}$

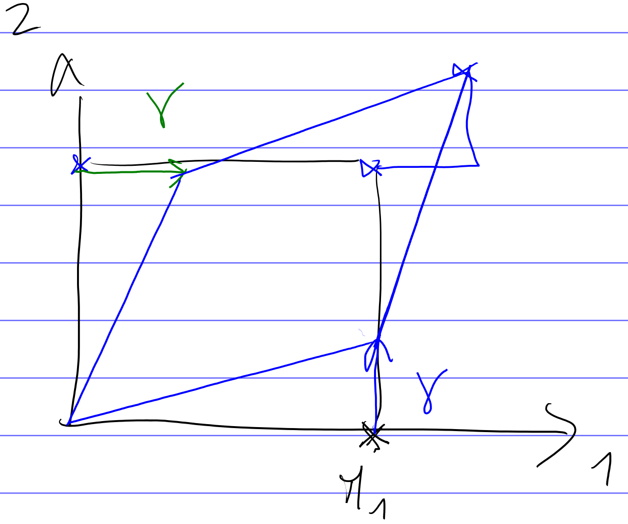
ε_{11} : allongement dans la direction $\underline{e}_1$

Si $\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ alors



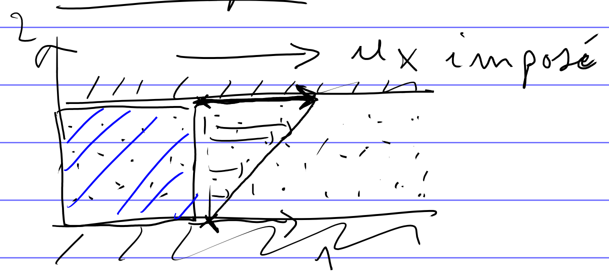
$\underline{\tilde{u}} = \underline{\underline{\varepsilon}} \cdot \underline{X}$ $\qquad$ $\underline{\tilde{u}}(M) - \underline{\tilde{u}}(O) \simeq \underline{\underline{\varepsilon}} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \varepsilon_{11} \\ 0 \\ 0 \end{pmatrix}$

Si $\underline{\underline{\varepsilon}}$ quelconque et $\underline{n}$ quelconque : $\boxed{\varepsilon_{nn} = (\underline{\underline{\varepsilon}} \cdot \underline{n}) \cdot \underline{n} \text{ est l'allongement suivant } \underline{n}}$



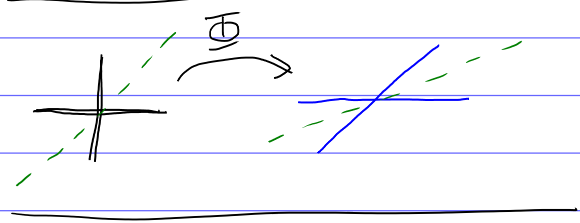
$$\underline{\underline{\varepsilon}} = \begin{array}{ccc} M_1 & & \\ \end{array}\begin{pmatrix} 0 & \gamma & 0 \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{(1,2,3)}$$

Exemple : cisaillement simple. cas (c)

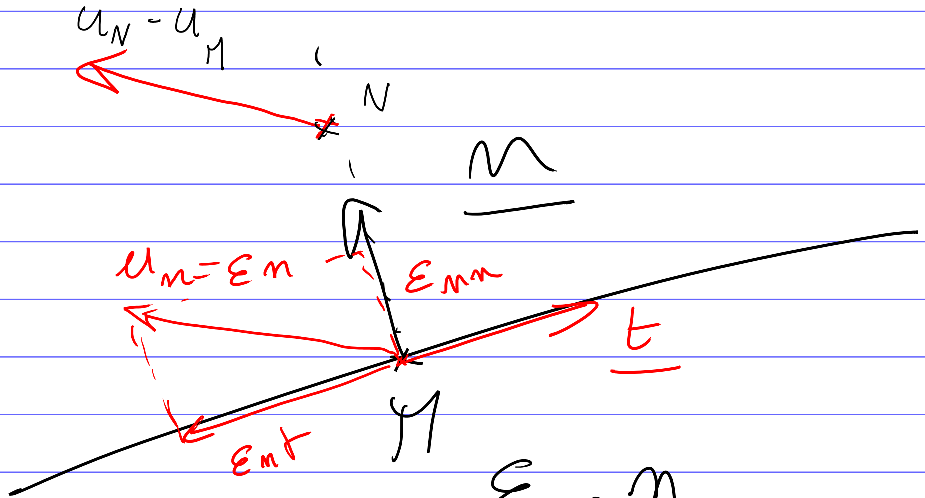


$$u = \begin{pmatrix} x_2 \dfrac{u_x}{h} \\ 0 \\ 0 \end{pmatrix}$$

$$\nabla u = \begin{pmatrix} 0 & \dfrac{u_x}{h} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{u_x}{2h} & 0 \\ -\dfrac{u_x}{2h} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & \dfrac{u_x}{2h} & 0 \\ \dfrac{u_x}{2h} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

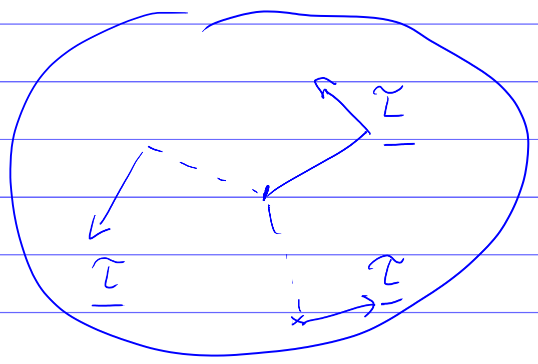


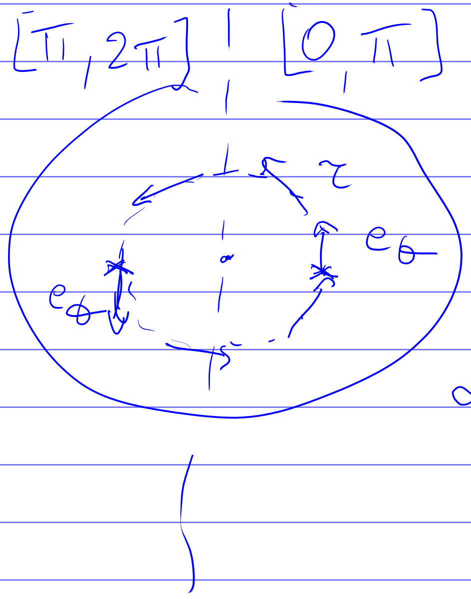
$$= \underbrace{\begin{pmatrix} 0 & \gamma/2 & 0 \\ -\gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\underline{\underline{\omega}}} + \underbrace{\begin{pmatrix} 0 & \gamma/2 & 0 \\ \gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\underline{\underline{\varepsilon}}}$$



$$\underline{u}_n = \underline{\underline{\varepsilon}} \cdot \underline{n}$$

$$\varepsilon_{nn} = \underline{u}_n \cdot \underline{n} = \underline{n} \cdot \underline{\underline{\varepsilon}} \cdot n$$





$$\int_S \alpha r \underline{e}_\theta \, ds$$

$$\alpha \int r e_\theta = \alpha \int_0^R r \int_0^{2\pi} e_\theta \, d\theta \, dr$$

$$= \alpha \int_0^R r \int_0^{\pi} (e_\theta - e_\theta) \, d\theta \, dr$$